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Leader-following Consensus Criterion for Multi-agent Systems with Probabilistic Self-delay

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Abstract—This paper proposes a delay-dependent leader-following consensus condition of multi-agent systems with both communication delay and probabilistic self-delay. The proposed methods employ a suitable piecewise Lyapunov-Krasovskii functional and the average dwell time approach. New consensus criterion for the systems are established in terms of linear matrix inequalities (LMIs) which can be easily solved by various effective optimization algorithms. Numerical example showed that the proposed method is effective.

Keywords—Multi-agent systems; probabilistic self-delay; consensus; Lyapunov method; LMI.

I. Introduction

MULTI-AGENT systems (MASs) have received considerable attentions due to their extensive applications in many fields such as biology, physics, robotics, power grid, and so on [1]-[2]. A prime concern in these systems is the agreement of a group of agents on their states of leader by interaction. Namely, this problem is a consensus problem. Specially, consensus problem with a leader is called a leader-following consensus problem or consensus regulation. Recently, this problem has been applied in various fields such as vehicle systems [3]-[4], groups of mobile autonomous agent [5], networked control systems [6], other applications [7]-[8].

MASs are being put to use in the consensus problem for time-delay which occurs due to the finite speed of information processing in the implementation of this system. Here, it is well known that time-delay often causes undesirable dynamic behaviors such as oscillation, performance degradation, and instability of the system. Thus, it is necessary to study the problems for MASs with time-delay. For instance, see [9]-[12]. However, the above mentioned literature mainly have addressed for the consensus conditions of the MASs with only communication delay. In implementation of many practical systems such as aircraft and electric circuits, there exist occasionally stochastic perturbations. The perturbations have influence on the stochastic occurrence of self-delay. It is no less important than the communication delay as a considerable factor affecting dynamics in the fields of network science and communication systems applications. Therefore, it should be pointed out that analyzing the consensus problem of the MASs with probabilistic self-delay can be regarded as investigating the stability of MASs.

Motivated by this mentioned above, in this paper, new delay-dependent consensus problem for MASs with both communication delay and probabilistic self-delay will be studied. The probabilistic self-delay is randomly occurring self-delay with Bernoulli sequence. Also, delay-dependent analysis has been paid more attention than delay-independent one because the sufficient conditions for delay-dependent analysis make use of the information on the size of time delay [13]. By construction of a suitable piecewise Lyapunov-Krasovskii functional and utilization of the average dwell time approach [14], a consensus criterion for MASs with both communication delay and probabilistic self-delay is derived in terms of LMIs which can be solved efficiently by use of standard convex optimization algorithms such as interior-point methods [15]. Using the average dwell time approach can be analyzed for the problem without the mathematical expectation operator $\mathbb{E}\{\cdot\}$ used in [16]-[17]. One numerical example is included to show the effectiveness of the proposed method.

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Notation: $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. For symmetric matrices X and Y, $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, nonnegative). $X^{\perp}$ denotes a basis for the null-space of X. I_n, 0_n and $0_{m\cdot n}$ denote $n \times n$ identity matrix, $n \times n$ and $m \times n$ zero matrices, respectively. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix norm. $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. $\otimes$ denotes the notation of Kronecker product.

II. Problem Statements

The interaction topology of a network of agents is represented using a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the set of nodes $\mathcal{V} = \{1, 2, \ldots, N\}$ and edges $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subset \mathcal{V} \times \mathcal{V}$. An adjacency matrix $\mathcal{A} = [a_{ij}]_{N\times N}$ of the digraph $\mathcal{G}$ is the matrix with nonnegative elements satisfying $a_{ii} = 0$ and $a_{ij} \geq 0$. If there is an edge between i and j, then the elements of matrix A described as $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$. The digraph $\mathcal{G}$ is said to be undirected if $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$ and $a_{ij} = a_{ji}$. A set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. A degree of node i is denoted by $deg(i) = \sum_{j\in\mathcal{N}_i} a_{ij}$. A degree matrix of digraph $\mathcal{G}$ is diagonal matrix defined as $\mathcal{D} = diag\{deg(1), \ldots, deg(N)\}$. The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. More details can be seen in [18].

