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Long-Range Dependence of Financial Time Series Data

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Abstract**—**This paper examines long-range dependence or long-memory of financial time series on the exchange rate data by the fractional Brownian motion (fBm). The principle of spectral density function in Section 2 is used to find the range of Hurst parameter (H) of the fBm. If 0< H <1/2, then it has a short-range dependence (SRD). It simulates long-memory or long-range dependence (LRD) if 1/2< H <1. The curve of exchange rate data is fBm because of the specific appearance of the Hurst parameter (H). Furthermore, some of the definitions of the fBm, long-range dependence and self-similarity are reviewed in Section II as well. Our results indicate that there exists a long-memory or a long-range dependence (LRD) for the exchange rate data in section III. Long-range dependence of the exchange rate data and estimation of the Hurst parameter (H) are discussed in Section IV, while a conclusion is discussed in Section V.

Keywords**—**Fractional Brownian motion, long-range dependence, memory, short-range dependence.

I. INTRODUCTION

THE fractional Brownian motion(fBm) is a generalisation of the Brownian motion(Bm) with the Hurst parameter(H) or the index of self-similarity. It is also known as the random walk process. A random walk is called the Brownian motion (Weiner Process) when H=1/2. But the fractional Brownian motion data sets where 0< H <1/2 and 1/2< H <1 simulate short-range dependence (SRD) and long-range dependence(LRD), respectively. Moreover, the fBm is defined as a Gaussian process with stationary increments as well.

An approach to model financial processes with long-memory is via the theory of stochastic differential equations derived by fractional Brownian motion (Dai and Heyde [7], Comte and Renault [5],[6], Norros, Valkeila and Virtamo [16], Alos, Mazet and Nualart [1]). In this approach, the effect of LRD can be obtained from the noise term. Recently, Heyde [11]) proposed a risky asset model with LRD through fractal activity time. The idea is to replace Brownian time in geometric Brownian motion by some process with stationary LRD increments and heavy tails.

The existence of long-memory behavior of exchange rates can be related to the dynamic properties of other economic variables. The purchasing power parity (PPP) hypothesis suggests that exchange rate fluctuations are tied to the movements of relative national prices (Cheung[4]). Furthermore, the researchers pay particular emphasis on the implications of long-memory for market efficiency. According to the market efficiency hypothesis in its weak form, asset prices incorporate all relevant information, rendering asset returns unpredictable. The price of an asset determined in an efficient market should follow a martingale process in which each price change is unaffected by its predecessor and has no memory. If the return series exhibit long memory, they display significant autocorrelation between distant observations. Therefore, the series realizations are not independent over time and past returns can help predict futures returns, thus violating the market efficiency hypothesis (Assaf and Cavalcante [2]).

Since the specific appearance of the Hurst parameter, the properties of the fBm which is referred to as long memory or long range dependence (LRD) is also studied in this paper.

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II. MATERIAL AND METHODS

Fractional Brownian Motion

Definition 1

For $H \in (0,1)$, a Gaussian process $B^H = \left\{ B_t^H \ , t \ge 0 \right\}$ is a fractional Brownian motion (fBm) if it has

Mean: $E(B_t^H) = 0$

Covariance:

$$E(B_t^H, B_s^H) = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H} \right),$$

for all t ,s $\in$ R.

Note that one may recover the standard Brownian motion by replacing H with 1/2.

Theorem 1

A fBm process $B^H = \left\{ B_t^H \ , t \ge 0 \right\}$ has the following properties:

1. if 0< H <1/2 then it has short-range dependence (SRD);
2. if H = 1/2 then it has independent increments or is standard Brownian motion;
3. if 1/2< H <1 then it has long-range dependence (LRD).

Note that if fBm has LRD, then it has memory.

