International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:13, No:8, 2019

Low-Level Modeling for Optimal Train Routing and Scheduling in Busy Railway Stations

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***Abstract*—This paper studies a train routing and scheduling problem for busy railway stations. Our objective is to allow trains to be routed in dense areas that are reaching saturation. Unlike traditional methods that allocate all resources to setup a route for a train and until the route is freed, our work focuses on the use of resources as trains progress through the railway node. This technique allows a larger number of trains to be routed simultaneously in a railway node and thus reduces their current saturation. To deal with this problem, this study proposes an abstract model and a mixed-integer linear programming formulation to solve it. The applicability of our method is illustrated on a didactic example.**

***Keywords*—Busy railway stations, mixed-integer linear programming, offline railway station management, train platforming, train routing, train scheduling.**

I. INTRODUCTION

NOWADAYS, the railway network in Europe and most areas in the world have a great demand for transport. It is necessary to make the best use of railway resources while satisfying commercial objectives without conflicts between trains and resources. In order to fully explore the capacity of railway infrastructure, searching for optimal platform stops and passing through busy railway stations is important. In most researches, two main problems are investigated: train routing and train scheduling [1].

The train routing problem is to assign each train to a route through the railway stations and to a platform in the station. The number of routings available to each train strongly affects the size of the problem and the time required to optimally solve it.

The train scheduling problem is to determine timing and ordering plans for all trains on the assigned train routes. The number of possible solutions can be very large depending on the network structure, the number and type of trains.

A train routing and scheduling problem in railway stations consists of assigning trains to platforms, so as to satisfy several constraints such as headway, dwell time and platform occupation. The schedule must satisfy some commercial objectives such as desired train arrival and departure times, platform stops, etc.

In case of simple railway structures with few lines, the problem is easy since there are few number of routes for each train. Some works dealing with train routing and scheduling problem focus mainly on low traffic densities within a reasonable computation time. Reference [2] proposes a mixed-integer program to find train routing which is concerned with assigning trains and train times for rail links, stations stop..., so as to avoid train conflicts while minimizing costs and satisfying travel demands. The numerical example in this paper has 10 nodes, 28 links and 10 trains that requires less than one minute to be solved. The strategy of scheduling is to find the route of trains one at a time until all trains are routed and if necessary, the route of trains can be rescheduled until a feasible solution is found. Reference [3], [4] investigate computational complexity of the problem of routing trains through railway station. They consider the reservation of a complete route which guarantees that each train can travel without interruptions along the reserved route. They also include shunting decisions and small deviations for preferred arrival time and departure time of trains. They prove that if each train has at most two routing possibilities, a solution can be computed in polynomial time.

The routing and scheduling problem becomes difficult in busy railway stations, having busy lines and several alternative platforms. Some research focus on busy complex railway stations. In [5], they propose a model for busy complex railway stations. Heuristic methods are developed according to train planners' objectives. The algorithm schedules each train one by one. For each train, they check feasible platforms and for each of these platforms, they check if there are any conflicts with other trains that are already scheduled. If there are conflicts, the arrival time and departure time of train are changed to resolve conflicts. The experiment example has 12 main platforms (with 34 sub-platforms) and 491 trains with 900 arrivals and departures. The computation times can be from a few seconds to several hours depending on the heuristic method and the train planners' objectives. Reference [6] proposes a model dealing with the routing and scheduling problem for busy complex railway stations by applying a hybrid algorithm combining branch-and-bound and heuristic algorithms. In this model, they consider the reservation of a complete path and the deviation of departure time in a similar way to [3], [4]. The experiment example has 250 trains divided in sub-groups, the biggest group has about 60 trains. The computation time is a few minutes with 182 minutes deviation of departure times of 37 trains that contains 3 trains postponed by more than 10 minutes, 8 trains by more than 6 minutes and 29 trains by less than 5 minutes. Reference [7] proposes a track-circuit based model dealing with perturbations. In this paper, all track-circuits belonging to a block must be reserved for trains. Reference [8] proposes a set packing model to deal with the problem of routing trains through railway junctions.

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The route locking and sectional release system is used in this model, a sequence of track sections must be reserved before the arrival of trains.

In view of the above, the reservation of a complete route is popularly used to solve the routing and scheduling problem in railway stations since it can guarantee that trains travel safely without interruptions. In this method, all sections in the route of trains are reserved until the trains release the complete route. One complete route can be reserved by only one train at a time. In principle, the reservation duration of each section of route can be calculated. It depends on the length and speed of train and the length of section. In this paper, we want to assess the interest and performance of a model considering the reservation of each section independently. This implies low-level modeling consideration with respect to the speed and length of train. A section can be reserved when a train arrives and it can be released after the train leaves it, so that the use of available resources can be more efficient. It allows the full exploitation of the capacity of railway infrastructures.

The paper is structured as follows. In the 2nd section, first of all, we specify the objectives of this work. Next, we propose the main concepts for describing the problem. In the 3rd section we propose a mathematical model allowing a resolution by a mixed integer programming approach. Section IV is an application of the proposed model to a case study to illustrate the feasibility of our approach. In the 5th section, we conclude with the lessons of this work and indicate its perspectives.

II. Description of the Problem

A. Research Objectives

The objectives of this paper is to propose a model which can solve two problems:

• Routing problem: the routing of trains passing through the railway station defined by a sequence of sections for a train from its origin to its destination. If trains must stop in platforms, we have to allocate appropriate platforms to these trains.

• Scheduling problem: the timetable of trains in order to avoid collisions between trains.

We chose to take into account separately the reservation of each section on a route of train to improve the capacity of railway infrastructure. That may lead to some interruptions on the route of trains, we must therefore minimize the number of interruptions and the duration of interruptions.

B. Topology and Working Hypotheses

We propose to study a topology based on two types of generic components: "section" and "connector".

A section is a segment of railway infrastructure that can contain only one train at a time.

The set of sections in a railway infrastructure is denoted by $\mathbb{S}$= $\{s_1, s_2, .., s_S\}$ where S is the cardinal number of $\mathbb{S}$.

A connector is a point which connects several sections.

The set of connectors in a railway infrastructure is denoted by $\mathbb{C}$= $\{c_1, c_2, .., c_C\}$ where C is the cardinal number of $\mathbb{C}$. **Relations between sections and connectors**: The topology we consider corresponds to a sequence of sections and connectors, see Fig. 1. Each section is bounded by only two connectors.

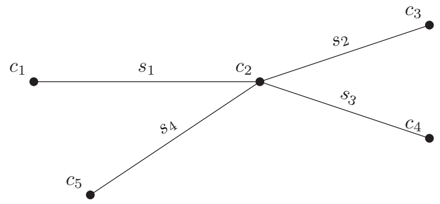

Fig. 1 Relations between sections and connectors

For every $c \in \mathbb{C}$, we denote the set of sections connected with connector c by $\mathbb{S}_c$. In Fig. 1, $\mathbb{S}_{c_2} = \{s_1, s_2, s_3, s_4\}$.

Sections doublet. (s_1, s_2) is a doublet of connector c_1 when s_1, $s_2 \in \mathbb{S}_{c_1}$ and trains can traverse from section s_1 to section s_2 by connector c_1.