Consider the following MASs with the dynamics of agent i

$$\dot{p}_i(t) = u_i(t) \ (i \in \mathcal{V} = 1, 2, \ldots, N), \tag{1}$$

where N is the number of agents, n is the number of states of agent i, $p_i(t) = [p_{i1}(t), \ldots, p_{in}(t)]^T \in \mathbb{R}^n$ and $u_i(t) = [u_{i1}(t), \ldots, u_{in}(t)]^T \in \mathbb{R}^n$ are the state vector and the consensus protocol vector of agent i, respectively.
According to the works [19], a consensus algorithm in agent can be described as

$$u_i(t) = -K \sum_{j \in \mathcal{N}_i} a_{ij}(p_i(t) - p_j(t)) - Kb_i(p_i(t) - p_0), \tag{2}$$

where $K = [k_{ij}] \in \mathbb{R}^{n \times n}$ is a protocol gain matrix, $p_0 \in \mathbb{R}^n$ is the state vector of leader, a_{ij} and b_i are the interconnection weights defining

$$\begin{cases} a_{ij} > 0, & \text{if agent } i \text{ is connected to agent } j, \\ a_{ij} = 0, & \text{otherwise,} \end{cases}$$

$$\begin{cases} b_i > 0, & \text{if agent } i \text{ is connected to the leader,} \\ b_i = 0, & \text{otherwise.} \end{cases}$$

Remark 1: The agents of the MASs (1) with the consensus algorithm (2) can be known in the sense that each agent needs information from its local neighborhood. In practice, N-unmanned vehicles or N-soccer robots move in a n-dimensional plan with identical first-order dynamics.

Assumption 1: The self delay is randomly occurring. This mean that, ϱ_t is a stochastic process representing the self delay occurring process; that is, let ϱ_t be a Bernoulli distributed sequence defined by

$$\varrho_t = \begin{cases} 1, & \text{if the self-delay occurs,} \\ 0, & \text{if the self-delay not occurs.} \end{cases}$$

From understanding the algorithm (2) and Assumption 1, a consensus algorithm with randomly occurring self-delay can be

$$\begin{aligned} u_i(t) = &-K_{\varrho_t} \sum_{j \in \mathcal{N}_i} a_{ij}(p_i(t - \varrho_t h) - p_j(t-h)) \\ &-K_{\varrho_t} b_i(p_i(t) - p_0), \end{aligned} \tag{3}$$

where $0 < h$ is a time-invariant delay.
Let us define $x_i(t) = p_i(t) - p_0$. From (1) and (3), since the following equality with $\dot{p}_0 = 0$ holds

$$\begin{aligned} \dot{x}_i(t) &= \dot{p}_i(t) = u_i(t) \\ &= -K_{\varrho_t} \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(t - \varrho_t h) - x_j(t-h)) \\ &\quad -K_{\varrho_t} b_i x_i(t), \end{aligned}$$

the MASs with the error dynamics of agent i can be rewritten as the matrix form by

$$\begin{aligned} \dot{x}(t) = &-(\mathcal{B} \otimes K_{\varrho_t})x(t) + (\mathcal{A} \otimes K_{\varrho_t})\, x(t-h) \\ &-(\mathcal{D} \otimes K_{\varrho_t})x(t - \varrho_t h), \end{aligned} \tag{4}$$

where

$$\mathcal{A} = [a_{ij}]_{N \times N}, \ \mathcal{B} = diag\{b_1, \ldots, b_N\},$$
$$\mathcal{D} = diag\left\{ \sum_{j \in \mathcal{N}_1} a_{1j}, \ldots, \sum_{j \in \mathcal{N}_N} a_{Nj} \right\}.$$