Definition 2

Let $\{X_t : t \in N\}$ be a time-series which is weakly stationary. The auto correlation function (ACF) for a weakly stationary time series is given by

$$\rho(k) = \text{cov}(X_t, X_{t+k}), \quad k = 0, \pm 1, \pm 2, ...$$

and the spectral density function can be defined as

$$f(\lambda) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \rho(k) e^{-ik\lambda}, -\pi < \lambda < \pi \qquad (1)$$

Definition 3

A fBm is a Gaussian process (denoted B_H) which has stationary increments and spectral density of the form

$$f(\lambda) = \frac{C}{|\lambda|^{2H+1}} \qquad (2)$$

where $C > 0$, $0 < H < 1$, $\lambda \in R$

Note that $H \in (0,1)$ is the so-called Hurst index or Hurst parameter.

Thus (2) is transformed to the linearly logarithm equation

$$\log f(\lambda) = -(2H+1)\log|\lambda| + \log C \qquad (3)$$

The relationship between the Hurst parameter and a slope in (3) is

$$H = \left| \frac{slope + 1}{2} \right| \qquad (4)$$

Algorithm of fBm

Step 1 Simulate the data sets in the determined time.
Step 2 Sketch the log-normal curve from step 1.
Step 3 Find the average of a slope value from the log-normal curve in the determined time.
Step 4 Calculate the Hurst parameter from

$$H = \left| \frac{slope + 1}{2} \right|.$$

Step 5 Classify a range of the Hurst parameter.

Long-Range Dependence and Self-Similarity

Long-range dependence (LRD) or long-memory has been investigated extensively in a variety of applied fields, especially in finance (Baillie [3], Granger and Ding [9], Comte and Renault [5],[6], Willinger, Taqqu and Teverovsky [17], Heyde and Liu [12]). Since the concept of long-range dependence is incompatible with the efficient market hypothesis, a key assumption in mathematical finance, it is still a controversial issue whether market models should include long-memory (Lo [14], Baillie [3], Willinger, Taqqu and Teverovsky [17]).

Long-range dependence can also be found in the fractionally integrated autoregressive moving average (ARFIMA) process of financial time series. Hosking [13] showed that the autocorrelation, $\rho(.)$, of an ARFIMA process satisfies $\rho(k) \propto k^{2d-1}$ as $k \to \infty$. Thus the memory property of a process depends on the value of d. When $d \in (0,0.5)$, the autocorrelations do not have a finite sum. When $d \le 0$, the autocorrelations have a finite sum; that is ARFIMA processes with $d \in (0,0.5)$ display long memory. Hence the existence of long memory can be determined by testing for the statistical significance of the sample differencing parameter d (Cheung [4]).

A second-order stationary process $\xi(t)$ with discrete time is said to possess long-range dependence if its covariance function $R(s) = \text{cov}(\xi(t), \xi(t+s)), s \in Z_+$ decays at a hyperbolic rate as $s \to \infty$. In particular, the covariance function R(s) of a process with LRD can be approximated as

$$R(s) \sim Ks^{2d-1}, |d| < \frac{1}{2}, s \to \infty \qquad (5)$$

for some constant K>0, or

$$R(s) \sim Ks \cos(\kappa s), s \to \infty \qquad (6)$$

where K>0, $|d| < \frac{1}{2}, \kappa = \cos^{-1}\phi \in [0,\pi], |\phi| \le 1$

The covariance function (5) decays slowly at a hyperbolic rate, while the covariance function (6) resembles a hyperbolically damped cosine wave.

