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Krylov Model Order Reduction of a Thermal Subsea Model

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***Abstract*—**A subsea hydrocarbon production system can undergo planned and unplanned shutdowns during the life of the field. The thermal FEA is used to simulate the cool down to verify the insulation design of the subsea equipment, but it is also used to derive an acceptable insulation design for the cold spots. The driving factors of subsea analyses require fast responding and accurate models of the equipment cool down. This paper presents cool down analysis carried out by a Krylov subspace reduction method, and compares this approach to the commonly used FEA solvers. The model considered represents a typical component of a subsea production system, a closed valve on a dead leg. The results from the Krylov reduction method exhibits the least error and requires the shortest computational time to reach the solution. These findings make the Krylov model order reduction method very suitable for the above mentioned subsea applications.

***Keywords*—**Model order reduction, Krylov subspace, subsea production system, finite element.

I. Introduction

A subsea production system can undergo planned and unplanned shutdowns during the life of field. The shutdown will result in cool down of the subsea equipment and after some time it will reach the ambient temperature. When the temperature drops below the hydrate formation temperature, hydrates can form in the parts of the system containing gas and water. To prevent this from happening different measures are taken such as injection of inhibitors, depressurization and insulating the subsea production system. The oil field operator sets a cool down requirement to which the subsea equipment should perform. Often this means that the equipment must be insulated. Valves, connectors and sensors are uninsulated effective cold spots that drain heat out of the system and should therefore be insulated.

Thermal FEA is used to simulate the cool down to verify the insulation design of the subsea equipment, but it is also used to derive an acceptable insulation design for the cold spots. The thermal FEA approach starts with the geometry which must be cleaned before meshing. In order to accurately model the heat transfer in complex subsea equipment a relatively fine mesh must be applied. The mesh density drives the simulation time. After defining contacts and boundary conditions the steady state thermal solution of the model is derived and used as the initial condition for the cool down. The transient cool down analysis is run as long as the field operator requires.

The following driving factors are encountered in the area of flow assurance and general subsea analyses:

System approach: moving from local to global analyses of the system, larger models are required by customers

Uncertainty control: quality requirements impose the need to have full control of the input and output parameters

Real-time/virtual modeling: allows to fully utilizing the potential of the subsea equipment over the life of field

Optimization: optimization of the insulation design of the subsea equipment is typically aiming at reducing cost, which is one of the keys to customers' success.

These challenges have been observed in other more mature industries, such as the automotive and machine tool industry. A solution is to create fast responding and accurate models.

The models are normally described as ordinary differential equations with possibly very high number of unknowns. The number of unknowns can easily exceed 10^7. Such a large number of unknowns effectively hinder usage of classical solution methods because of the computational time. The system level simulations are crucial in the modeling of micro-electronic and micro-electro-mechanic devices [1] or in virtual machine tools simulations [2], [3]. Another field in need of fast computation of system responses is optimization, where many iterations are required and thus the simulation time becomes critical.

There have been attempts to reduce the number of unknowns almost since the advent of the Finite element method (FEM).The first reduction method was static condensation proposed by Irons [4] and Guyan [5]. This method was introduced for structural mechanics problems but it is also valid for thermal analysis and other analyses regardless of the underlying physics. The static condensation method was the first of the model order reduction methods. However this method is of questionable quality when using it for dynamic thermal analyses as was shown in [6].

To remedy the insufficiencies in the static condensation method the component mode synthesis (CMS, [7]) was proposed by Craig and Bampton. CMS has become widely used by the engineering community. CMS was used to efficiently conduct large-scale structural eigenanalysis [8], but

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also for transient heat conduction analysis [9] and heat conduction/convection analysis [10]. Another field of application of CMS is coupled physics simulations. The weakly coupled thermo-mechanical models were studied in [11]. There is still active research regarding improvement of CMS [12].

