

# Lithium-Ion Battery State of Charge Estimation Using One State Hysteresis Model with Nonlinear Estimation Strategies

Mohammed Farag, Mina Attari, S. Andrew Gadsden, Saeid R. Habibi

**Abstract**—Battery state of charge (SOC) estimation is an important parameter as it measures the total amount of electrical energy stored at a current time. The SOC percentage acts as a fuel gauge if it is compared with a conventional vehicle. Estimating the SOC is, therefore, essential for monitoring the amount of useful life remaining in the battery system. This paper looks at the implementation of three nonlinear estimation strategies for Li-Ion battery SOC estimation. One of the most common behavioral battery models is the one state hysteresis (OSH) model. The extended Kalman filter (EKF), the smooth variable structure filter (SVSF), and the time-varying smoothing boundary layer SVSF are applied on this model, and the results are compared.

**Keywords**—State of charge estimation, battery modeling, one-state hysteresis, filtering and estimation.

## I. INTRODUCTION

IN the last ten years, battery management systems (BMS) have garnered lots of attention from many researchers. Accurate BMS increases the life of a battery and reduces fast ageing-effects, thermal runaways, and performance ceasing. It is, therefore, vital and fundamental for the BMS to accurately predict and estimate the SOC, among other critical parameters. Several methods have been implemented for SOC estimation; starting from very abstract models dealing with batteries as a black-box, to very detailed electrochemical models which are used to capture the battery internal physical behavior [1].

The most popular battery chemistry in use today is the lithium-ion batteries [2]-[4]. Li-Ion batteries are often found in portable electronic devices due to their lightweight and ability to recharge relatively well. During operation, the BMS estimate parameters that affect the battery packs and their operating conditions [5]-[7]. A number of surveys have been performed on BMS modeling and estimation [8]-[11]. Parameters of interest include terminal voltage (typically measured), battery state of charge (SOC), state of health (SOH), power and capacity fade, and instantaneous power. These parameters must be estimated using a filtering strategy such as the Extended Kalman filter (EKF).

In Section II, the EKF, SVSF, and VBL-SVSF filtering methods are briefly described. Section III provides an overview of the one state hysteresis (OSH) model. In Section IV, three

nonlinear estimation strategies are applied on the OSH battery model and compared. The paper then concludes in the final section.

## II. ESTIMATION STRATEGIES

This section provides an overview of the three nonlinear estimation strategies used in this paper.

### A. Extended Kalman Filter

Rudolph Kalman introduced the Kalman filter (KF) in the 1960s. Since then, KF has been one of the most commonly used state and parameter estimation strategies. KF strategies calculate a statistically optimal gain to correct predicted the system state estimates [12]. For Kalman filter to be optimal many strict assumptions must be followed, such as the system and measurements functions must be linear and known, and the noise must be white and Gaussian-distributed [13].

Many variations for Kalman filter have been introduced. Andrews et al. [14] formulated the extended Kalman filter (EKF) for the case of nonlinear systems and measurements. The EKF method uses the first-order Taylor series approximations to linearize the nonlinearities about the operating point. The EKF equations are similar to the KF, except for the linearization. The prediction phase of the EKF begins as follows [14]:

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) \quad (1)$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \quad (2)$$

Note that the update phase is defined by the following set of equations [14]:

$$e_{z,k+1|k} = z_{k+1} - h(\hat{x}_{k+1|k}) \quad (3)$$

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \quad (4)$$

$$K_{k+1} = P_{k+1|k} H_{k+1}^T S_{k+1}^{-1} \quad (5)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \quad (6)$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} \quad (7)$$

The first-order Taylor series approximation is used to linearize the non-linear state and measurement equations as follows:

$$F_k = \left. \frac{\partial f(x)}{\partial x} \right|_{x=\hat{x}_{k|k}, u_k} \quad (8)$$

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$$H_{k+1} = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_{k+1|k}} \quad (9)$$

For a complete list of the nomenclature and the corresponding definitions, please refer to the Appendix.