The set of doublets of a connector c is denoted by $\mathbb{K}_c = \{(s_1, s_1'), (s_2, s_2'), .., (s_K, s_K')\}$ where K is the cardinal number of $\mathbb{K}_c$.

We must remark that a doublet of connectors represents only one travel direction. For example, a doublet (s_1, s_2) of connector c_1 represents the travel direction from section s_1 to section s_2 by connector c_1. The reverse exists only in case that we have another doublet (s_2, s_1) for connector c_1.

For example, in Fig. 1, if trains can only traverse from sections s_1, s_4 to s_2, s_3, then $\mathbb{K}_{c_2} = \{(s_1, s_3), (s_1, s_2), (s_4, s_2), (s_4, s_3)\}$.

Two kinds of sections can exist: sections with one way and two way directions.

For every $s \in \mathbb{S}$, we denote the set of reachable sections from section s by $\mathbb{S}_s$. In Fig. 1, $\mathbb{S}_{s_1} = \{s_2, s_3\}$.

For every $s \in \mathbb{S}$, we denote the set of reachable sections to section s by $\hat{\mathbb{S}}_s$. In Fig. 1, $\hat{\mathbb{S}}_{s_2} = \{s_1, s_4\}$.

For every $s \in \mathbb{S}$, for every $s' \in \mathbb{S}_s$, it exists only one connector denoted as $c_{ss'}$ between these two reachable sections. In Fig. 1, $c_{s_1 s_3}$ is c_2.

A bordering connector is a connector surrounding the railway infrastructure where trains can enter or leave railway infrastructure.

The set of bordering connectors in a railway infrastructure is denoted by $\mathbb{B}$= $\{b_1, b_2, .., b_B\}$ where B is the cardinal number of $\mathbb{B}$. Thus, $\mathbb{B} \subset \mathbb{C}$.

An external section is a section surrounding the railway infrastructure, represented by a line which connects from a bordering connector to the outside of the infrastructure where trains can enter or leave railway infrastructure. The set of external sections in a railway infrastructure is denoted by $\mathbb{E}$= $\{e_1, e_2, .., e_E\}$ where E is the cardinal number of $\mathbb{E}$. Thus, $\mathbb{E} \subset \mathbb{S}$.

A platform is a section which is used for passengers that can await, board or unboard from trains. Train can usually stop long-time in platforms.

The set of platforms in a railway infrastructure is denoted by $\mathbb{P}$= $\{p_1, p_2, .., p_P\}$ where P is the number of platforms. Thus, $\mathbb{P} \subset \mathbb{S}$ and $\mathbb{P} \cap \mathbb{E} = \varnothing$.

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
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An internal section is a section inside railway infrastructure where trains can pass through. The internal sections are not platforms. The set of internal sections in a railway infrastructure is denoted by $\mathbb{I}$= $\{i_1, i_2, .., i_I\}$ where I is the cardinal number of $\mathbb{I}$. Thus, $\mathbb{I} \subset \mathbb{S}$, $\mathbb{I} \cap \mathbb{E} = \varnothing$, $\mathbb{I} \cap \mathbb{P} = \varnothing$ and $\mathbb{S} = \mathbb{I} \cup \mathbb{E} \cup \mathbb{P}$.

An example of the railway infrastructure is represented in Fig. 2 and the correspondences between sections of this figure are listed in Table I.

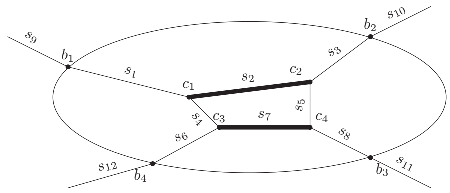

Fig. 2 An example of railway infrastructure

TABLE I
THE CORRESPONDENCES BETWEEN SECTIONS OF FIG. 2

Section	External section	Internal section	Platform
s_1		i_1	
s_2			p_1
s_3		i_2	
s_4		i_3	
s_5		i_4	
s_6		i_5	
s_7			p_2
s_8		i_6	
s_9	e_1		
s_{10}	e_2		
s_{11}	e_3		
s_{12}	e_4		

C. Trains' Activities

Train: The traffic in the railway infrastructure is defined by a set of trains $\mathbb{T}$= $\{t_1, t_2, .., t_T\}$ where T is the number of trains.

Routing of trains. A train passing through the railway station is given two external sections and need to be assigned to a route. The external sections of train t are denoted by e^t_{in}, $e^t_{out} \in \mathbb{E}$. The train enters the railway station from the external section e^{in}_t, arrives at a platform, after that the train departs from the platform and leaves the railway station by the external section e^t_{out}.

Train platform. We must allocate one and only one platform to train t, it is denoted as $p_t \in \mathbb{P}$. A route for the train passing through the railway station must be determined with the condition that the train arrives at and departs from the same platform p_t.

Circulation is an operation of a train which travel from one section to another.

Every train $t \in \mathbb{T}$ consists of a set of ordered circulations $\mathbb{L}^t = \{l^t_1, l^t_2, ..., l^t_{L^t}\}$ where L^t is the cardinal number of $\mathbb{L}^t$.

Three types of circulation are defined:

- **Entering circulation** is a circulation of a train which travels from an external section to a platform, see Fig. 3. The set of entering circulations is denoted by $\mathbb{L}_{ent}$.

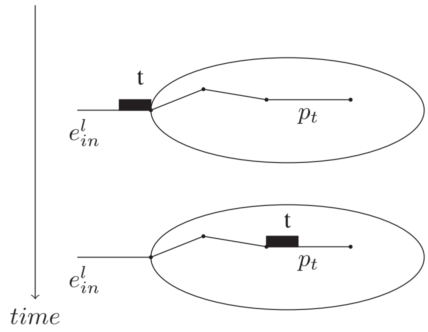

Fig. 3 Entering circulation

- **Leaving circulation** is a circulation of a train which travels from a platform to an external section, see Fig. 4. The set of leaving circulations is denoted by $\mathbb{L}_{leav}$.

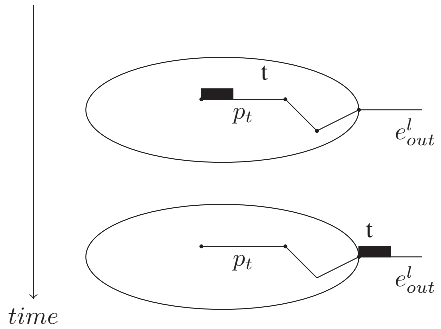

Fig. 4 Leaving circulation

- **Crossing circulation** is a circulation of a train which passes through the railway station from an external section to another external section and does not stop at any platform, see Fig. 5. The set of crossing circulations is denoted by $\mathbb{L}_{pass}$.