Another improved reduction method is the Improved reduced system (IRS) proposed by O'Callahan in [13]. Later, Friswell developed iterated version of IRS in [14]. The static condensation, CMS and IRS can be viewed as engineering approaches to reduce the number of equations.

Global error bounds and preservation of passivity and stability are important questions posed on MOR methods in a more mathematical point of view. One of the MOR techniques proposed in accordance to these questions are Krylov subspace reduction [15] and Balanced truncation [15]. Balanced truncation methods [16] have a great advantage because there exists an a priori global error bound. But it also has a great disadvantage in that the Lyapunov equation [17] needs to be solved in order to reduce the system. Thus the usage of Balanced truncation in reduction of large-scale systems is limited.

Krylov subspace methods [15], [18], [19], [20] are very interesting because of their iterative nature which allows reduction of large scale problems. Also passivity and stability preservation has been achieved using Krylov MOR methods [21], [15]. A second order structure preserving Krylov algorithm has been presented in [22]. The Krylov subspace MOR method has become widely used for microscale electro-mechanical system (MEMS) simulations [23], [1] in addition to RLC networks simulations [24]. Handling of nonlinear convection coefficient was studied in [25]. Reduction of coupled physic problems was studied in [26] for the case of a thermo-mechanical model of packages and in [27] for the case of structural-acoustic coupled models. Krylov subspace MOR was also successfully used in optimization of MEMS devices [28] and sensitivity analysis of structural frequency response [29]. One of the most important directions in development on Krylov base reductions is parametric model order reduction (PMOR). PMOR allows preservation of parameters which the system depends on [30]. The dependence of parameters may be either linear or nonlinear.

The comparison of different model order reduction methods has been discussed in [31] where thehe Krylov subspace MOR method was found to be one of the best methods. The comparison of Krylov, CMS and Balanced truncation can be found in [32]. Krylov is also found to be very robust and efficient method of MOR.

The Krylov subspace reduction method has been found to be the most suitable for subsea industry applications. The Krylov subspace reduction method was therefore applied as an alternative to the conventional way of performing thermal analyses. Krylov subspace reduction produces, just as other MOR methods, only an approximation of the system. It is therefore necessary to assess the level of approximation. The following requirements have to be satisfied in order to use Krylov subspace reduction as a replacement to the usual procedure:

- Low error in approximation
- Fast computation

A low error in the approximated system is required as the objective is to replace results obtained by conventional methods with results obtained with Krylov MOR. Reducing computational time is the key to success in the field of uncertainty control, real-time modeling and optimization in the subsea thermal analyses.

The article is organized in the following way: section I contains the introduction and the motivation of work; the model setup and problem description is considered in section II; a description of the usual solution methods by means of sparse direct and iterative solvers is given in section III; section IV contains the description of Krylov subspace reduction; section V contains comparison of the results obtained by different methods and section VI contains the conclusion and suggestions on future work.

II. Model Description

The model considered here represents a symmetric valve and manifold dead leg where production fluid is running through the header. The branch going from the header to the closed valve contains stagnant production fluid. During both steady state production and cool down the branch together with the valve will drain heat from the header. Branches like these are in the subsea industry called dead legs. Since the model is symmetric, only half of it is modeled. The model consists of a production fluid domain in a steel pipe covered by insulation, see Fig. 1, 2 and 3.

Fig. 1 Insulation domain

Fig. 2 Pipe/Valve domain

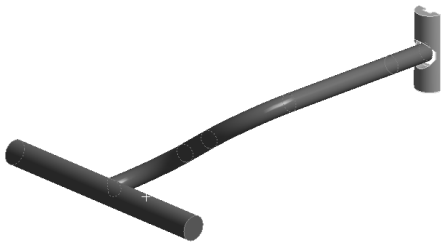

Fig. 3 Production fluid domain and valve cavity

The FEA approach does not account for convective heat transfer. Hence, the production fluid is modeled as a solid. Artificial thermal conductivities are therefore used in order to model the convection in the fluid domains. The mesh consist of 5 322 721 elements and 956 488 nodes, see Fig. 4.