### B. Smooth Variable Structure Filter (SVSF)

Saeid Habibi introduced the smooth variable structure filter (SVSF) in 2007 [15]. The SVSF is a relatively new estimation strategy with respect to the KF and EKF. The SVSF uses the sliding mode concepts in calculating the correction gain. Formulaically, the SVSF is a predictor-corrector estimator; however, its gain is fundamentally different from KF gain. The theory behind the SVSF estimation process is shown in Fig. 1.

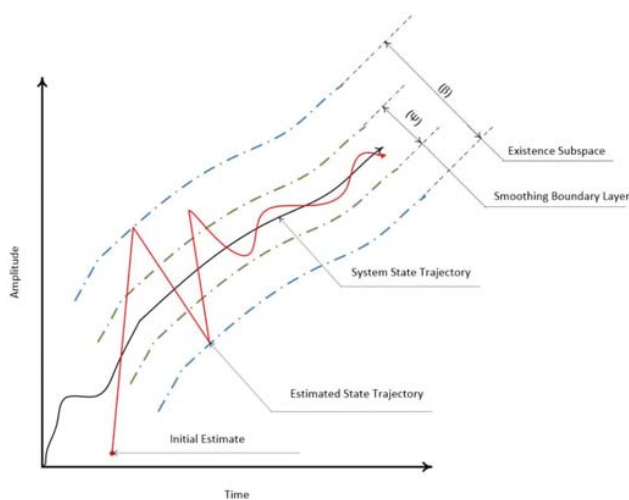


Fig. 1 The SVSF estimation concept

The prediction phase of the SVSF is similar to the EKF, and can be described as follows:

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) \quad (10)$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \quad (11)$$

$$e_{z,k+1|k} = z_{k+1} - h(\hat{x}_{k+1|k}) \quad (12)$$

The SVSF gain is defined by (13).

$$K_{k+1} = C^+ \text{diag} \left[ \left( |e_{z,k+1|k}|_{Abs} + \gamma |e_{z,k|k}|_{Abs} \right) \circ \text{sat} \left( \bar{\psi}^{-1} e_{z,k+1|k} \right) \right] \text{diag} \left( e_{z,k+1|k} \right)^{-1} \quad (13)$$

The update phase is defined as follows:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \quad (14)$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} (I - K_{k+1} H_{k+1})^T + K_{k+1} R_{k+1} K_{k+1}^T \quad (15)$$

$$e_{z,k+1|k+1} = z_{k+1} - h(\hat{x}_{k+1|k+1}) \quad (16)$$

Note that the SVSF gain is a function of: (i) a priori measurement error, (ii) a posteriori measurement error, (iii) SVSF memory, (iv) and a smoothing boundary layer term. The smoothing boundary layer term is utilized to decrease the magnitude of chattering created by the switching term in (13).

The existence subspace shown in Fig. 1 represents the estimation processes' amount of uncertainties [16]. This value is defined in terms of modeling errors or measurement uncertainties, and is often tuned by trial and error based on the amount of system or measurement noise. The width of the existence subspace  $\beta$  is time variant and is correlated to inaccuracy of the filter and battery model as well as the measurement model [15]. The existence subspace value is not known, however prior knowledge of the system is helpful to set an upper limit.

### C. Time-Varying Smoothing Boundary Layer (VBL SVSF)

Gadsden et al. [17] introduced the time-varying smoothing boundary layer formulation of the SVSF in 2012 in order to increase the estimation accuracy and avoid the chattering effect. On the one hand, The SVSF estimation accuracy is reduced by the chattering effect caused by the SVSF gain definition. On the contrary, the chattering effect drastically enhances filter robustness and stability against modeling errors and uncertainties [15].

