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Large-Dimensional Shells under Mining Tremors from Various Mining Regions in Poland

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***Abstract*—**In the paper a detailed analysis of the dynamic response of a cooling tower shell to mining tremors originated from two main regions of mining activity in Poland (Upper Silesian Coal Basin and Legnica-Glogow Copper District) was presented. The representative time histories registered in the both regions were used as ground motion data in calculations of the dynamic response of the structure. It was proved that the dynamic response of the shell is strongly dependent not only on the level of vibration amplitudes but on the dominant frequency range of the mining shock typical for the mining region as well. Also a vertical component of vibrations occurred to have considerable influence on the total dynamic response of the shell. Finally, it turned out that non-uniformity of kinematic excitation resulting from spatial variety of ground motion plays a significant role in dynamic analysis of large-dimensional shells under mining shocks.

***Keywords*—**Cooling towers, dynamic response, mining tremors, non-uniform kinematic excitation

I. Introduction

NATURAL seismic events in Poland, which is located in a low seismicity zone, occur rarely and their energy is relatively low. But there is an urgent need to protect existed and designed engineering structures against mining tremors occurring in two main mining activity regions in Poland: the Upper Silesian Coal Basin and the Legnica-Glogow Copper District.

The evaluation of risk resulting from mining shocks to surface structures is complex and became a task of recent studies in Poland [1]-[4]. Most papers concern influences of mining tremors on residential buildings. The recognition of mining related influences on other engineering structures is still insufficient. In particular effects of mining shocks on large-dimensional structures are not well recognized. Typical spatially spread structures which are commonly erected in the industrial mining areas are cooling towers.

In calculations of dynamic response of these structure to mining tremor several factors have to be considered. Energy of shocks and level of vibration amplitudes are, of course, major factors which have an essential influence on the dynamic response of the shell, but dominant frequencies of mining events are of a great interest as well.

Also vertical component of vibrations which is usually neglected in seismic analysis may cause significant increase of internal forces as far as mining tremors are considered. Eventually, spatial variety of kinematic excitation should be taken into account. In typical analysis of dynamic response of a structure to kinematic excitation, spatial variation of ground motion is commonly neglected although mining shocks present high variability in space. In practice it means that calculations of dynamic response of a structure to shock are based on the assumption that movements of every point of the ground beneath the structure are identical. However, the influence of spatial variety of excitation on the dynamic response of cooling towers may be significant. These structures are exposed to spatially different ground vibrations, since dimensions of their foundations are comparable with the length of the wave propagating in ground.

The influence of non-uniform seismic excitation on the dynamic response of large-dimensional structures was considered in many papers [5]-[8]. Generally authors claim that dynamic response to non-uniform kinematic excitation is smaller than dynamic response to uniform excitation. The decrease of the dynamic response is caused by reduction of average amplitudes of kinematic excitation. On the other hand, the authors mention that occurrence of quasi-static effects that result from differences in excitation in particular points of foundations may lead to an increased global response.

This paper presents complex evaluation of the influence of mining tremors from two main mining regions in Poland on the dynamic response of the cooling tower shell. In the analysis dominant frequencies of mining tremors, effects of taking vertical component into consideration as well as non-uniformity of kinematic excitation were taken into account.

II. Model of Non-uniform Kinematic Excitation

The equation of motion of a general multi-degree of freedom structure under kinematic non-uniform excitation can be formulated as follows [9]:

$$\begin{bmatrix} M_{ss} & M_{sg} \\ M_{gs} & M_{gg} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_s^t \\ \ddot{u}_g \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sg} \\ C_{gs} & C_{gg} \end{bmatrix} \cdot \begin{Bmatrix} \dot{u}_s^t \\ \dot{u}_g \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sg} \\ K_{gs} & K_{gg} \end{bmatrix} \cdot \begin{Bmatrix} u_s^t \\ u_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_g \end{Bmatrix} \quad (1)$$

Where s, g – degrees of freedom of structure and ground respectively,

$[M]$, $[C]$, $[K]$ – mass, damping, stiffness matrix respectively,

$\{\ddot{u}_s^t\}$, $\{\dot{u}_s^t\}$, $\{u_s^t\}$ – vectors of total accelerations, velocities, and displacements of structure respectively,

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$\{\ddot{u}_g\}$, $\{\dot{u}_g\}$, $\{u_g\}$ – vectors of accelerations, velocities, and displacements of ground motion respectively, $\{F_g\}$ – vector of reaction forces.