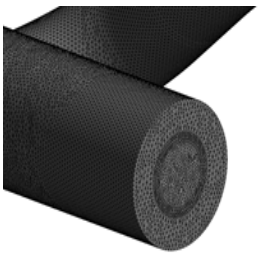

Fig. 4 Computational mesh - header

All external surfaces are exposed to an ambient sea temperature of 5 °C and a heat transfer coefficient of 1000 W/m^{2}K is applied. The initial temperature in the header is set to 50 °C, see Fig. 5.

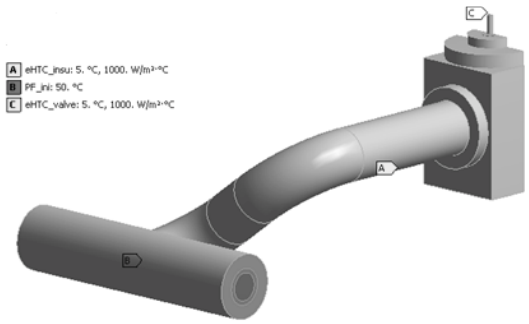

Fig. 5 Boundary and initial conditions

Adiabatic boundaries are assumed on the remaining outer faces of the model, the header ends and the symmetry plane of the valve.

III. Solution of the Heat Equations

In simulation of numerical heat transfer problems the model is discretized on a finite element (FE) computational grid where the heat equation is solved on each element. The solution may be obtained directly or iteratively using a finite difference time discretization scheme to achieve the solution for each step in a given time period.

A. The Heat Equations

The heat equation for transient heat conduction of solid material with isotropic thermal conductivity reads [33].

$$\rho c\dot{T} - k\nabla^2 T = F, \tag{1}$$

where ρ is the density, c is the heat capacity and k the thermal conductivity of the material. The time derivative of the temperature, $T(x,y,z,t)$, is denoted by the superimposed dot and the applied loads are represented by the time independent $F(x,y,z)$. To close the boundary value problem, initial and boundary conditions must be applied.
Using the FE method [34] on the problem (1) produces the semi-discrete equation

$$C\dot{T} + KT = Q \tag{2}$$

In which the heat capacity and conductivity matrices, C and K, are symmetric and positive definite. T is the unknown vector of nodal temperatures

B. Time Discretization

A widely used finite difference scheme for solving (2) first order equations like is the trapezoidal (or theta) rule [35], [36]. It approximates an equation

$$\frac{\partial u}{\partial t} = G \tag{3}$$

by

$$\frac{u^{n+1} - u^n}{\Delta t} = \theta G^{n+1} + (1-\theta)G^n \tag{4}$$

where u is the unknown, G is a function of u and the time t and θ is the transient integration parameter. Using this on (2) we get

$$\left(\frac{1}{\theta\Delta t}C + K\right)T^{n+1} = \frac{1}{\theta}Q + \left(\frac{1}{\theta\Delta t}C - \frac{1-\theta}{\theta}K\right)T^n, \tag{5}$$

where the nodal temperature solution is unknown at time step $n+1$. This can be expressed as the linear system

$$Ax = b \tag{6}$$

where x is the unknown T_{n+1}, and the load vector b and coefficient matrix A are known. Consequently, the time dependent system can be solved using a direct or iterative solver.

In ANSYS [37] the transient integration parameter is limited to

$$\frac{1}{2} \leq \theta \leq 1, \tag{7}$$

where θ=1/2 and θ =1 corresponds respectively to the Crank-Nicholson method and the backward Euler method. The initial values for the system can be set by the user as a known load or solution vector in the setup of the transient analysis, or it can be retrieved by a steady state analysis.