To obtain the VBL formulation, a correlation between the partial derivative of the trace of a posteriori covariance and the smoothing boundary layer term was introduced to the filter gain derivation. The VBL-SVSF prediction phase is similar to (10)-(12). The time-varying smoothing boundary layer (VBL) is calculated using the following three equations:

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \quad (17)$$

$$A_{k+1} = |e_{z,k+1|k}|_{Abs} + \gamma |e_{z,k|k}|_{Abs} \quad (18)$$

$$\Psi_{k+1} = (\bar{A}_{k+1}^{-1} H_{k+1} P_{k+1|k} H_{k+1}^T S_{k+1}^{-1})^{-1} \quad (19)$$

The VBL-SVSF gain is then used to update the state estimates and state error covariance matrix, as follows:

$$K_{k+1} = H_{k+1}^{-1} \bar{A}_{k+1} \Psi_{k+1}^{-1} \quad (20)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \quad (21)$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} (I - K_{k+1} H_{k+1})^T + K_{k+1} R_{k+1} K_{k+1}^T \quad (22)$$

$$e_{z,k+1|k+1} = z_{k+1} - h(\hat{x}_{k+1|k+1}) \quad (23)$$

Both VBL-SVSF and SVSF works in a predictor-corrector fashion, the main difference is the equations used to calculate the filters gain. The main drawback of the SVSF is the fact that its conservative fixed smoothing boundary layer is fixed throughout the operation. This reduces the overall estimation accuracy. The VBL-SVSF calculates a near-optimal value for the boundary layer, to improve the estimation accuracy.

### III. ONE STATE HYSTERESIS MODEL

Various methods for battery modeling exist in literature. Plett et al. [6] developed the most common used behavioral models: the combined model, simple model, zero and one state hysteresis models, and enhanced self-correcting model. All of the models have the terminal voltage as an output and the SOC as a system state. The one state hysteresis (OSH) is a popular behavioral model and has been selected for use in this paper. The hysteresis phenomena is very important to increase the system performance and improve the SOC estimation accuracy [8]. The description of the one state hysteresis model shown here may be found in details in [6].

In the OSH model, the terminal voltage is calculated as follows:

$$y_k = OCV(z_k) - s_k M(z_k) - Ri_k \quad (24)$$

where  $s_k$  represents the sign of the current. For some sufficiently small and positive value  $\varepsilon$ , one has:

$$s_k = \begin{cases} +1 & i_k > \varepsilon \\ -1 & i_k < -\varepsilon \\ s_k - 1 & |i_k| \leq \varepsilon \end{cases} \quad (25)$$

where  $M(z_k)$  is a constant value and equal to half the difference between the charge and discharge values [6]. As shown in the previous equation, the hysteresis state is not a function of time, but of SOC. The state-space representation of the OSH model is defined as follows:

$$\begin{bmatrix} h_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} F(i_k) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_k \\ z_k \end{bmatrix} + \begin{bmatrix} 0 & 1 - F(i_k) \\ -\frac{\eta_i \Delta t}{C} & 0 \end{bmatrix} \begin{bmatrix} i_k \\ M(z, \dot{z}) \end{bmatrix} \quad (26)$$

$$y_k = OCV(z_k) - Ri_k + h_k \quad (27)$$

Furthermore, note that  $F(i_k) = \exp(-|\eta_i i(t) \gamma / C_n|)$ . The parameters vector for the OSH is shown in (28) and the values for the parameters is presented in Appendix V.

$$\theta = [R^+, R^-, M^+, M^-, \gamma] \quad (28)$$

### IV. ESTIMATION PROBLEM AND RESULTS

This section discusses the estimation problem and the results of applying the non-linear estimation strategies for the purposes of estimating the Li-Ion state of charge.

#### A. Problem Setup

In this paper, the simulation data was collected from AVL CRUISE software. This software is used to mimic a real-time scenario for vehicles and power-trains modeling. The vehicle simulation model was subjected to an urban dynamometer driving schedule (UDDS) cycle. The vehicle velocity profile for the UDDS cycle is shown in Fig. 2, and the corresponding battery current profile is shown in Fig. 3. The main parameters of interest include the terminal voltage (measurement) and the state of charge (SOC).