The simplest model of a stationary process with LRD and covariance function (5) was first proposed by Granger and Joyeux [10] and Hosking [13]. This process can be defined by the difference equation

$$(1-B)^d \xi(t) = \varepsilon(t), t \in Z, |d| < \frac{1}{2} \qquad (7)$$

where $\varepsilon(t)$ is white noise with $E\varepsilon(t) = 0, E\varepsilon^2(t) = \sigma^2 > 0$. The covariance function of this process can be approximated by (5) and the spectral density has the form:

$$f_d(\lambda) = \frac{\sigma^2}{2\pi}|1 - \exp(-i\lambda)|^{-2d} = \frac{\sigma^2}{2\pi}\left(2\sin\frac{\lambda}{2}\right)^{-2d}, \lambda \in [-\pi, \pi) \qquad (8)$$

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In continuous time, a fundamental process which may exhibit long-memory is fractional Brownian motion. For this reason, models with long-range dependence are often formulated in terms of self-similar process. Fractional Brownian motion is a typical example of self-similar process whose increments exhibit long-range dependence. For any H in (0,1) fractional Brownian motion (fBm) with Hurst index H is a centered Gaussian process $B^H = \left\{ B_t^H \ , t \geq 0 \right\}$ with covariance

$$E(B_s^H, B_t^H) = \frac{V_H}{2}\left(s^{2H} + t^{2H} - |t-s|^{2H}\right),$$

where V_H is a normalizing constant given by

$$V_H = \frac{\Gamma(2-2H)\cos(\pi H)}{\pi H(1-2H)}$$

(Mandelbrot and Ness [15]). It is a process starting from zero with stationary increments, $E(B_t^H - B_s^H)^2 = V_H |t-s|^{2H}$, and is self-similar, that is, $B_{\alpha t}^H$ has the same distribution as $\alpha^H B_t^H$ (Decreusefond and Ustunel [8], Alos, Mazet and Nualart[1]). The constant H determines the sign of the covariance of the future and past increments. This covariance is positive when $H > \frac{1}{2}$ and negative when $H < \frac{1}{2}$. The case $H = \frac{1}{2}$ corresponds to the ordinary Brownian motion. Furthermore, as the covariance between increments at a distance u decreases to zero as U^{2H-2}, fBm exhibits long-range dependence when $H > \frac{1}{2}$.

III. Results

The daily changes of the French Franc(FRF) against the US-Dollar (USD) from 2 January 1975 to 15 January 1999(N=5529), the Deutsch Mark(DM) against the US-Dollar from 4 January 1971 to 30 April 1996(N=6350), the JPY/USD from 4 January 1971 to 18 June 2007 (N=9131) and the Euro(EUR) against the US-Dollar from 4 January 1999 to 18 June 2007(N=2124) are studied for long-range dependence. Fig. 1 shows the daily changes of the FRF/USD, Fig. 2 shows the daily changes of the DM/USD, Fig. 3 shows the daily changes of the JPY/USD from 4 January 1971 to 18 June 2007 (N=9131) and Fig. 4 shows the daily changes of the EUR/USD from 4 January 1999 to 18 June 2007(N=2124), respectively. As can be seen in Fig. 5, the slope of the spectral density of FRF/USD is equal to -2.8567, the slope of the spectral density of the DM/USD in Fig. 6 is equal to -2.3433, the slope of the spectral density of the JPY/USD in Fig. 7 is equal to -2.1359 and the slope of the spectral density of the EUR/USD in Fig. 8 is equal to -2.1082, respectively.

IV. Discussion

As can be seen in Fig. 5, the slope of the spectral density of the FRF/USD is equal to -2.8567. Since it is equal to –(2H+1) in (3), then the Hurst index (H) is equal to 0.9284. Since the slope of the spectral density of the DM/USD in Fig. 6 is equal to -2.3433, then the Hurst index (H) is equal to 0.6716. Similarly, the slope of the spectral density of the JPY/USD in Fig. 7 and the EUR/USD in Fig. 8 are equal to -2.1359 and -2.1082. Thus the Hurst index (H) of the JPY/USD and the Euro/USD are equal to 0.5680 and 0.5541, respectively. It might be a standard Brownian motion rather than a long-range dependence(LRD) for the JPY/USD and the EUR/USD because the Hurst indices(H) approach to 0.5. But the Hurst indices(H) of the FRF/USD and the DM/USD approach to 1 rather than 0.5. Therefore, the FRF/USD and the DM/USD have memory due to H>1/2. While Cheung [4], taking monthly data from January 1974 through December 1989, found some evidence for long memory in the French Franc/US Dollar and some marginal evidence for the UK Pound/US Dollar, but no apparent departure from martingale behavior for the German Mark, Swiss Franc, or Japanese Yen.