For each ϱ_t, the system (4) can be rewritten as the following switched systems by

$$\dot{x}(t) = F_{\sigma(t)}x(t) + G_{\sigma(t)}x(t-h), \tag{5}$$

where $\sigma(t) : [0, \infty) \to L = \{1, 2, \ldots, l\}$ is the switching signal which in deterministic, piecewise constant and right continuous, corresponding to it, the switching sequence $\{x(t_0); (i_0, t_0), \ldots, (i_l, t_l), \ldots, | i_l \in L, l = 1, 2, \ldots\}$, which means that the i_lth subsystem is activated when $t \in [t_l, t_{l+1})$, and

$$\begin{aligned} &F_1 = -(\mathcal{B} \otimes K_1), \ G_1 = -(\mathcal{L} \otimes K_1) \ \text{ if } \varrho_t = 1, \\ &F_2 = -((\mathcal{D} + \mathcal{B}) \otimes K_2), \ G_2 = (\mathcal{A} \otimes K_2) \ \text{ if } \varrho_t = 0. \end{aligned}$$

The aim of this paper is to investigate the consensus analysis (in other word, stability analysis) of the MASs (5). This means that the protocol $u_i(t)$ solves the consensus problem, if and only if the states of agents satisfy

$$\lim_{t \to \infty} \|p_i(t) - p_0\| = \lim_{t \to \infty} \|x_i(t)\| = 0, \quad \forall i \in \mathcal{V}.$$

In order to do this, we introduce the following definitions and lemma.

Definition 1: System (5) is said to be asymptotically stable under switching signal $\sigma(t)$ if the solution $x(t)$ of system (5) satisfies

$$\|x(t)\| \le \alpha \|x(t_0)\|_{c1}, \quad \forall t \ge t_0, \tag{6}$$

for $\alpha \ge 1$, where $\|x(t_0)\|_{c1} = \sup_{-h \le s \le 0}\{\|x(t_0 + s)\|, \|\dot{x}(t_0 + s)\|\}$.

Definition 2: [14] For any $T > t \ge 0$, let $N_\sigma(t, T)$ denote the switching number of σ on an interval (t, T), if

$$N_\sigma(t, T) \le N_0 + \frac{T - t}{\tau_a} \tag{7}$$

holds for given $N_0 \le 0$ and $\tau_a > 0$. Then the constant τ_a is called the average dwell time and N_0 is the chatter bound. Without loss of generality, we choose $N_0 = 0$ in this paper.

Lemma 1 (Finsler lemma): [20] Let $\zeta \in \mathbb{R}^n$, $\Phi = \Phi^T \in \mathbb{R}^{n \times n}$, and $\Upsilon \in \mathbb{R}^{m \times n}$ such that $rank(\Upsilon) < n$. The following statements are equivalent:
(i) $\zeta^T \Phi \zeta < 0$, $\forall \Upsilon \zeta = 0$, $\zeta \neq 0$,
(ii) ${\Upsilon^{\perp}}^T \Phi \Upsilon^{\perp} < 0$,
(iii) $\exists \mathcal{F} \in \mathbb{R}^{n \times m} : \Phi + \mathcal{F}\Upsilon + \Upsilon^T \mathcal{F}^T < 0$.

III. Main Results

In this section, we propose new stability and stabilization criteria for system (5). For simplicity of matrix representation, $e_i \in \mathbb{R}^{3Nn \times Nn}$ are defined as block entry matrices; e.g.,