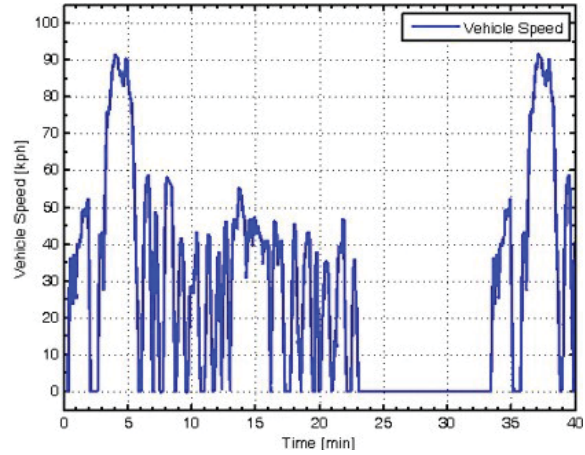


Fig. 2 The UDDS velocity profile

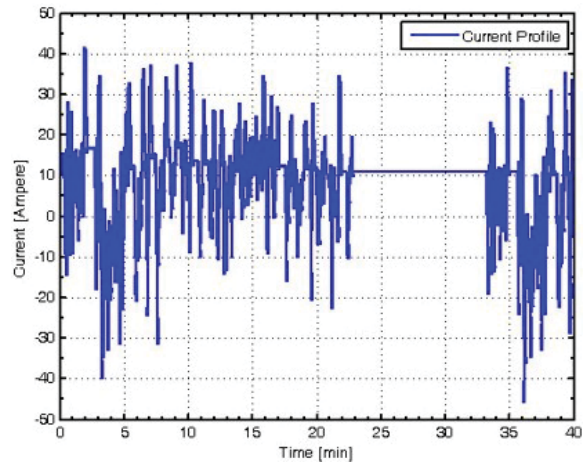


Fig. 3 The UDDS current profile

The system and measurement noise covariance matrices used by the EKF, and VBL-SVSF for state estimation are defined as follows:

$$Q = \text{diag}([0.05 \quad 5 \times 10^{-3}]) \quad (29)$$

$$R = 0.1 \quad (30)$$

The SVSF memory or forgetting factor is set as  $\gamma = 0.3$ , and the fixed smoothing boundary layer width are set to  $\psi = 2$  for the state estimates. These values were selected based on designer knowledge of Q and R, and also by trial-and-error, in an effort to improve the estimation accuracy. Note also that the sample rate of the simulation is  $\delta T = 100$  milliseconds.

#### B. Estimation Results

The three filters (EKF, SVSF, and VBL-SVSF) are used to estimate the state of charge for the model discussed in Section II.

Fig. 4 illustrates the SOC estimation results for the three filters. The true SOC is a result of coulomb counting that is used as the basis to compare the estimation performance for the filters. As it is shown in Fig. 4 the VBL-SVSF has the

best performance when compared to other filters in estimating the SOC followed by SVSF and EKF. Particularly, the better performance is more noticeable after the idle interval between minutes 22 and 33. This improvement in SOC estimation is significant because it solves the SOC estimation accuracy problem for the battery systems of vehicles after they start from a stop or idle condition.

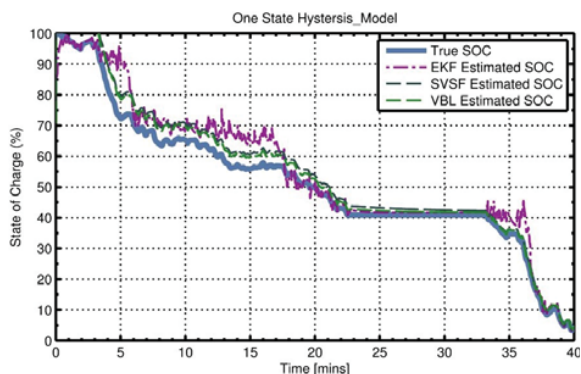


Fig. 4 State of charge estimation using the three filters

Fig. 5 displays the corresponding terminal voltage estimates using the three filters. All filters are capable of providing estimates of terminal voltage.