Vector of total displacements of a structure $\{u_s^t\}$ (as well as vectors of total velocities and accelerations) consists of two parts: dynamic component $\{u_s^d\}$ and quasi-static component $\{u_s^p\}$, i.e.: $\{u_s^t\} = \{u_s^d\} + \{u_s^p\}$.

The quasi-static component equals:

$$\{u_s^p\} = [R] \cdot \{u_g\} \tag{2}$$

where: $[R] = -[K_{ss}^{-1}] \cdot [K_{sg}]$.

After taking into consideration (2) and assuming small damping, (1) is equivalent to:

$$\begin{gathered}[M_{ss}] \cdot \{\ddot{u}_s^d\} + [C_s^s] \cdot \{\dot{u}_s^d\} + [K_s^s] \cdot \{u_s^d\} = \\ \left(-[M_{ss}] \cdot [R] - [M_{sg}]\right) \cdot \{\ddot{u}_g\}\end{gathered} \tag{3}$$

Since dynamic response of a structure to kinematic excitation is obtained by numerical integration of (3), it depends on ground accelerations vector $\{\ddot{u}_g\}$. Individual components of this vector represent time histories of ground accelerations at particular supports of a structure. In mining activity regions in Poland time histories of accelerations are registered at selected points. So the application of the formula (3) requires additionally an assumption of a model of kinematic excitation. On the basis of this model ground accelerations at particular supports could be specified.

There are three phenomena responsible for non-uniformity of excitation [10]: wave passage effect (difference in time when a wave reaches various points of the structure foundation), incoherence effect (loss of coherence resulting from reflections and refractions of a wave), local soil effects (differences in ground conditions in particular points of subsoil beneath a structure). Authors enumerate also fourth reason which is responsible for non-uniformity of kinematic excitation. It is an attenuation effect – a decrease of vibration amplitudes caused by geometrical damping and energy wave absorbtion [11].

In this paper a model of non-uniform kinematic excitation was adopted. It was assumed that subsequent points of ground on the way of the wave propagation repeat the same movement with a certain time delay dependent on wave velocity. The decrease of vibration amplitudes resulting from increasing distance from epicentre was also taken into account. Neither loss of coherence nor changes of amplitudes with regard to different local ground conditions were taken into consideration. Hence, the model of non-uniform kinematic excitation required: time histories of vibrations registered at one point in three directions, wave velocity in the ground and a formula of the amplitude decrease with the increasing distance from the epicentre.

III. Characteristics of Mining Tremors from Various Mining Activity Regions in Poland

A. Data of Registered Mining Shocks

Fig. 1 represents three components of a representative mining shock registered in the Upper Silesian Coal Basin [3].

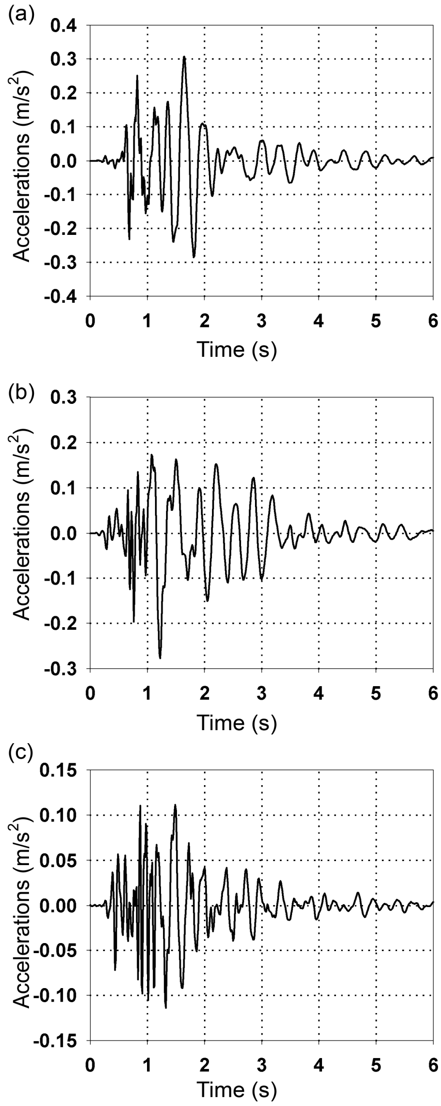

Fig. 1 Ground accelerations resulting from mining shock in Upper Silesian Coal Basin: (a) horizontal direction X, (b) horizontal direction Y, (c) vertical direction Z

Fig. 2 shows three components of a selected tremor registered in the Legnica-Glogow Copper District [2]. The energy of the tremor registered in the Upper Silesian Coal Basin was about $1 \cdot 10^7$ J; the tremor belonged to the strongest events ever measured in this region. The energy of the tremor in the Legnica-Glogow Copper District was about $5 \cdot 10^7$ J.