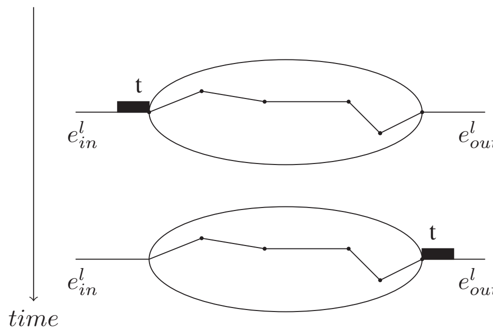

Fig. 5 Crossing circulation of a passing train

Hereafter, we only consider trains consist of two circulations l^t_1, l^t_2, one entering circulation and one leaving circulation. Passing trains will be considered in a future work.

Note: Trains can stop at only one platform but they are allowed to traverse other platforms. Crossing circulations do not stop at any platform but they can traverse platforms to go through the railway infrastructure.

Reference time. An entering circulation $l \in \mathbb{L}_{ent}$ is associated to a reference time A^t. This reference time A^t is the preferred arrival time of occupation of the platform by the train t.

Trains can arrive late to platform within a permissible deviation time. The maximum permissible deviation is denoted by L.

Stopping time. The time taken for trains remaining stopped at a platform to take passengers onboard is denoted by D^t.

Route. The route of a circulation is a sequence of reachable sections from one to another that the train can use for this circulation. One circulation can have many routes and we have to determine which one is the most appropriate.

A route of a circulation l^t of train t denoted by r consists of a set of ordered reachable sections $\mathbb{S}^r$=$\{s_1^r, s_2^r, .., s_{S^r}^r\}$ where S^r is the cardinal number of $\mathbb{S}^r$.

In this study, we consider that every train t consists of two circulations $l_1 \in \mathbb{L}_{ent}$, $l_2 \in \mathbb{L}_{leav}$. Trains can stop at only one platform and they are allowed to traverse other platforms (they do not stop at these platforms).

The external section of the entering circulation $e_{in}^{l_1}$ of train t is the external section given e_{in}^t of this train. The external section of the leaving circulation $e_{out}^{l_2}$ of train t is the external section given e_{out}^t of this train.

III. Mixed-Integer Linear Programming Model

In this section, we propose a mathematical model as a mixed-integer linear program with the parameters and hypotheses we presented in the previous section.

A. Parameters

For every train $t \in \mathbb{T}$, we have some corresponding parameters below:

TABLE II
The Parameters of Trains' Activities

Circulation	External section	Reference time
$l_1 \in \mathbb{L}_{ent}$	e_{in}^l	A^t, D^t, L
$l_2 \in \mathbb{L}_{leav}$	e_{out}^l	

The constant time taken to traverse section s by circulation l is denoted by Δ_s^l. These constants depend on the length of sections and the speed of trains, they can be given as:

$$\Delta_s^l = \frac{\text{length of section s}}{\text{speed of circulation l}}$$

The time taken for a circulation l going through a connector is denoted by Θ^l. These constants depend on the length and the speed of trains, they can be given as:

$$\Theta^l = \frac{\text{length of train}}{\text{speed of circulation l}}$$

Note: We assume that the speed of train does not change during a circulation.

H is a sufficiently large constant.

B. Decision Variables

The function $\delta(Q)$ is an indicator such that $\delta(Q) = 1$ if the condition Q is valid, otherwise 0.

- S_s^l: boolean variable, represents the passage of circulation l going through section s. $S_s^l = \delta$(circulation l passes through section s).
- C_c^l: boolean variable, represents the passage of circulation l going through connector c. $C_c^l = \delta$(circulation l passes through connector c).
- $Y_s^{ll'}$**:** boolean variable, represents the chronological order of two circulations l, l' using routes containing a common section s. $Y_s^{ll'} = \delta$(circulation l passes through section s before circulation l').
- $X_c^{ll'}$**:** boolean variable, represents the chronological order of two circulations l, l' using routes containing a common connector c. $X_c^{ll'} = \delta$(circulation l passes through connector c before circulation l').
- $Z_{ss'}^l$: boolean variable, represents the passage from section s to section s' in the route of circulation l. $Z_{ss'}^l = \delta$(circulation l travels from section s to section s').

The time interval occupation of sections and connectors are represented in Fig. 6:

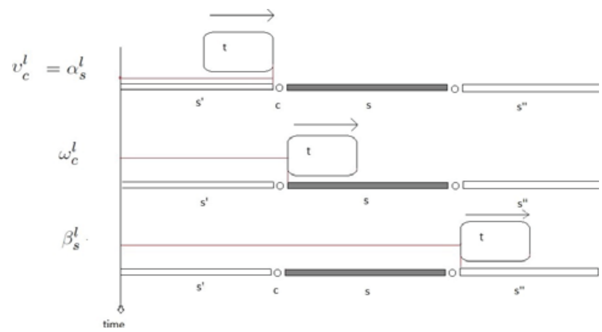

Fig. 6 Occupation time variables

Note: A section is reserved when trains arrive at the connector connected with this section and the section is released when trains leave the other connector connected with this section.

- $[\alpha_s^l,\beta_s^l]$: integer variables, the actual time interval of occupation of section s by circulation l.
- $[v_c^l, \omega_c^l]$: integer variables, the actual time interval of occupation of connector c by circulation l.
- W_s^l: integer variables, the time taken for circulation l remaining stopped at section s.
- P_p^l: boolean variables, represents the stopping platform of circulation l. $P_p^l = \delta$(platform p is allocated to circulation l as a stopping platform).

C. Constraints

Routing constraints. This section presents constraints which ensure that circulations can travel from their origin to their destination.

- If the doublet (s, s') does not exist, it means that section s and section s' are not reachable. Thus, $Z_{ss'}^l$ is equal to 0:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \notin \mathbb{S}_s \qquad Z_{ss'}^l = 0 \qquad (1)$$

• If a circulation passes from section s to s', it cannot pass from section s' to s:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \qquad Z^l_{ss'} + Z^l_{s's} \leq 1 \quad (2)$$

Route of circulation:

• If a circulation enters a section, this circulation must pass through this section:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \qquad \sum_{s' \in \hat{\mathbb{S}}_s} Z^l_{s's} = 1 \Rightarrow S^l_s = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \qquad S^l_s \geq \sum_{s' \in \hat{\mathbb{S}}_s} Z^l_{s's} \quad (3)$$

• If a circulation leaves a section, this circulation must pass through this section:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \qquad \sum_{s' \in \mathbb{S}_s} Z^l_{ss'} = 1 \Rightarrow S^l_s = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \qquad S^l_s \geq \sum_{s' \in \mathbb{S}_s} Z^l_{ss'} \quad (4)$$

Note: The inequality in constraints (3) and (4) represents the case of external sections and platforms. For example, a circulation can pass through an external section but it cannot enter this external section in case that this external section is the first section in the route of this circulation.