C. Sparse Direct Solve

The direct solver available in ANSYS is the sparse direct solver. Direct solvers are based on direct elimination of a set of equations. The following system is considered

$$KT = Q \tag{8}$$

where K is the conductivity matrix, T is the vector of nodal temperatures which are unknown and Q is the heat flow. To solve this problem matrix K must be decomposed into lower and upper triangular matrices, so that

$$K = LU \tag{9}$$

and

$$LUT = Q, \tag{10}$$

where L is the lower triangular matrix and U is the upper triangular matrix. To solve T forward and backward substitutions of L and U are made. If the K matrix is sparse symmetric and positive definite Cholesky factorization can be used so that [38]

$$K = LL^T \tag{11}$$

and

$$LL^T T = Q, \tag{12}$$

where L is a lower triangular matrix where the entities on the diagonal are positive.

The sparse direct solver is designed to handle only nonzero entities in matrix K. In the decomposition nonzero coefficients appear in matrix L where matrix K had zero entities. This fill-in is minimized by the sparse direct solver algorithm by reordering the equation numbers in matrix K. The sparse direct solver uses two different reordering schemes, the Minimum Degree ordering [39] and the METIS [40]. The solver algorithm automatically chooses the appropriate method in order to achieve the least amount of fill-in [37].

When the solution depends on time the decomposition must be done for every time step. But when the time step is constant the left hand side (LHS) of equation (2) only needs one decomposition. Hence, the decomposition of the LHS is reused for all time steps. If the following is considered

$$\Delta t = constant for n, n+1, \ldots \tag{13}$$

Then

$$\underbrace{\left(\frac{1}{\theta \Delta t} C + K\right)}_{\text{constant}} T^{n+1} = \frac{1}{\theta} Q + \left(\frac{1}{\theta \Delta t} C - \frac{1-\theta}{\theta} K\right) T^n \tag{14}$$

$$LL^T = \left(\frac{1}{\theta \Delta t} C + K\right) \tag{15}$$

$$LL^T T^{n+1} = \frac{1}{\theta} Q + \left(\frac{1}{\theta \Delta t} C - \frac{1-\theta}{\theta} K\right) T^n \quad \text{for n, n+1, ...} \tag{16}$$

This means that only back substitutions are necessary to solve T^{n+1}.

For infinite machine precision the sparse direct solver would produce exact results. For real computers the results would be exact except for rounding errors.

D. Iterative Sparse Solver: The Incomplete Cholesky Conjugate Gradient (ICCG) Method

Iterative solvers are computationally efficient, but less robust alternatives to the direct solvers for large sparse linear systems.

Guessing an initial solution vector x_0, an iterative solution method produces approximations $x_1, \ldots, x_m$, where x_k is closer to the solution than the previous. The method terminates it reaches the specified convergence tolerance. The CG method is a fast converging algorithm for solving large *nxn* systems of linear equations. Were it not for computational round off errors, the method would produce the exact solution in n steps [41].

The CG method expands the solution in a series of n mutually conjugate search directions p that span the Krylov subspace $K(r_0,A)$ where r*0* is the first residual $r_0=Ax_0-b$ of equation (6). In iteration number $j+1$ we have

$$x_{j+1} = x_j + \alpha_j p_j \tag{17}$$

for some scalar α_j, that minimizes the residual in the approximated solution at step $j+1$. The corresponding residual becomes

$$r_{j+1} = b - Ax_{j+1} = r_j - \alpha_j A p_j, \tag{18}$$

where the next solution search direction p_j is built of the current residue. We demand that the residuals are orthogonal, i.e. the inner product $(r_{(j+1)}, r_j)=0$. The search direction p_j may be computed using only the previous $p_{(j-1)}$ when A is symmetric and positive definite

$$p_{j+1} = r_j + \beta_j p_j \tag{19}$$

For the FE discretized heat equation the coefficient matrix A is sparse and ill-conditioned. In order to improve the convergence of the CG method for this problem a preconditioner should be used. The Incomplete Cholesky decomposition is a well-known and frequently used preconditioner for the CG method. The preconditioner