Hence, the energy of this event was 5 times bigger than the energy of the shock recorded in the Upper Silesian Coal Basin.

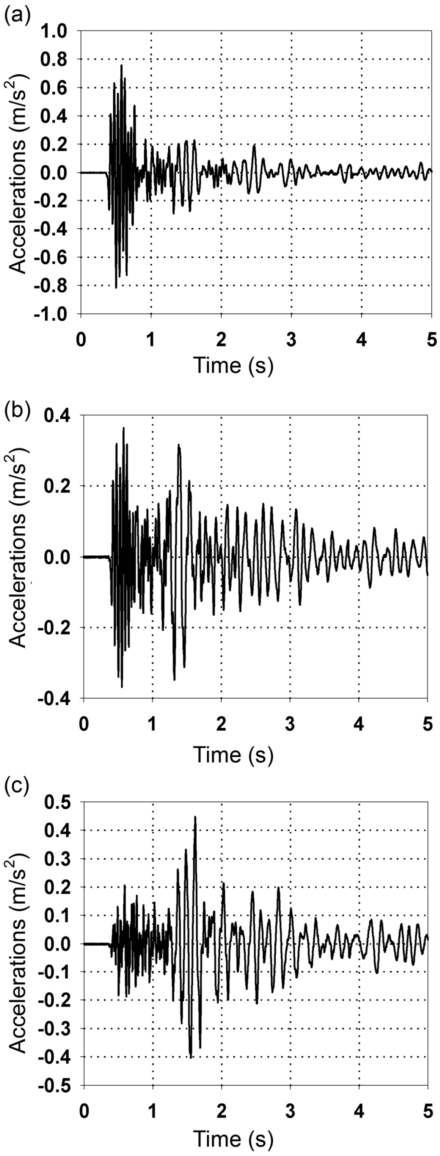

Fig. 2 Ground accelerations resulting from mining shock in Legnica-Glogow Copper District: (a) horizontal direction X, (b) horizontal direction Y, (c) vertical direction Z

B. Comparison of Components of Ground Motion

The first factor, the influence of which was analyzed in this studies, was the effect of vertical component of ground motion on the dynamic response of the structure to mining shock.

In case of calculations of dynamic response of structures to earthquake horizontal component of ground motion parallel to the direction of wave propagation plays central role. This component results from the Rayleigh wave propagation. Other components are usually found non-essential and they are rarely taken into account in seismic analyses.

In case of mining shocks the situation is different. As the epicenter of the shock is located relatively close to the analyzed structure different types of waves, i.e. P, S and surface waves, reach the structure at the same time. In typical time history of a mining shock registered in a short distance from the epicenter values of amplitudes in three directions are comparable. Vertical amplitudes of ground motion can even be bigger than horizontal components.

It could be observed in Figs 1 and 2 that the maximal amplitudes of accelerations in horizontal directions are greater than the maximal vertical amplitudes. But still the values of the vertical and horizontal component are comparable. Hence, all three components of ground vibrations resulting from the mining tremors have to be considered in the dynamic analysis.

C. Differences in Frequency Spectra

The dominant frequency contents are different for various mining regions in Poland [2].

Fig. 3 represents the frequency spectrum of the horizontal component in direction X of the mining shock from the Upper Silesian Coal Basin. The dominant frequencies of the shock in horizontal direction X are located in the range from 1.6 to 4.8 Hz with a noticeable peak at 3.5 Hz. In horizontal direction Y and in vertical direction Z a similar range of dominant frequencies can be observed.

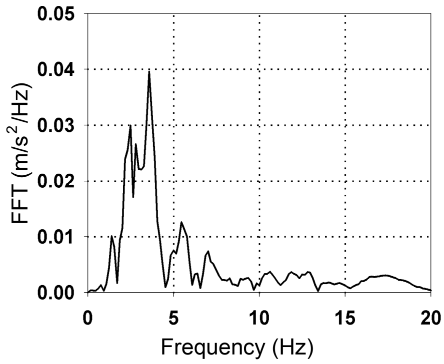

Fig. 3 Frequency spectrum of accelerations in horizontal direction X of mining shock in Upper Silesian Coal Basin

Fig. 4 shows the frequency spectrum of the horizontal component in direction X of the mining shock from the Legnica-Glogow Copper District. The amplitudes show maxima at the dominant frequencies of about 7 and 20 Hz.