• If a circulation travels from section s to section s', it must use the connector $c_{ss'}$ between these two sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \qquad Z^l_{ss'} = 1 \Rightarrow C^l_{c_{ss'}} = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \qquad Z^l_{ss'} \leq C^l_{c_{ss'}} \quad (5)$$

Constraints of external sections:

• Entering circulation l must pass through and leave the external section given e^l_{in}:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent} \qquad S^l_{e^l_{in}} = 1 \quad (6)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent} \qquad \sum_{s' \in \mathbb{S}_{e^l_{in}}} Z^l_{e^l_{in}s'} = 1 \quad (7)$$

• This entering circulation l must not pass through others external sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall s \in \mathbb{E} \backslash \{e^l_{in}\} \qquad S^l_s = 0 \quad (8)$$

• Leaving circulation l must enter and pass through the external section given e^l_{out}:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav} \qquad S^l_{e^l_{out}} = 1 \quad (9)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav} \qquad \sum_{s' \in \hat{\mathbb{S}}_{e^l_{out}}} Z^l_{s'e^l_{out}} = 1 \quad (10)$$

• This leaving circulation l must not pass through others external sections:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav}, \forall s \in \mathbb{E} \backslash \{e^l_{out}\} \qquad S^l_s = 0 \quad (11)$$

Constraints of internal sections: If a circulation enters an internal section, it must leave this internal section. Conversely, if this circulation leaves this internal section, it must enter this internal section.

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \qquad \sum_{s' \in \hat{\mathbb{S}}_s} Z^l_{s's} = \sum_{s'' \in \mathbb{S}_s} Z^l_{ss''} \quad (12)$$

Constraints of non-stopping platforms: We consider that trains can pass through some platforms but might not stop at these platforms. If a circulation enters an non-stopping platform, it must leave this platform. Conversely, if this circulation leaves this platform, it must enter this platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad P^l_p = 0 \Rightarrow \sum_{s' \in \mathbb{S}_p} Z^l_{ps'} = \sum_{s'' \in \hat{\mathbb{S}}_p} Z^l_{s''p}$$

These constraints are expressed using the linear constraints below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P}$$
$$\begin{cases} \sum_{s' \in \mathbb{S}_p} Z^l_{ps'} - \sum_{s'' \in \hat{\mathbb{S}}_p} Z^l_{s''p} \leq H \cdot P^l_p \\ \sum_{s'' \in \hat{\mathbb{S}}_p} Z^l_{s''p} - \sum_{s' \in \mathbb{S}_p} Z^l_{ps'} \leq H \cdot P^l_p \end{cases} \quad (13)$$

Note: If $P^l_p = 0$, the inequation (13) implies that $\sum_{s' \in \mathbb{S}_p} Z^l_{ps'} - \sum_{s'' \in \hat{\mathbb{S}}_p} Z^l_{s''p} \leq 0$ and $\sum_{s'' \in \hat{\mathbb{S}}_p} Z^l_{s''p} - \sum_{s' \in \mathbb{S}_p} Z^l_{ps'} \leq 0$, it means that $\sum_{s' \in \mathbb{S}_p} Z^l_{ps'} = \sum_{s'' \in \hat{\mathbb{S}}_p} Z^l_{s''p}$. If $P^l_p = 1$, the inequation is always true because H is a big constant.

Constraints of stopping platforms:

• There is only one stopping platform for a circulation:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t \qquad \sum_{p \in \mathbb{P}} P^l_p = 1 \quad (14)$$

• An entering circulation and a leaving circulation of a same train must have the same platform:

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \qquad P^l_p = P^{l'}_p \quad (15)$$

• An entering circulation must enter a stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P} \qquad P^l_p = 1 \Rightarrow \sum_{s \in \hat{\mathbb{S}}_p} Z^l_{sp} = 1$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P} \qquad \sum_{s \in \hat{\mathbb{S}}_p} Z^l_{sp} \geq P^l_p \quad (16)$$

• An entering circulation must not leave the stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P} \qquad P^l_p = 1 \Rightarrow \sum_{s \in \mathbb{S}_p} Z^l_{ps} = 0$$

The constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P} \qquad \sum_{s \in \mathbb{S}_p} Z^l_{ps} \leq (1 - P^l_p) \quad (17)$$

• A leaving circulation must leave a stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav}, \forall p \in \mathbb{P} \qquad \sum_{s \in \mathbb{S}_p} Z^l_{ps} \geq P^l_p \tag{18}$$

• A leaving circulation must not enter the stopping platform:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav}, \forall p \in \mathbb{P} \qquad \sum_{s \in \hat{\mathbb{S}}_p} Z^l_{sp} \leq (1 - P^l_p) \tag{19}$$

Constraints of relations between sections and connectors: If a circulation l passes through a connector c, there must be two sections, connected to this connector, which are in the route of circulation l:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \qquad C^l_c = 1 \Rightarrow \sum_{s \in \mathbb{S}_c} S^l_s = 2$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \qquad \begin{cases} \sum_{s \in \mathbb{S}_c} S^l_s - 2 \leq H \cdot (1 - C^l_c) \\ 2 - \sum_{s \in \mathbb{S}_c} S^l_s \leq H \cdot (1 - C^l_c) \end{cases} \tag{20}$$

Note: We remind that $\mathbb{S}_c$ is a set of sections connected with connector c. We consider that circulations are not allowed to pass through a connector many time in this model. Circulations can pass a connector only one time.

Actual time interval of occupation of sections and connectors.

The actual time interval of occupation of a section $s \in \mathbb{S}$ by a circulation l is defined by $[\alpha^l_s, \beta^l_s]$ and the time taken for circulation l remaining stopped at section s is defined by variables W^l_s.

The actual time interval of occupation of a connector $c \in \mathbb{C}$ by a circulation l is defined by $[\upsilon^l_c, \omega^l_c]$.

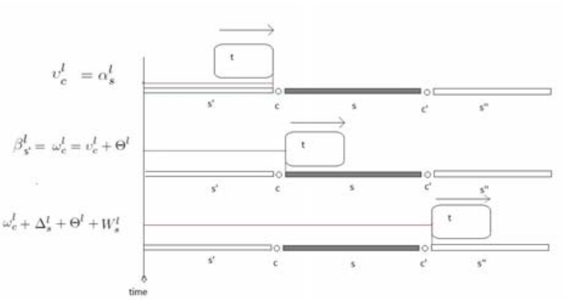

Fig. 7 The actual time intervals of occupation of sections and connectors

• The actual time intervals of occupations of sections and connectors are represented in Fig. 7. The constraints of all connectors are expressed as follows:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \qquad \omega^l_c = \upsilon^l_c + \Theta^l \tag{21}$$

• The constraints of all sections which are not the stopping platform are expressed as follows:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \beta^l_s = \alpha^l_s + \Delta^l_s + 2\Theta^l + W^l_s \tag{22}$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{E} \quad \beta^l_s = \alpha^l_s + \Delta^l_s + \Theta^l + W^l_s \tag{23}$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P}$$
$$P^l_p = 0 \Rightarrow \beta^l_p = \alpha^l_p + \Delta^l_p + 2\Theta^l + W^l_p$$

This constraint is expressed using the linear constraints below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P}$$
$$\begin{cases} \alpha^l_p + \Delta^l_p + 2\Theta^l + W^l_p - \beta^l_p \leq H \cdot P^l_p \\ \beta^l_p - \alpha^l_p - \Delta^l_p - 2\Theta^l - W^l_p \leq H \cdot P^l_p \end{cases} \tag{24}$$

Note: If $P^l_p = 0$, the inequation (24) implies that $\alpha^l_p + \Delta^l_p + 2\Theta^l + W^l_p - \beta^l_p \leq 0$ and $\beta^l_p - \alpha^l_p - \Delta^l_p - 2\Theta^l - W^l_p \leq 0$, it means that $\beta^l_p = \alpha^l_p + \Delta^l_p + 2\Theta^l + W^l_p$. If $P^l_p = 1$, the inequation is always true.