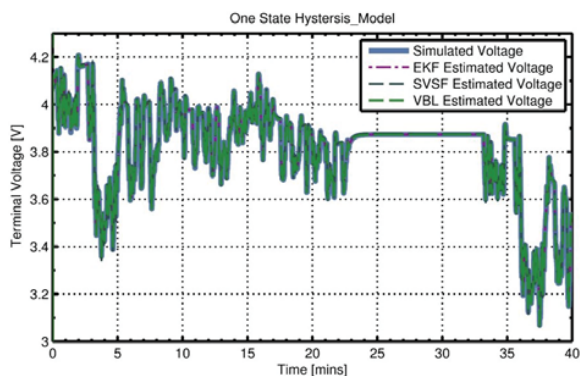


Fig. 5 Battery terminal voltage estimation using the three filters

For better comparison, Fig. 6 shows the root mean squared error (RMSE) in the state of charge calculated for three estimators. The VBL-SVSF provides the best estimation accuracy followed by the SVSF and EKF. The superior performance of SVSF based estimation comes from the specific design of the corrective gain in this strategy that makes it more robust to modeling uncertainties. Furthermore, VBL-SVSF inherits the robustness from SVSF and estimation accuracy from its optimal gain design.

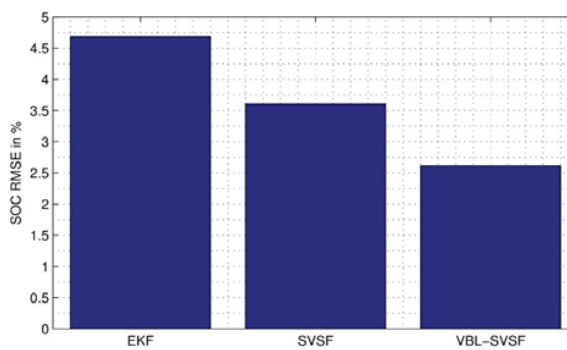


Fig. 6 Terminal voltage RMSE using the three filters

## V. CONCLUSION

This paper presented the results of applying three nonlinear estimation strategies on a Li-Ion battery model. The one state hysteresis model was used as a standard benchmark for three filters: the extended Kalman filter (EKF), the smooth variable structure filter (SVSF), and the time-varying smoothing boundary layer formulation of the SVSF (VBL-SVSF). It was found that the VBL-SVSF yielded the best results in terms of the SOC estimation accuracy. This was to be expected based on the formulation of the gain.

Future work will involve applying additional filtering strategies, such as the unscented Kalman filter (UKF), cubature Kalman filter (CKF), and the particle filter (PF). In addition, a number of other popular battery models will be studied; such as behavioural, equivalent circuit, and electrochemical models.

## NOMENCLATURE

Symbol	Description	Unit
$x$	State vector or values	—
$z$	Measurement (system output) vector or values	—
$w$	System noise vector	—
$v$	Measurement noise vector	—
$F$	Linearized system transition matrix	—
$H$	Linearized measurement (output) matrix	—
$A$	SVSF error vector (or matrix)	—
$K$	Filter gain matrix	—
$P$	State error covariance matrix	—
$Q$	System noise covariance matrix	—
$S$	Innovation covariance matrix	—
$e_z$	Measurement (output) error vector	—
$\gamma$	SVSF memory or convergence rate	—
$\psi$	SVSF smoothing boundary layer	—
$\text{diag}[a]$ or $\bar{a}$	Diagonal of some vector or matrix $a$	—
$\text{sat}()$	Saturation function	—
$ a $	Absolute value of $a$	—
$\bar{a}$	Diagonal matrix of some vector $a$	—
$T$	Transpose of a vector	—
$+$	Pseudoinverse of some non-square matrix	—
$\circ$	Denotes a Schur product	—

## APPENDIX

TABLE I  
A LIST OF THE MODEL PARAMETERS

Parameter	Value	Unit
$R^+$	0.0022	$\Omega$
$R^-$	0.0018	$\Omega$
$M^-$	0.0105	—
$M^+$	-0.016	—
$\gamma$	0.1	—