The presented frequency spectra form two analyzed mining regions differ from each other as far as dominant frequencies are concerned. Dominant frequency range of mining tremors in the Upper Silesian Coal Basin generally consists of higher frequencies than in the Legnica-Glogow Copper District.

This means that similar engineering structures located in these particular mining regions could differ in dynamic responses to ground motions of similar values of maximal amplitudes. The difference may be caused by the frequency contents of the shocks.

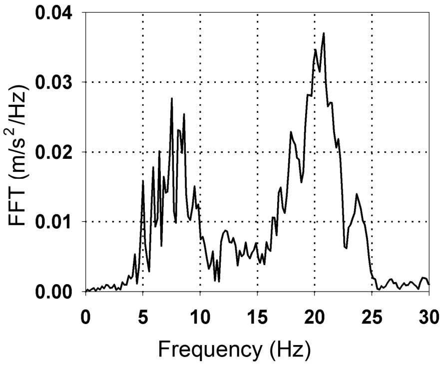

Fig. 4 Frequency spectrum of accelerations in horizontal direction X of mining shock in Legnica-Glogow Copper District

D. Spatial Variability of Ground Motion

The decrease of amplitudes with the increasing distance from the epicentre is more rapid in case of mining shocks than in case of seismic events. Hence the reduction of amplitudes is not negligible as far as tremors are concerned. In this studies the decrease of vibration amplitudes was adopted on the basis of empirical dependence of the level of vibration on the distance from epicenter. Such relations were elaborated by seismologists for the analyzed mining regions [2].

The empirical formula which describes the decrease of amplitudes with the distance from the epicentre is as follows:

$$a(r) = a_e \cdot H(r) \qquad (4)$$

where:

$a(r)$ – accelerations at a distance r,

a_e – accelerations in epicentre zone,

r – distance from epicentre, [km],

$H(r)$ – empirical function which describes the decrease of amplitudes which results from geometrical damping outside the epicentre zone:

– for the Upper Silesian Coal Basin:

$$H(r) = 1{,}53 \cdot r^{0{,}155} \cdot e^{-0{,}65 \cdot r} + 0{,}014 \qquad (5)$$

– for the Legnica-Glogow Copper District:

$$H(r) = 0{,}8575 \cdot r^{-1{,}0098} \qquad (6)$$

Fig. 5 presents charts of $H(r)$ function for the Upper Silesian Coal Basin and for the Legnica-Glogow Copper District. On the basis of known registered accelerations and taking advantage of (4), (5), and (6) values of amplitudes at any distance from the epicentre can be calculated.

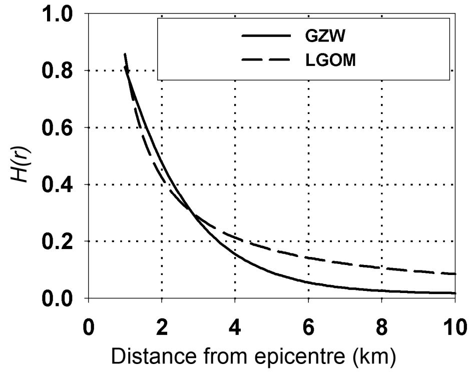

Fig. 5 Empirical function *H(r)* which served to calculate the decrease of vibration amplitudes with the distance from epicentre

IV. Data Of Analyzed Cooling Tower Shell

A. Numerical Model of the Structure

A detailed analysis of the dynamic response to mining shocks from the different mining regions of Poland was performed for a hiperboloidal cooling tower of a height of 130 m. This is a typical structure erected in industrial mining regions. Fig. 6 presents the main dimensions of the cooling tower. The thickness of the shell varies from 0.66 m at the lower edge and 0.26 m at the upper edge to 0.14 m in the middle of the shell. The shell is supported by V-shaped columns. A foundation ring is located on a very firm subsoil so soil-structure interaction was not taken into consideration. The material data of the structure are given in Table I.