There is only one connector connected with external sections. Thus, the constraint (23) applies for all external sections.

Succession of sections:

The actual time intervals of occupations of two consecutive sections are represented in Fig. 8.

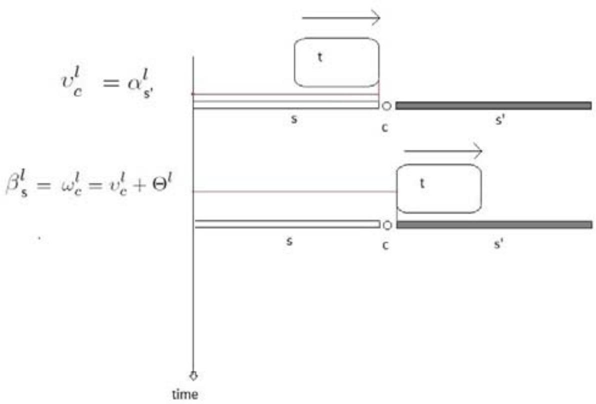

Fig. 8 The actual time intervals of occupation of two consecutive sections.

• If circulation l travels from section s to section s' by connector c, we consider that the section s' is reserved when connector c is occupied by circulation l. Thus, we have the constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \qquad Z^l_{ss'} = 1 \Rightarrow \upsilon^l_{c_{ss'}} = \alpha^l_{s'}$$

This constraint is expressed using the linear constraints below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s$$
$$\begin{cases} \upsilon^l_{c_{ss'}} - \alpha^l_{s'} \leq H \cdot (1 - Z^l_{ss'}) \\ \alpha^l_{s'} - \upsilon^l_{c_{ss'}} \leq H \cdot (1 - Z^l_{ss'}) \end{cases} \tag{25}$$

• If a circulation travels from section s to section s', we have the constraint for the time interval of occupation of these two sections below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \;\; Z^l_{ss'} = 1 \Rightarrow \beta^l_s = \alpha^l_{s'} + \Theta^l$$

This constraint is expressed using the linear constraints below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s$$
$$\begin{cases} \alpha^l_{s'} + \Theta^l - \beta^l_s \leq H \cdot (1 - Z^l_{ss'}) \\ \beta^l_s - \alpha^l_{s'} - \Theta^l \leq H \cdot (1 - Z^l_{ss'}) \end{cases} \tag{26}$$

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:13, No:8, 2019

Note: If $Z^l_{ss'} = 1$, the inequation (26) implies that $\alpha^l_{s'} + \Theta^l - \beta^l_s \leq 0$ and $\beta^l_s - \alpha^l_{s'} - \Theta^l \leq 0$, it means that $\beta^l_s = \alpha^l_{s'} + \Theta^l$. If $Z^l_{ss'} = 0$, the inequation is always true.

Actual time interval of occupation of stopping platform: The time interval of occupation of stopping platform of an entering circulation must respect the preferred arrival time A_t which can be adjusted within a time interval L.

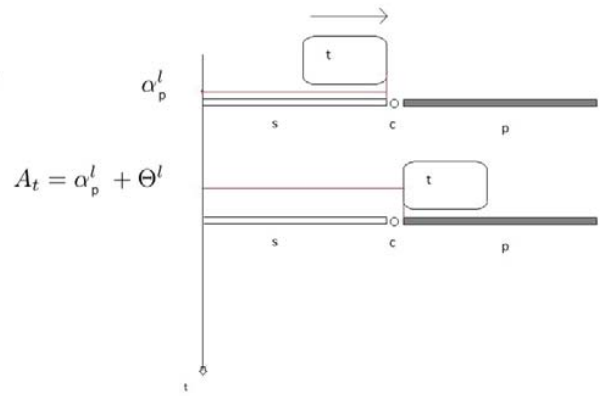

Fig. 9 The actual time interval of occupation of a platform

The actual time interval of occupation of a platform is represented in Fig. 9, we assume that the entering circulation of a train arrives at a stopping platform when the train leaves the connector connected with the platform. Hence, the entering circulation allows passengers to board or unboard the train. After that, the leaving circulation of this train will pass through and leaves the platform.

• The time interval of occupation of a stopping platform of an entering circulation must respect the preferred arrival time of platform A_t:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P}$$
$$P^l_p = 1 \Rightarrow A^t \leq \alpha^l_p + \Theta^l \leq A^t + L$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P}$$
$$\begin{cases} A^t - \alpha^l_p - \Theta^l \leq H \cdot (1 - P^l_p) \\ \alpha^l_p + \Theta^l - A^t \leq L + H \cdot (1 - P^l_p) \end{cases} \quad (27)$$

• The time interval of occupation of stopping platform of an entering circulation must respect the stopping time at platform D_t:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P} \qquad P^l_p = 1 \Rightarrow \beta^l_p = \alpha^l_p + \Theta^l + D^t$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{ent}, \forall p \in \mathbb{P}$$
$$\begin{cases} \beta^l_p - \alpha^l_p - \Theta^l - D^t \leq H \cdot (1 - P^l_p) \\ \alpha^l_p + \Theta^l + D^t - \beta^l_p \leq H \cdot (1 - P^l_p) \end{cases} \quad (28)$$

• The constraint of the time interval of occupation of a stopping platform of an entering circulation and a leaving circulation of the same train is expressed below:

$$\forall t \in \mathbb{T}, \forall l_1 \in \mathbb{L}^t_{ent}, \forall l_2 \in \mathbb{L}^t_{leav}, \forall p \in \mathbb{P} \; P^{l_1}_p = 1 \Rightarrow \alpha^{l_2}_p = \beta^{l_1}_p$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, l_1 \in \mathbb{L}^t_{ent}, l_2 \in \mathbb{L}^t_{leav}, \forall p \in \mathbb{P}$$
$$\begin{cases} \beta^{l_1}_p - \alpha^{l_2}_p \leq H \cdot (1 - P^{l_1}_p) \\ \alpha^{l_2}_p - \beta^{l_1}_p \leq H \cdot (1 - P^{l_1}_p) \end{cases} \quad (29)$$

• The constraint of the time interval of occupation of stopping platform of a leaving circulation is expressed below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav}, \forall p \in \mathbb{P}$$
$$P^l_p = 1 \Rightarrow \beta^l_p = \alpha^l_p + \Delta^l_p + \Theta^l + W^l_p$$

This constraint is expressed using the linear constraint below:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t_{leav}, \forall p \in \mathbb{P}$$
$$\begin{cases} \alpha^l_p + \Delta^l_p + \Theta^l + W^l_p - \beta^l_p \leq H \cdot (1 - P^l_p) \\ \beta^l_p - \alpha^l_p - \Delta^l_p - \Theta^l - W^l_p \leq H \cdot (1 - P^l_p) \end{cases} \quad (30)$$

Security constraints. The security constraints ensure that two circulation cannot pass the same section or the same connector at the same time. We use the actual time interval variables and ordering variables defined previously to express these constraints.