$e_2 = [0_{Nn}, I_{Nn}, 0_{Nn}]^T$. The notations of several matrices are defined as:

$$\begin{aligned}\zeta(t) &= [x^T(t), x^T(t-h), \dot{x}^T(t)]^T,\\ \Upsilon_1 &= [-\mathcal{B}\otimes K_1, -\mathcal{L}\otimes K_1, -I_{Nn}],\\ \Upsilon_2 &= [-(\mathcal{D}+\mathcal{B})\otimes K_2, \mathcal{A}\otimes K_2, -I_{Nn}],\\ \Xi_1 &= sym\{e_1(I_N\otimes P)e_3^T\}\\ &\quad +e_1(I_N\otimes Q)e_1^T - e_2(I_N\otimes Q)e_2^T\\ &\quad +e_3(I_N\otimes h^2R)e_3^T - (e_1-e_2)(I_N\otimes R)(e_1-e_2)^T\\ \Xi_2 &= e_1(I_N\otimes hS_1)e_1^T + e_3(I_N\otimes hS_2)e_3^T\\ &\quad +e_1(I_N\otimes M)e_1^T - e_2(I_N\otimes M)e_2^T,\\ \Phi &= \Xi_1+\Xi_2. \end{aligned} \tag{8}$$

Now, we have the following Theorem 1.

Theorem 1: For given scalars $0 < h$ and the gain K_l, the agents in the system (5) converge to the state of leader, if there exist positive definite matrices $P \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{n\times n}$, $S_a \in \mathbb{R}^{n\times n}(a = 1, 2)$ and any symmetric matrix $M \in \mathbb{R}^{n\times n}$, satisfying the following LMIs:

$$[\Upsilon_l^{\perp}]^T \Phi [\Upsilon_l^{\perp}] < 0 \ (l = 1, 2), \tag{9}$$

$$\left[\begin{array}{c|c} I_N\otimes S_1 & I_N\otimes M\\ \hline \star & I_N\otimes S_2\end{array}\right] > 0. \tag{10}$$

Then, system (5) is asymptotically stable for switching signal with occurrence probability, $\Pr\{\varrho_t\} = \varrho_0$, of self-delay and average dwell time satisfying $\tau_a > 0$.

Proof: Let us consider the following Lyapunov-Krasovskii functional candidate as

$$V(x(t)) = V_1 + V_2, \tag{11}$$

where

$$\begin{aligned} V_1 &= x(t)(I_N\otimes P)x(t)\\ &\quad +\int_{t-h}^{t} x^T(s)(I_N\otimes Q)x(s)ds\\ &\quad +h\int_{t-h}^{t}\int_{s}^{t}\dot{x}^T(u)(I_N\otimes R)\dot{x}(u)duds,\\ V_2 &= \int_{t-h}^{t}\int_{s}^{t}(x^T(u)(I_N\otimes S_1)x(u)\\ &\quad +\dot{x}^T(u)(I_N\otimes S_2)\dot{x}(u))duds. \end{aligned}$$

By Jensen inequality [21], the time-derivative of V_1 is calculated as

$$\begin{aligned} \dot{V}_1 &= 2x^T(t)(I_N\otimes P)\dot{x}(t) + x^T(t)(I_N\otimes Q)x(t)\\ &\quad -x^T(t-h)(I_N\otimes Q)x(t-h)\\ &\quad +\dot{x}^T(t)(I_N\otimes(h^2R))\dot{x}(t)\\ &\quad -\left(\int_{t-h}^{t}\dot{x}(s)ds\right)^T (I_N\otimes R_1)\left(\int_{t-h}^{t}\dot{x}(s)ds\right)\\ &\le \zeta^T(t)\Xi_1\zeta(t). \end{aligned} \tag{12}$$

Next, the $\dot{V}_2$ is calculated as

$$\begin{aligned} \dot{V}_2 &= x^T(t)(hI_N\otimes S_1)x(t) + \dot{x}^T(t)(hI_N\otimes S_2)\dot{x}(t)\\ &\quad -\int_{t-h}^{t}\xi^T(s)\left[\begin{array}{c|c} I_N\otimes S_1 & 0_{Nn}\\ \hline \star & I_N\otimes S_2\end{array}\right]\xi(s)ds. \end{aligned} \tag{13}$$

where $\xi(t) = [x^T(t), \dot{x}^T(t)]^T$.