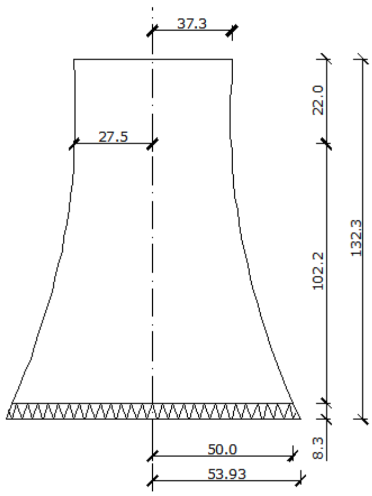

Fig. 6 Main dimensions of hiperboloidal cooling tower

TABLE I
Material Data of Cooling Tower

Part of the structure	Elasticity modulus [GN · m^{-2}]	Poisson ratio [-]	Mass density [kg · m^{-3}]
Shell	20	0.167	2500
Columns	23	0.167	2500
Foundation ring	23	0.167	2600

A finite element model of the cooling tower consisted of all parts of the structure, points selected for further dynamic analysis as well as internal force symbols are presented in Fig. 7. For calculations the ABAQUS program was used – a general-purpose system for calculations of engineering structures based on FEM.

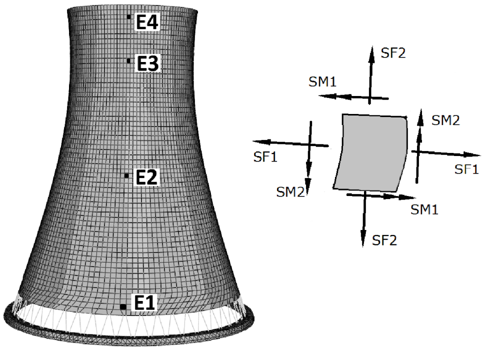

Fig. 7 Finite element model of the cooling tower, points selected for dynamic analysis and internal force symbols

B. Dynamic Characteristics of Cooling Tower

The evaluation of natural frequencies and modes of vibrations was the first step of dynamic analysis. Figs 8 and 9 show selected modes of vibrations. The mode of natural vibrations which occurs at the lowest frequency (see Fig. 8) is connected with the circumferential deformation of the shell.

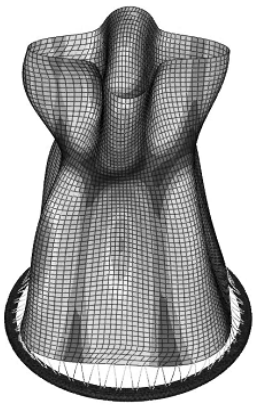

Fig. 8 Mode of natural vibrations, the lowest frequency 0.83 Hz

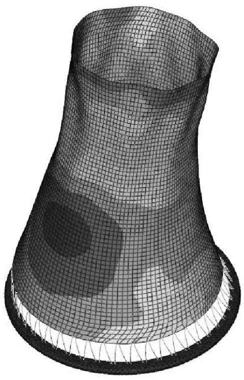

Fig. 9 Mode of natural vibrations, frequency 2.80 Hz

The mode of natural vibration shown in Fig. 9 is a basic mode which could be executed by horizontal kinematic excitation. Similar values of natural frequencies and modes of vibrations were obtained by other authors who analyzed large-dimensional cooling towers [12]-[14]. It should be pointed out that the natural frequency 2.80 Hz at which this mode occurs is located within the range of dominant frequencies of the mining shock from the Upper Silesian Coal Basin.

For further dynamic analysis a model of Rayleigh damping was adopted:

$$[C] = \alpha \cdot [M] + \beta \cdot [K] \tag{7}$$

Rayleigh damping coefficients α and β were determined from the following relations:

$$2\,\xi_1 = \frac{\alpha}{2\pi \cdot f_1} + \beta \cdot 2\pi \cdot f_1 \tag{8}$$

$$2\,\xi_2 = \frac{\alpha}{2\pi \cdot f_2} + \beta \cdot 2\pi \cdot f_2 \tag{9}$$

where ξ_1, ξ_2 are critical damping fractions referring to frequencies f_1 and f_2 respectively. The critical damping ratios ξ_1, ξ_2 were assumed 5%. As f_1 in (8) the first natural frequency of the shell equaled 0.83 Hz was assumed. As f_2 in (9) the natural frequency of the mode of vibrations shown in Fig. 9 equaled 2.80 Hz was specified.

V. Dynamic Response of Cooling Tower Shell to Selected Mining Shocks

A. Influence of Vertical Component of Kinematic Excitation

In order to evaluate the influence of the vertical component of the mining tremor on the dynamic response of the shell calculations for the excitation data registered at the Upper Silesian Coal Basin were performed. Figs 10 and 11 present comparisons of circumferential forces SF1 and moments SM2 respectively, caused by the horizontal components of the shock only and by the horizontal and vertical components.