$$A \approx M = LDL^T \quad (20)$$

is an approximation of A where D is a diagonal matrix and L is the lower triangular factorization matrix forced to have the same sparsity pattern as A and thus saving storage space but also introducing a small error in the approximate solution. The preconditioned system is written

$$M^{-1}Ax = M^{-1}b \quad (21)$$

to which the CG method is applied. M is computed once and directions p are computed successively only using the previous search dir. The ICCG solver is the preferred iterative solver for large sparse linear (ill-conditioned and transient problems) systems as it uses a more sophisticated preconditioner than other iterative solvers implemented in ANSYS [37].

IV. Krylov Subspace Reduction

Krylov subspace reduction has lately become widely used in several fields [23], [1], [25], [29], [21]. Its main advantage lies in computational efficiency and excellent approximation performance. It was therefore chosen to be used in this study in order to assess possible advantages over classical methods of solution of transient thermal problems.

In this section only the basics behind Krylov reductions will be described. The reader is encouraged to read an excellent mathematical description of Krylov based reductions in [15]. An overview of reduction methods is given in [16]. Although optimal Krylov based reduction algorithms are available [42] a simpler and possibly more computational efficient method will be used - a block rational Krylov method [24].

Let's consider single-input / single-output (SISO) linear time invariant (LTI) system in state space form

$$\begin{aligned} C\dot{T}(t) + KT(t) &= Qu(t) \\ y(t) &= L^T T(t), \end{aligned} \quad (22)$$

where $C \in R^{NxN}$ and $K \in R^{NxN}$ are specific heat matrix and conductivity matrix, N is the dimension of the system. $Q \in R^N$ is the input vector and $L \in R^N$ is the output vector. $T \in R^N$ is the state vector (temperature). $y(t) \in R$ is the output function and $u(t) \in R$ is the input function. In this case $u(t) = 1$.

Let's consider the coordinate transformation

$$\begin{aligned} T(t) &= \tilde{T}(t) + T_0, \\ \dot{T}(t) &= \dot{\tilde{T}}(t). \end{aligned} \quad (23)$$

Substituting (23) into (22) we get

$$\begin{aligned} C\dot{\tilde{T}} + \mathrm{K}\tilde{T} &= \mathrm{Q} - \mathrm{KT}_0 \\ \tilde{\mathrm{T}}(0) &= 0, \end{aligned} \quad (24)$$

therefore we can only consider zero initial conditions (IC) because using transformation (23) the nonzero IC is moved to the right hand side (RHS) of (24). The treatment of a nonzero IC was introduced in [43].

The Laplace transform of (24) has the form

$$H(s) = L^T(sC + K)^{-1}Q. \quad (25)$$

H(s) is transfer function of system (22). The MacLaurin series of (25) has following form

$$H(s) = \sum_{i=0}^{\infty} m_i s^i, \quad (26)$$

where m^l are the so-called moments of the transfer function:

$$m_l = L^T r_l, \quad (27)$$

where

$$\begin{aligned} r_0 &= K^{-1}Q \\ r_1 &= K^{-1}Cr_0 \\ r_l &= K^{-1}\mathrm{C}r_0, \qquad l > 1. \end{aligned}$$

The vectors r_l span Krylov space

$$K_n = span(r_0, \cdots, r_{n-1}). \quad (28)$$

Let V_n be the orthonormal basis of K_n

$$K_n = span(V_n), \qquad V_n^T V_n = I, \quad V_n \in R^{Nxn}. \quad (29)$$

The projection of state coordinates T onto K_n using V_n is called generalized state coordinates $q \in R^n$

$$T = V_n q + \epsilon. \quad (30)$$

The error $\epsilon \in R^n$ in the projection rises while performing projection of x onto K_n (Fig. 6).