Inspired by the work of [22], the following zero equality with any symmetric matrix M is considered

$$\begin{aligned} 0 &= x^T(t)(I_N\otimes M)x(t) - x^T(t-h)(I_N\otimes M)x(t-h)\\ &\quad -2\int_{t-h}^{t} x^T(s)(I_N\otimes M)\dot{x}(s)ds. \end{aligned} \tag{14}$$

By use of Eqs. (14), we get

$$\begin{aligned} \dot{V}_2 &= \zeta^T(t)\Xi_2\zeta(t)\\ &\quad -\int_{t-h}^{t}\xi^T(s)\left[\begin{array}{c|c} I_N\otimes S_1 & I_N\otimes M\\ \hline \star & I_N\otimes S_2\end{array}\right]\xi(s)ds. \end{aligned} \tag{15}$$

Here, if the inequality (10) holds, then the $\dot{V}$ has an upper bound as follows

$$\dot{V} \le \zeta^T(t)(\Xi_1+\Xi_2)\zeta(t). \tag{16}$$

Also, the system (5) with the augmented vector $\zeta(t)$ can be rewritten as $\Upsilon_l\zeta(t) = 0$ $(l = 1, 2)$. Then, a consensus condition for the system (5) is

$$\zeta^T(t)(\Xi_1+\Xi_2)\zeta(t) < 0 \tag{17}$$

subject to $\Upsilon_l\zeta(t) = 0$.

From Lemma 1 (iii), the inequality (17) is equivalent to the following condition

$$(\Xi_1+\Xi_2) + \mathcal{F}\Upsilon_l + (\mathcal{F}\Upsilon_l)^T < 0 \ (l = 1, 2), \tag{18}$$

where $\mathcal{F}$ is any matrix with appropriate dimension.

According to the works [23]-[24], define the following piecewise Lyapunov-Krasovskii functional candidate as

$$V(x(t)) = V_{\sigma(t)}(x(t)). \tag{19}$$

When $t \in [t_l, t_{l+1})$, (10) and (18) give

$$V(x(t)) = V_{\sigma(t)}(x(t)) \le V_{\sigma(t_l)}(x(t_l)). \tag{20}$$

Using (19), at the switching instant t_l, we get

$$V_{\sigma(t_l)}(x(t_l)) \le V_{\sigma(t_l^-)}(x(t_l^-)). \tag{21}$$

Therefore, from (20), (21) and the relation $l = N_\sigma(t_0, t)$, it follows

$$V(x(t)) \le V_{\sigma(t_l^-)}(x(t_l^-)) \le V_{\sigma(t_0)}(x(t_0)). \tag{22}$$

Furthermore, for any $t \in [0, h]$, from (19), the following inequalities hold

$$a\|x(t)\|^2 \le V(x(t)) \le V_{\sigma(t_0)}(t_0) \le b\|x(t_0)\|_{c1}^2, \tag{23}$$

where

$$\begin{aligned} a &= \lambda_{min}\{P\},\\ b &= \lambda_{max}\{P\} + h\lambda_{max}\{Q\} + \frac{h^3}{2}\lambda_{max}\{R\}\\ &\quad +\frac{h^2}{2}\Big(\lambda_{max}\{S_1\} + \lambda_{max}\{S_2\}\Big), \end{aligned}$$

which means

$$\|x(t)\| \le \sqrt{\frac{b}{a}}\|x(t_0)\|_{c1}. \tag{24}$$

By Definition 1, we know that system (5) is asymptotically stable.
Finally, by utilizing Lemma 1 (ii), the condition (18) is equivalent to the following inequality

$$[\Upsilon_l^{\perp}]^T(\Xi_1+\Xi_2)[\Upsilon_l^{\perp}] < 0 \ (l = 1, 2). \qquad (25)$$

From the inequality (25), if the LMIs (9) and (10) hold, then stability condition (17) is satisfied. This completes our proof.

IV. Numerical Examples

In this section, two numerical examples will be shown to illustrate the effectiveness of the proposed Theorem 2.