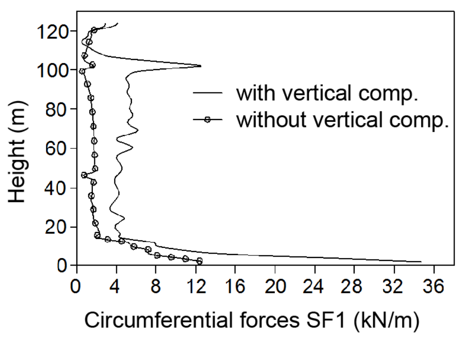

Fig. 10 Comparison of maximal circumferential forces SF1 caused by horizontal components only and by horizontal and vertical components

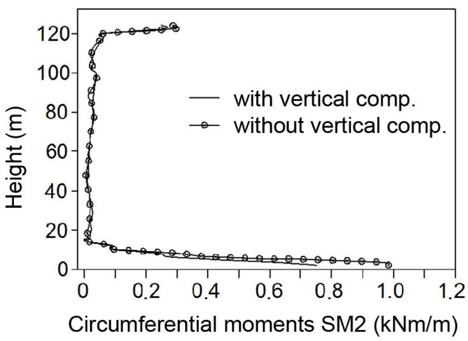

Fig. 11 Comparison of maximal circumferential moments SM2 caused by horizontal components only and by horizontal and vertical components

The comparisons presented in Figs 10 and 11 show that the internal forces and moments obtained in case of including the vertical component of the excitation were about 30% greater than in case of considering only horizontal components. At the lower edge zone the vertical component strengthen the boundary effect. Since the values of the vertical and horizontal components of the selected mining tremor are comparable all components of ground vibrations have to be considered in the dynamic analysis.

B. Influence of Dominant Frequencies of Mining Shock

For evaluation of the influence of dominant frequencies of the mining tremor on the dynamic response of the shell a comparative analysis of internal forces obtained for the shocks from different mining regions was performed.

Figs 12 and 13 present comparisons of maximal longitudinal forces SF2 and maximal longitudinal moments SM1 respectively obtained for shocks from different regions. It could be observed that the amplitudes of accelerations of the mining tremor registered in the Legnica-Glogow Copper District were almost three times greater than the amplitudes of accelerations from the Upper Silesian Coal Basin (comp. Figs 1 and 2). Despite this, the values of the longitudinal forces SF2 as well as the values of the longitudinal moments SM1were greater in case of the tremor from the Upper Silesian Coal Basin, especially in the lower edge zone of the shell.

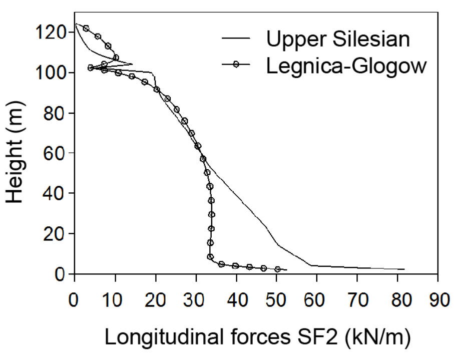

Fig. 12 Comparison of maximal longitudinal forces SF2 obtained for shocks from different mining regions

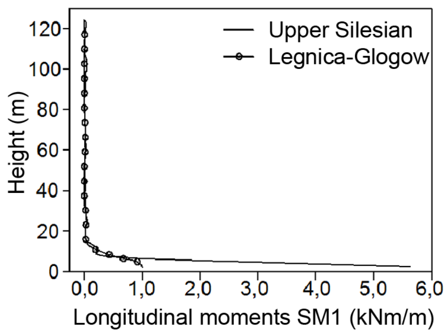

Fig. 13 Comparison of maximal longitudinal moments obtained for shocks from different mining regions

It indicates that the dynamic response of the shell is strongly dependent on the dominant frequency range of the kinematic excitation. In case of the shock from the Upper Silesian Coal Basin the frequency band includes the natural frequency of the shell associated with the basic mode executed by horizontal excitation (see Fig. 9).

C. Influence of Non-uniformity of Kinematic Excitation

For calculations of the dynamic response of the cooling tower to mining shock from the Upper Silesian Coal Basin the model of non-uniform kinematic excitation was applied. In order for evaluate the dependence of the dynamic response on the wave velocity in the ground calculations of the response to kinematic excitation propagating at two velocities were performed: 400 and 800 m/s. For comparison the case of uniform kinematic excitation that corresponds with wave propagation velocity $v = \infty$ was also evaluated. The decrease of vibration amplitudes, which is not negligible as far as mining shocks are considered, was adopted on the basis of empirical formulae (4), (5) and (6). It was assumed that the structure is located 1000 m from the epicentre.