## REFERENCES

- [1] M. Farag, M. Fleckenstein, and S. Habibi, "Continuous piecewise-linear, reduced-order electrochemical model for lithium-ion batteries in real-time applications," *Journal of Power Sources*, vol. 342, pp. 351–362, feb 2017.
- [2] B. Bhangu, P. Bentley, D. Stone, and C. Bingham, "Nonlinear observers for predicting state-of-charge and state-of-health of lead-acid batteries for hybrid-electric vehicles," *IEEE Trans. Veh. Technol.*, vol. 54, no. 3, pp. 783–794, may 2005.
- [3] A. Vasebi, S. Bathaee, and M. Partovibakhsh, "Predicting state of charge of lead-acid batteries for hybrid electric vehicles by extended kalman filter," *Energy Conversion and Management*, vol. 49, no. 1, pp. 75–82, jan 2008.
- [4] T. Okoshi, K. Yamada, T. Hirasawa, and A. Emori, "Battery condition monitoring (BCM) technologies about lead-acid batteries," *Journal of Power Sources*, vol. 158, no. 2, pp. 874–878, aug 2006.
- [5] G. L. Plett, "Extended kalman filtering for battery management systems of lipb-based hev battery packs: Part 1. background," *Journal of Power sources*, vol. 134, no. 2, pp. 252–261, 2004.
- [6] —, "Extended kalman filtering for battery management systems of lipb-based hev battery packs: Part 2. modeling and identification," *Journal of power sources*, vol. 134, no. 2, pp. 262–276, 2004.
- [7] —, "Extended kalman filtering for battery management systems of lipb-based hev battery packs: Part 3. state and parameter estimation," *Journal of power sources*, vol. 134, no. 2, pp. 277–292, 2004.
- [8] M. Farag, S. Gadsden, S. Habibi, and J. Tjong, "A comparative study of li-ion battery models and nonlinear dual estimation strategies," in *2012 IEEE Transportation electrification conference and expo (ITEC)*. IEEE, 2012, pp. 1–8.
- [9] M. Farag, M. Fleckenstein, and S. R. Habibi, "Li-ion battery SOC estimation using non-linear estimation strategies based on equivalent circuit models," in *SAE Technical Paper Series*. SAE International, apr 2014.
- [10] X. Hu, S. Li, and H. Peng, "A comparative study of equivalent circuit models for li-ion batteries," *Journal of Power Sources*, vol. 198, pp. 359–367, 2012.
- [11] J. Kim, S. Lee, and B. H. Cho, "Complementary cooperation algorithm based on DEKF combined with pattern recognition for SOC/capacity estimation and SOH prediction," *IEEE Transactions on Power Electronics*, vol. 27, no. 1, pp. 436–451, jan 2012.
- [12] B. D. O. Anderson, J. B. Moore, and M. Eslami, "Optimal filtering," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 12, no. 2, pp. 235–236, 1982.
- [13] W. L. Brogan, "Applied optimal estimation (arthur gels, ed.)," *SIAM Rev.*, vol. 19, no. 1, pp. 172–175, jan 1977.
- [14] M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice with MATLAB*. JOHN WILEY & SONS INC, 2014. (Online). Available: [http://www.ebook.de/de/product/23151381/mohinder\\_s\\_grewal\\_angus\\_p\\_andrews\\_kalman\\_filtering\\_theory\\_and\\_practice\\_with\\_matlab.html](http://www.ebook.de/de/product/23151381/mohinder_s_grewal_angus_p_andrews_kalman_filtering_theory_and_practice_with_matlab.html)
- [15] S. Habibi, "The smooth variable structure filter," *Proceedings of the IEEE*, vol. 95, no. 5, pp. 1026–1059, may 2007.
- [16] M. A. Al-Shabi, S. A. Gadsden, and S. R. Habibi, "The toeplitz-observability smooth variable structure filter," in *2013 IEEE Jordan Conference on Applied Electrical Engineering and Computing Technologies (AEECT)*. Institute of Electrical & Electronics Engineers (IEEE), dec 2013.
- [17] S. A. Gadsden and S. R. Habibi, "A new robust filtering strategy for linear systems," *J. Dyn. Sys., Meas., Control*, vol. 135, no. 1, p. 014503, oct 2012.