The chronological order of two circulations l, l' passing through a common section s is denoted by $Y^{ll'}_s$.

• With two circulations using the same section, one circulation must be scheduled before the other:

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall s \in \mathbb{S} \quad Y^{ll'}_s + Y^{l'l}_s = 1 \quad (31)$$

Occupation of sections: Two circulations passing through a common section cannot be scheduled during the same time interval:

$$\forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S}$$
$$\begin{cases} \beta^l_s \leq \alpha^{l'}_s + H \cdot (3 - S^l_s - S^{l'}_s - Y^{ll'}_s) \\ \beta^{l'}_s \leq \alpha^l_s + H \cdot (3 - S^l_s - S^{l'}_s - Y^{l'l}_s) \end{cases} \quad (32)$$

Note: If section s is in the route of both circulations l and l', so that $S^l_s = 1, S^{l'}_s = 1$ and either $Y^{ll'}_s = 1$ or $Y^{l'l}_s = 1$. It means that $3 - S^l_s - S^{l'}_s - Y^{ll'}_s = 0$ or $3 - S^l_s - S^{l'}_s - Y^{l'l}_s = 0$. In the first case, we have $\beta^l_s \leq \alpha^{l'}_s$, it means that circulation l leaves section s before the arriving of circulation l' at section s. The second constraint is trivially verified ($Y^{l'l}_s = 0$). In the other case, we have $\beta^{l'}_s \leq \alpha^l_s$, it means that circulation l' leaves section s before the arriving of circulation l at section s.

The chronological order of two circulations l, l' passing through a common connector c is denoted by $X^{ll'}_c$.

• With two circulations using the same connector, one circulation must be scheduled before the other:

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall c \in \mathbb{C} \quad X^{ll'}_c + X^{l'l}_c = 1 \quad (33)$$

Occupation of connectors: Two circulations passing through a common connector cannot be scheduled during the same time interval:

$$\forall t,t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \quad \begin{cases} \omega_c^l \leq \upsilon_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'}) \\ \omega_c^{l'} \leq \upsilon_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l}) \end{cases} \quad (34)$$

Note: If connector c is in the route of both circulations l and l', so that $C_c^l = 1, C_c^{l'} = 1$ and either $X_c^{ll'} = 1$ or $X_c^{l'l} = 1$. It means that $3 - C_c^l - C_c^{l'} - X_c^{ll'} = 0$ or $3 - C_c^l - C_c^{l'} - X_c^{l'l} = 0$. In the first case, we have $\omega_c^l \leq \upsilon_c^{l'}$, it means that circulation l leaves connector c before the arriving of circulation l' at connector c. The second constraint is trivially verified ($X_c^{l'l} = 0$). In the other case, we have $\omega_c^{l'} \leq \upsilon_c^l$, it means that circulation l' leaves connector c before the arriving of circulation l at connector c.

List of inequations and equations of all constraints:

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \notin \mathbb{S}_s \quad Z_{ss'}^l = 0 \quad (1)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l + Z_{s's}^l \leq 1 \quad (2)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l \quad (3)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S} \quad S_s^l \geq \sum_{s' \in \mathbb{S}_s} Z_{ss'}^l \quad (4)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad Z_{ss'}^l \leq C_{c_{ss'}}^l \quad (5)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad S_{e_{in}^l}^l = 1 \quad (6)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t \quad \sum_{s' \in \mathbb{S}_{e_{in}^l}} Z_{e_{in}^l s'}^l = 1 \quad (7)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall s \in \mathbb{E} \backslash \{e_{in}^l\} \quad S_s^l = 0 \quad (8)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad S_{e_{out}^l}^l = 1 \quad (9)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t \quad \sum_{s' \in \hat{\mathbb{S}}_{e_{out}^l}} Z_{s' e_{out}^l}^l = 1 \quad (10)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall s \in \mathbb{E} \backslash \{e_{out}^l\} \quad S_s^l = 0 \quad (11)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \sum_{s' \in \hat{\mathbb{S}}_s} Z_{s's}^l = \sum_{s'' \in \mathbb{S}_s} Z_{ss''}^l \quad (12)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad \begin{cases} \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l - \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l \leq H \cdot P_p^l \\ \sum_{s'' \in \hat{\mathbb{S}}_p} Z_{s''p}^l - \sum_{s' \in \mathbb{S}_p} Z_{ps'}^l \leq H \cdot P_p^l \end{cases} \quad (13)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t \quad \sum_{p \in \mathbb{P}} P_p^l = 1 \quad (14)$$

$$\forall t \in \mathbb{T}, \forall l, l' \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad P_p^l = P_p^{l'} \quad (15)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \geq P_p^l \quad (16)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \leq (1 - P_p^l) \quad (17)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \mathbb{S}_p} Z_{ps}^l \geq P_p^l \quad (18)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \sum_{s \in \hat{\mathbb{S}}_p} Z_{sp}^l \leq (1 - P_p^l) \quad (19)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \begin{cases} \sum_{s \in \mathbb{S}_c} S_s^l - 2 \leq H \cdot (1 - C_c^l) \\ 2 - \sum_{s \in \mathbb{S}_c} S_s^l \leq H \cdot (1 - C_c^l) \end{cases} \quad (20)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall c \in \mathbb{C} \quad \omega_c^l = \upsilon_c^l + \Theta^l \quad (21)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{I} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + 2\Theta^l + W_s^l \quad (22)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{E} \quad \beta_s^l = \alpha_s^l + \Delta_s^l + \Theta^l + W_s^l \quad (23)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall p \in \mathbb{P} \quad \begin{cases} \alpha_p^l + \Delta_p^l + 2\Theta^l + W_p^l - \beta_p^l \leq H \cdot P_p^l \\ \beta_p^l - \alpha_p^l - \Delta_p^l - 2\Theta^l - W_p^l \leq H \cdot P_p^l \end{cases} \quad (24)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad \begin{cases} \upsilon_{c_{ss'}}^l - \alpha_{s'}^l \leq H \cdot (1 - Z_{ss'}^l) \\ \alpha_{s'}^l - \upsilon_{c_{ss'}}^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \quad (25)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall s \in \mathbb{S}, \forall s' \in \mathbb{S}_s \quad \begin{cases} \alpha_{s'}^l + \Theta^l - \beta_s^l \leq H \cdot (1 - Z_{ss'}^l) \\ \beta_s^l - \alpha_{s'}^l - \Theta^l \leq H \cdot (1 - Z_{ss'}^l) \end{cases} \quad (26)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \begin{cases} A^t - \alpha_p^l - \Theta^l \leq H \cdot (1 - P_p^l) \\ \alpha_p^l + \Theta^l - A^t \leq L + H \cdot (1 - P_p^l) \end{cases} \quad (27)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{ent}^t, \forall p \in \mathbb{P} \quad \begin{cases} \beta_p^l - \alpha_p^l - \Theta^l - D^t \leq H \cdot (1 - P_p^l) \\ \alpha_p^l + \Theta^l + D^t - \beta_p^l \leq H \cdot (1 - P_p^l) \end{cases} \quad (28)$$