Fig. 14 presents the time history of circumferential forces SF1 at the point E3 resulting from: uniform excitation, non-uniform excitation with wave velocity 800 m/s and non-uniform excitation with wave velocity 400 m/s.

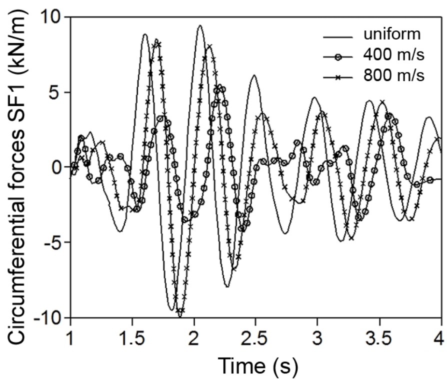

Fig. 14 Time history of circumferential forces SF1 at point E3 (middle zone of shell) for: uniform and non-uniform excitation (wave velocity 800 and 400 m/s)

It could be noticed that in case of point E3 belonging to the middle part of the shell the greatest circumferential forces SF1 were obtained at the assumption of wave velocity $v = \infty$ corresponding to uniform kinematic excitation. Lower values were obtained for wave velocity equaled 800 m/s and the lowest for 400 m/s. The dynamic analysis was carried out at all points shown in Fig. 7. Analogous conclusions were learned from calculations of internal forces and moments at the points E2 and E3 belonging to the middle part of the shell.

Fig. 15 presents the time history of circumferential moments SM2 at the point E1 belonging to the lower part of the shell, whereas Fig. 16 shows the time history of longitudinal forces SF2 at the point E4 belonging to the upper part of the shell. Again internal forces were obtained at the assumption of: uniform excitation, non-uniform excitation with wave velocity 800 m/s and non-uniform excitation with wave velocity 400 m/s.

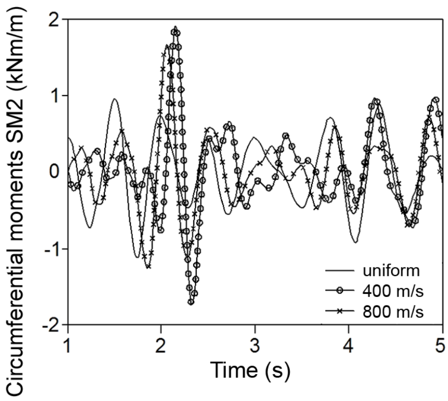

Fig. 15 Time history of circumferential moments SM2 at point E1 (lower zone of shell) for: uniform and non-uniform excitation (wave velocity 800 and 400 m/s)

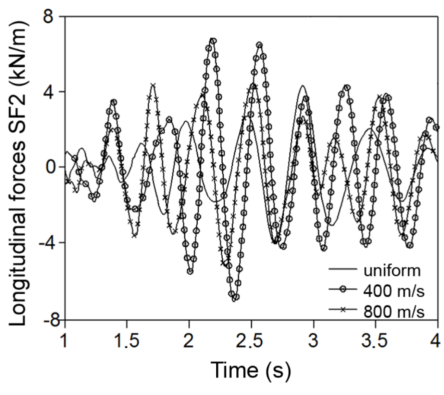

Fig. 16 Time history of longitudinal forces SF2 at point E4 (upper zone of shell) for: uniform and non-uniform excitation (wave velocity 800 and 400 m/s)

As far as the points E1 and E4 located in both edge zones were concerned the results of the dynamic analysis were opposite than in case of the points E2 and E3. The internal forces and moments calculated on the assumption of wave velocity 400 m/s were greater than the internal forces and moments calculated at the assumption of velocity 800 m/s. The assumption of uniform kinematic excitation ($v = \infty$) resulted in the lowest values.

VI. Conclusion

In the paper results of calculations of the dynamic response of the cooling tower shell to mining tremors originated from two main regions of mining activity in Poland are presented. The following conclusions as well as some general remarks for engineering practice could be formulated:

1) The registered values of vertical and horizontal components of mining tremors are comparable. The negligence of vertical component of kinematic excitation in calculations may cause underestimation of the dynamic response of the shell up to 30%. In case of mining shocks all components of ground vibrations have to be considered in dynamic analysis.
2) The comparative analysis of the dynamic response of the shell to shocks representative for various mining regions showed that not only values of vibration amplitudes but also frequency range of excitation may influence significantly the dynamic response of the shell. When the dominant frequency range includes basic frequencies of natural vibrations, the obtained values of maximal forces can be bigger than the values obtained in result of applying excitation of higher amplitudes but of frequency range not including basic natural frequencies. This means that dynamic performances of the same cooling tower located in various mining regions may differ even though values of maximal amplitudes of vibrations are of the same level. The differences may be caused by frequency range of shock which is a characteristic feature for particular region.
3) The global dynamic response of the shell is subjected to changes up to 30% at the assumption of various wave velocities in the ground. The decrease or the increase of the dynamic response can occur in the different parts of the shell.