V. Conclusion

Since the Hurst indices (H) of the JPY/USD and the EUR/USD approaches to 0.5, there are no memory for these cases. But it is assumed that the JPY/USD and the EUR/USD might be the Standard Brownian motion rather than the long-range dependence (LRD). It is a clear evidence that there exists memory for the FRF/USD and the DM/USD because the Hurst indices (H) of the FRF/USD and the DM/USD approach to 1 rather than 0.5. If 1/2< H <1 then it has long-range dependence (LRD) in the property of the fractional Brownian motion (fBm). Therefore, our results indicate that there exists long-range dependence (LRD) for these cases. If it has LRD, then it has memory which predictable in Time Series method for the next step. Finally, the principle of fBm is aimed to use in several financial time series data in our future work.

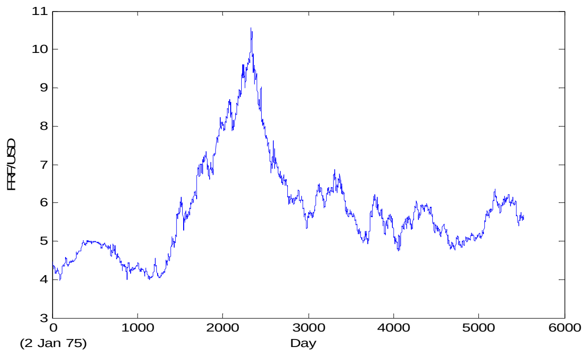

Fig. 1 The daily changes of the FRF/USD from 2 January 1975 to 15 January 1999(N=5529)

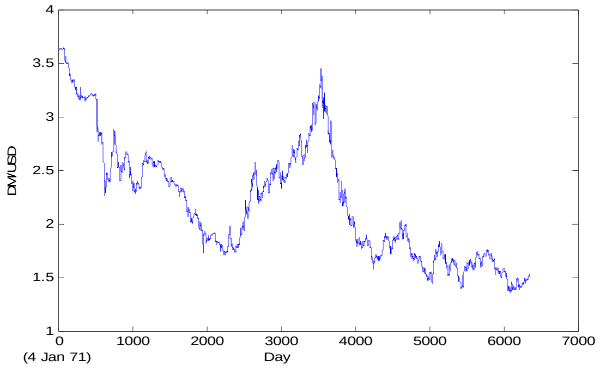

Fig. 2 The daily changes of the DM/USD from 4 January 1971 to 30 April 1996(N=6350)

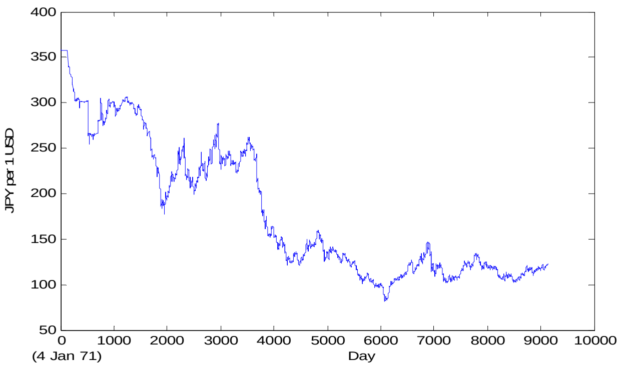

Fig. 3 The daily changes of JPY/USD from 4 January 1971 to 18 June 2007 (N=9131)