$$\forall t \in \mathbb{T}, l_1 \in \mathbb{L}_{ent}^t, l_2 \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \begin{cases} \beta_p^{l_1} - \alpha_p^{l_2} \leq H \cdot (1 - P_p^{l_1}) \\ \alpha_p^{l_2} - \beta_p^{l_1} \leq H \cdot (1 - P_p^{l_1}) \end{cases} \quad (29)$$

$$\forall t \in \mathbb{T}, \forall l \in \mathbb{L}_{leav}^t, \forall p \in \mathbb{P} \quad \begin{cases} \alpha_p^l + \Delta_p^l + \Theta^l + W_p^l - \beta_p^l \leq H \cdot (1 - P_p^l) \\ \beta_p^l - \alpha_p^l - \Delta_p^l - \Theta^l - W_p^l \leq H \cdot (1 - P_p^l) \end{cases} \quad (30)$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall s \in \mathbb{S} \quad Y_s^{ll'} + Y_s^{l'l} = 1 \tag{31}$$

$$\forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall s \in \mathbb{S}: \begin{cases} \beta_s^l \leq \alpha_s^{l'} + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{ll'}) \\ \beta_s^{l'} \leq \alpha_s^l + H \cdot (3 - S_s^l - S_s^{l'} - Y_s^{l'l}) \end{cases} \tag{32}$$

$$\forall t, t' \in \mathbb{T}, \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, l \neq l', \forall c \in \mathbb{C} \quad X_c^{ll'} + X_c^{l'l} = 1 \tag{33}$$

$$\forall t, t' \in \mathbb{T}, t \neq t', \forall l \in \mathbb{L}^t, \forall l' \in \mathbb{L}^{t'}, \forall c \in \mathbb{C} \quad \begin{cases} \omega_c^l \leq \upsilon_c^{l'} + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{ll'}) \\ \omega_c^{l'} \leq \upsilon_c^l + H \cdot (3 - C_c^l - C_c^{l'} - X_c^{l'l}) \end{cases} \tag{34}$$

IV. Numerical Experiments

In this experiment, our topology is depicted in Fig. 10 and the correspondences between sections of this figure are listed in Table III.

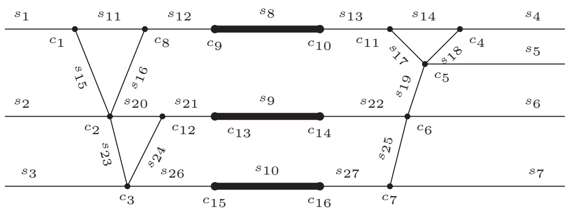

Fig. 10 The topology of the railway station

TABLE III
The Correspondences between Sections of Fig. 10

Section	External section	Platform
s_1	e_1	
s_2	e_2	
s_3	e_3	
s_4	e_4	
s_5	e_5	
s_6	e_6	
s_7	e_7	
s_8		p_1
s_9		p_2
s_{10}		p_3

Seven external sections (e_1 to e_7) are considered for the arrival and departure of trains. There are three platforms (p_1 to p_3) which are used for the boarding or unboarding of passengers. There are a total of 27 sections and 16 connectors in this railway station. We assume that all sections are sections with two-way directions. For example, the set of doublets of connector c_1 is $\mathbb{K}_{c_1} = \{(s_1, s_{11}), (s_{11}, s_1), (s_1, s_{15}), (s_{15}, s_1)\}$. All pairs of sections are not reachable (they are not doublets) even if two sections of these pairs are connected with a same connector. These pairs of unreachable sections are as follows: (s_{15}, s_{11}), (s_{11}, s_{16}), (s_{15}, s_{16}), (s_{15}, s_2), (s_{16}, s_{20}), (s_{16}, s_{23}), (s_{20}, s_{23}), (s_{20}, s_{24}), (s_{23}, s_3), (s_{23}, s_{24}), (s_{24}, s_{26}), (s_{14}, s_{18}), (s_{14}, s_{17}), (s_{17}, s_{18}), (s_{18}, s_5), (s_{17}, s_{19}), (s_{19}, s_6), (s_{22}, s_{25}), (s_{25}, s_7). For example, the pair of unreachable sections (s_{15}, s_{16}) means that trains are not allowed to travel from section s_{15} to section s_{16} and from s_{16} to s_{15}.

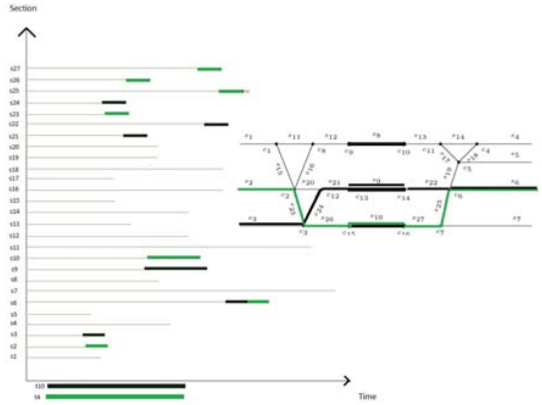

Fig. 11 The occupation of sections of train 4 and train 10

We run the experiments for 10 trains which correspond to 20 circulations. The data of each train are presented in Table IV.

TABLE IV
Example of Problem

Train	Circulation	Type	External section	A^t	D^t
1	1	ent	e_1	180	30
1	2	leav	e_6		
2	3	ent	e_2	275	40
2	4	leav	e_4		
3	5	ent	e_2	315	20
3	6	leav	e_4		
4	7	ent	e_2	125	30
4	8	leav	e_6		
5	9	ent	e_1	190	40
5	10	leav	e_4		
6	11	ent	e_4	220	40
6	12	leav	e_1		
7	13	ent	e_5	140	30
7	14	leav	e_2		
8	15	ent	e_5	285	40
8	16	leav	e_1		
9	17	ent	e_3	245	40
9	18	leav	e_7		
10	19	ent	e_3	125	40
10	20	leav	e_6		

The following constants are used:

- Maximum permissible deviation for A^t: L=3
- Duration to traverse section by circulation Δ=20 for all.
- Duration to traverse connector by circulation Θ=2 for all.

Note: Times are counted by seconds.

We run the experiments with the objective function of minimizing the total of waiting time $\sum_{l \in \mathbb{L}} \sum_{s \in \mathbb{S}} W_s^l$ and minimizing the total travel time $\sum_{l \in \mathbb{L}} \sum_{s \in \mathbb{S}} \Delta_s^l * S_s^l$.

We consider minimizing the total time of interruptions is more important than the total travel time. Thus, we affect the weight of the total waiting time to 10 and the weight of total travel time to 1.