The assumption of non-uniformity of excitation causes the decrease of the dynamic response as far as the middle part of the shell is considered. It results from the reduction in the average values of the excitation amplitudes. So called quasi-static effects are negligibly small in this part of the shell. The dynamic response depends less on geometrical changes of the structure than on effects of inertia. However, also the increase of the dynamic response with the decrease of wave velocity may occur, especially in the lower part of the shell. This is due to quasi-static effects which result from changes of the subsoil geometry during the shock and affects mainly the lower part of the shell. As the dynamic response of the shell depends on the wave velocity in the ground correct recognition of wave velocity in the ground is a necessary condition of further dynamic calculations. In final remarks it also should be pointed out that values of internal forces obtained in calculations of dynamic response of cooling tower to mining shocks are on the level of about 20% of values obtained in typical static analyses (dead load, wind).

References

[1] S. Lasocki, "Probabilistic seismic hazard analysis for mining-induced seismicity," in *Controlling Seismic Hazard and Sustainable Development of Deep Mines. Proc. 7th International Symposium on Rockburst and Seismicity in Mines*, Dalian, China, 2009, pp. 59-72.

[2] T. Tatara, *An influence of surface mining-related vibration on low-rise buildings*, Scientific Notebooks of Cracow University of Technology, no. 74, Krakow, 2002, pp. 61-70 (in Polish).

[3] G. Mutke, K. Stec, "Seismity in the Upper Silesian Coal Basin, Poland: Strong regional seismic events," in *Proc. 4th International Symposium on Rockbursts and Seismcity in Mines*, Krakow, Poland, 1997, pp. 213-217.

[4] R. Ciesielski, "Dynamic surface effects of underground copper ore mining in the Legnica Copper District, Poland," in *Engineering Geology of Underground Movements. Proc. of the 23rd Annual Conference of the Engineering Group of the Geological Society*, London, UK, 1988, pp. 435-444.

[5] A. Zerva, *Spatial Variation of Seismic Ground Motions: Modeling and Engineering Applications*, CRC Press,Taylor&Francis Group, FL, 2009.

[6] J. Dulinska, "In situ investigations of dynamic response of earth dam to non-uniform kinematic excitation," *Archives of Civil Engineering*, vol. 52, no. 1, pp. 37-58, Jan. 2006.

[7] G. Zanardo, H. Hao, C. Modena, "Seismic response of multi-span simply supported bridges to a spatially varying earthquake ground motion," *Earthquake Engineering and Structural Dynamics*, vol. 31, no. 6, pp. 1325-1345, June 2002.

[8] G. Mylonakis, D. Papastamatiou, J. Psycharis, K. Mahmoud, "Simplified modeling of bridge response on soft soil to nonuniform seismic excitation," *Journal of Bridge Engineering*, vol. 6, no. 6, pp. 587-597, 2001.

[9] R. W. Clough, J. Penzien, *Dynamics of structures*, New York: McGraw-Hill, Inc., 1975, pp. 575-578.

[10] M. D. Trifunac, M. I. Todorovska, "Spectra for differential motion of columns," *Earthquake Engineering and Structural Dynamics*, vol. 26, no. 2, pp. 251-268, Feb. 1997.

[11] A. Der Kiureghian, "A coherency model for spatially varying ground motion," *Earthquake Engineering and Structural Dynamics*, vol. 25, no. 1, pp. 99-111, Jan. 1996,

[12] D. Makovicka, "Response analysis of an RC cooling tower under seismic and windstorm effects," *Acta Polytechnica*, vol. 46, no. 6, pp. 17-21, Nov. 2006.

[13] T. Hara, "Dynamic analysis of R/C cooling tower shells under earthquake loading," in *Proc. 5th International Symposium on Natural-Draught Cooling Towers*, Istanbul, Turkey, 2004, pp. 283-291.

[14] T. Hara, P. L. Gould, "Local-global analysis of cooling tower with cutouts," *Computers&Structures*, vol. 80, no. 27-30, pp. 2157-2166, Nov. 2002.