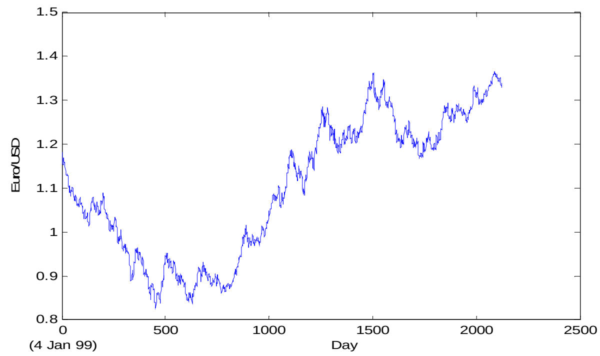

Fig. 4 The daily changes of the EUR/USD from 4 January 1999 to 18 June 2007(N=2124)

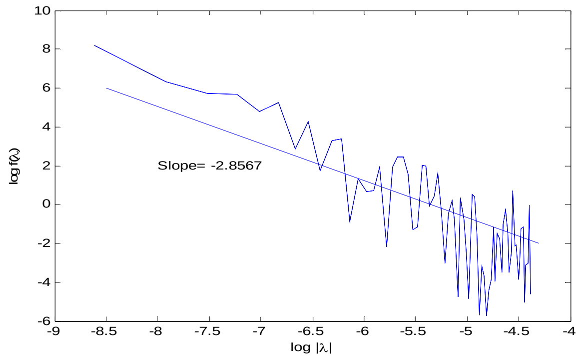

Fig. 5 The slope of the spectral density of the FRF/USD

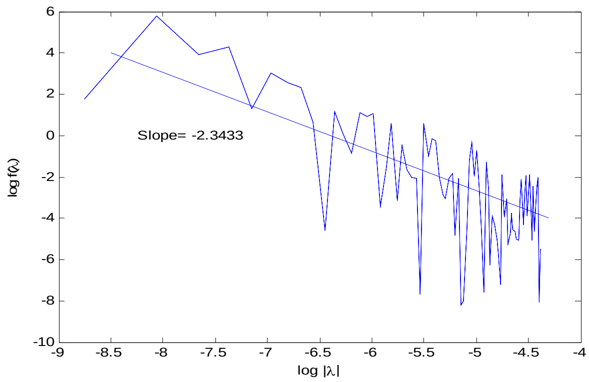

Fig. 6 The slope of the spectral density of the DM/USD

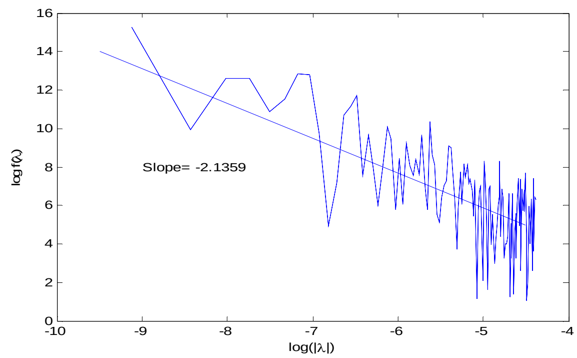

Fig. 7 The slope of the spectral density of the JPY/USD

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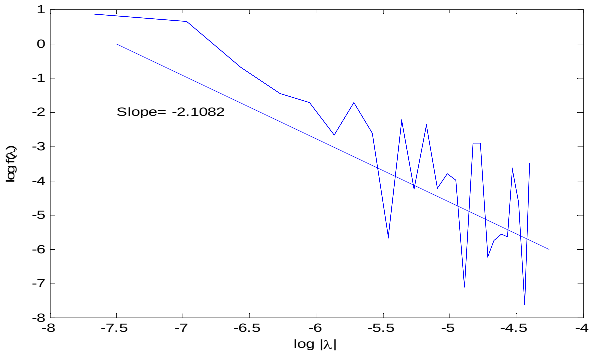

Fig. 8 The slope of the spectral density of the EUR/USD

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