Objective function: MIN $\sum_{l \in \mathbb{L}} \sum_{s \in \mathbb{S}} (10 * W_s^l + \Delta_s^l * S_s^l)$

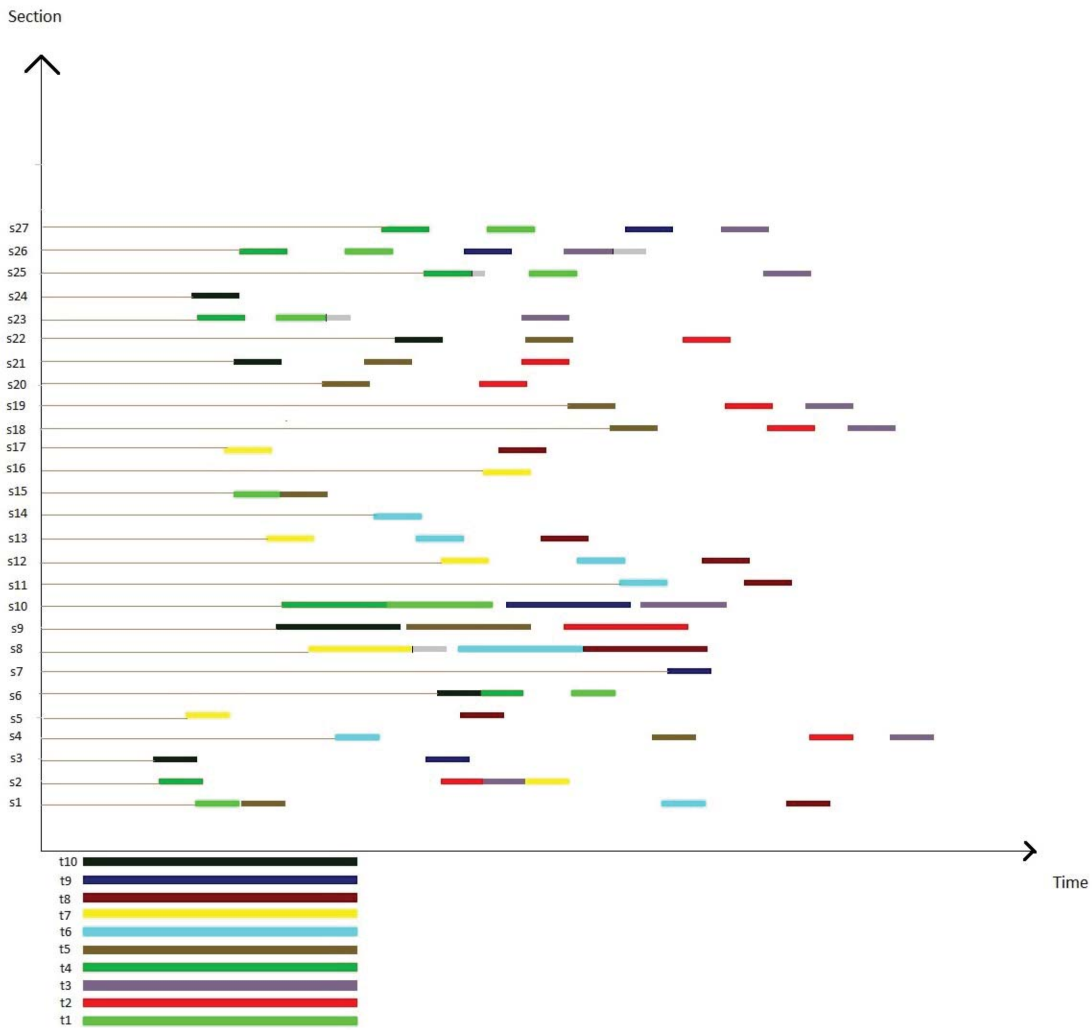

Fig. 12 Graphic timetable

The computation study is conducted under C++ in Visual Studio 2017 and CPLEX version 12.8. The hardware architecture is Windows x64, with Intel i7-870 CPU at 2.93 GHz and 4GB memory of RAM. The results are presented in Table V and the graphic timetable (Fig. 12). The time needed to solve the problem is 1.44 seconds. The results show that there are 4 interruptions of trains with a total waiting time of 55 seconds. The model considered has 1332 constraints and 2416 variables.

We can see clearly the occupation of sections of two trains t_4 and t_{10} in Fig. 11. Train 4 has an interruption at section s_{25}, it must wait train 10 leave section s_6 to enter this section. We must remark that train 10 use switch at connector c_3 and c_{12} to go to platform 2 (section s_9) and train 4 use switch at connector c_2 and c_3 to go to platform 3 (section s_{10}), in the future experiment we should propose an appropriate objective function to avoid the use of switch unnecessarily.

In Fig. 12, train 3 must wait at section s_{26} before entering the platform 3 (section s_{10}) to satisfy the reference arrival time A^t. Train 7 must wait to enter section s_2 but it does not stop

TABLE V
RESULTS OF PROBLEM

Train	Circ	Route	Platform	$[\alpha_p, \beta_p]$
1	1	$s_1, s_{15}, s_{23}, s_{26}, s_{10}$	p_3	[181,213]
1	2	$s_{10}, s_{27}, s_{25}, s_6$		[213,235]
2	3	s_2, s_{20}, s_{21}, s_9	p_2	[273,315]
2	4	$s_9, s_{22}, s_{19}, s_{18}, s_4$		[315,337]
3	5	$s_2, s_{23}, s_{26}, s_{10}$	p_3	[313,335]
3	6	$s_{10}, s_{27}, s_{25}, s_{19}, s_{18}, s_4$		[335,357]
4	7	$s_2, s_{23}, s_{26}, s_{10}$	p_3	[126,158]
4	8	$s_{10}, s_{27}, s_{25}, s_6$		[158,180]
5	9	$s_1, s_{15}, s_{20}, s_{21}, s_9$	p_2	[191,233]
5	10	$s_9, s_{22}, s_{19}, s_{18}, s_4$		[233,255]
6	11	s_4, s_{14}, s_{13}, s_8	p_1	[218,260]
6	12	s_8, s_{12}, s_{11}, s_1		[260,282]
7	13	s_5, s_{17}, s_{13}, s_8	p_1	[141,173]
7	14	s_8, s_{12}, s_{16}, s_2		[173,211]
8	15	s_5, s_{17}, s_{13}, s_8	p_1	[283,325]
8	16	s_8, s_{12}, s_{11}, s_1		[325,347]
9	17	s_3, s_{26}, s_{10}	p_3	[243,285]
9	18	s_{10}, s_{27}, s_7		[285,307]
10	19	s_3, s_{24}, s_{21}, s_9	p_2	[123,165]
10	20	s_9, s_{22}, s_6		[165,187]

at the consecutive section before section s_2 (section s_{16}). It stops at section s_8. Even though the duration of interruption is not changed, we should consider if trains must stop at the consecutive section before the section which is occupied in a future work.

V. Conclusion

In this article, we propose a mathematical model and a mixed-integer linear programming formulation to solve the optimal train routing and scheduling for busy railway stations. The main goal is to find a routing and scheduling decision support tool that is able to improve the capacity of railway infrastructure. This mathematical model is assessed by the an experiment with 10 trains consisting of 20 circulations. The computation time to solve this problem is short.

In a future work, we will increase the size of problem and implement some appropriate objective function to compare the results with the models of other researches dealing with the reservation of route. An implementation of heuristic methods must be considered to solve problems of larger size. A study of some strategies to reduce the number of constraints of the model should be taken into account.

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