Example 1: Consider the MASs (1) with 4-vehicles (agents) in 2-dimensional plan, i.e., $N = 4$, $n = 2$, and the switching interconnection topology described in Figure 1.

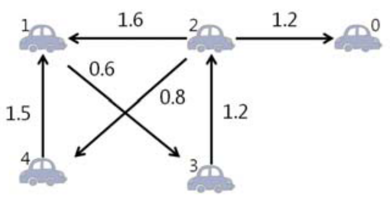

Fig. 1. The interconnection topology

From Figure 1, $\mathcal{A}$, $\mathcal{D}$ and $\mathcal{B}$ are

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0.6 & 0 \\ 1.6 & 0 & 0 & 0.8 \\ 0 & 1.2 & 0 & 0 \\ 1.5 & 0 & 0 & 0 \end{bmatrix},$$
$$\mathcal{D} = diag\{0.6, 2.4, 1.2, 1.5\},$$
$$\mathcal{B} = diag\{0, 1.2, 0, 0\}.$$

The occurrence probability, ϱ_0, of self-delay is 0.7. Then, the operation modes of switching signal with $\tau_a = 1$ are shown in Figure 2.

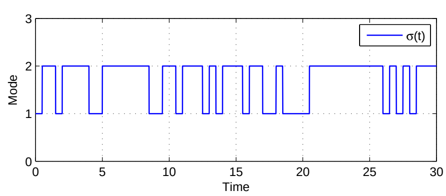

Fig. 2. The curve of the operation modes. (Example 1)

For the system mentioned above, the maximum bound of time-delay with the following consensus protocol gain K_l by Theorem 1 are 0.5.

$$K_1 = diag\{1, 1\}, \ K_2 = diag\{0.0001, 0.0001\} \times 10^{-320}.$$

In order to confirm this result, we set the condition on time-delay as (C1) $h = 0.5$. It assumed that the state of leader set by $z_0 = [2, -2]^T$. Then, the simulation results are given in Figure 3. These figures show that the each agent with the responses converge to the state of leader under the switching signal $\sigma(t)$ for given initial states of the agents $z_1(0) = [3, -1]^T$, $z_2(0) = [1, -1]^T$, $z_3(0) = [1, -3]^T$ and $z_4(0) = [3, -3]^T$.

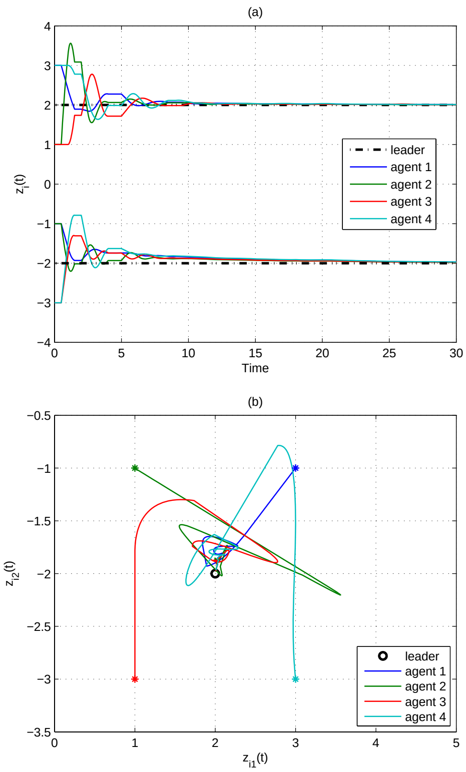

Fig. 3. State trajectories of the agents and leader with the condition (C1): (a) Time-$z_i(t)$ plan; (b) $z_{i1}(t)$-$z_{i2}(t)$ plan. (Example 1)

V. Conclusion

In this paper, new delay-dependent leader-following consensus criterion for MASs with both communication delay and probabilistic self-delay is proposed. To do this, a suitable piecewise Lyapunov-Krasovskii functional and the average dwell time approach are used to obtain the feasible region of consensus criterion. Numerical example has been given to show the effectiveness of the presented criterion.

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