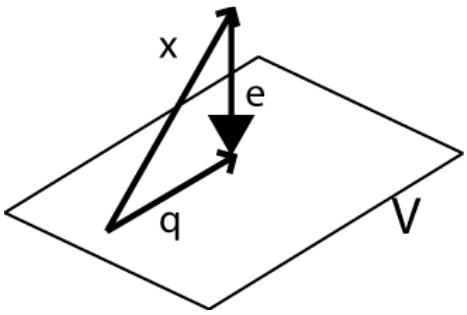

Fig. 6 Projection onto K_n

The system equation (22) in generalized coordinates has the form

$$CV_n\dot{q} + KV_n q = Q. \quad (31)$$

Using Galerkin method

$$CV_n\dot{q} + KV_nq - Q = \mathrm{R} \perp \mathrm{K_n} \tag{32}$$

where R is the vector of residual forces due to the error in the approximation ϵ in (30)(30). Using the Galerkin method leads to the reduced system of equations

$$\begin{aligned} C_n\dot{q} + K_nq &= Q_n \\ \tilde{y}(t) &= L_n^T q(t), \end{aligned} \tag{33}$$

where

$$\begin{aligned} K_n &= V_n^T K V_n \\ C_n &= V_n^T C V_n \\ Q_n &= Q_n^T F. \end{aligned} \tag{34}$$

The transfer function of the reduced system (33) has the form

$$H_n(s) = L_n^T(sC_n + K_n)^{-1}Q_n. \tag{35}$$

The above procedure assures that the first n moments of the transfer function (25) of the full system equals to the first n moments of the transfer function (35) of the reduced system [15].

The error induced by the projection (30) in the output function y(t) has the form

$$\epsilon = \max_{t>0}|y - \tilde{y}| \tag{36}$$

An a priori expression for error norm (36) is not known although there exist algorithms minimizing the error (42], [44]. The algorithm used in this paper to produce the reduced order systems is the block Arnoldi algorithm [24].

There exist wide possibilities to improve computational performance of Krylov methods. One of most obvious options is parallelization [45]. Another is to use an iterative algorithm to solve the system [46]. The presented case is of medium size and it is therefore suitable to use the direct sparse solver [38].

The procedure is easily extended to a multi-input / multi-output case where$Q \in R^{Nxl}$and$L \in R^{Nxm}$ are matrices. The size of the reduced system is determined by the size of Q and L. However it is possible to use the superposition property [47] to keep the matrices small.

V. Results

The main task is to assess the viability of MOR in comparison with classical approaches. To realize this task, 15 hour cool down simulation of the considered model (section II) has been computed using three different methods:

1. Sparse direct solver (section III.C) with time stepping strategies:
   a. Initial time step, minimal time step, maximum time step = [1s, 1s, 5400s], labeled T5400
   b. [1s,1s,540s], labeled T540
   c. [1s,1s,54s], labeled T54
2. ICCG solver (section III.D) with time stepping strategies:
   a. [1s,1s,5400s], labele $T5400_{iccg}$
   b. [1s,1s,540s], labeled $T540_{iccg}$
   c. [1s,1s,54s], labeled $T54_{iccg}$
3. Krylov MOR (section IV) – with time step 1s, order of reduction *n=100*, labeledTr_{100}.

In order to assess approximation qualities of MOR the following error norm is suggested

$$\begin{aligned} \varepsilon = \max_{i\epsilon timestep}(\max|T_i - T54_i|) \\ \mathrm{T_i} \in \{\mathrm{T540}, \mathrm{T5400}, \mathrm{T54_{iccg}}, \mathrm{T540_{iccg}}, \\ \mathrm{T5400_{iccg}}, \mathrm{Tr_{100}}\} \end{aligned} \tag{37}$$

The error norm compares the results of all the approaches against the result of the sparse direct solver with the finest time step. This case was chosen as the reference case because it is considered the most converged of all the results.

The resulting error norms for the selected time steps are shown in Table I and in Fig. 7. Krylov MOR displays the best performance of the compared approaches except for first few time steps.

Another crucial aspect of MOR is computational efficiency. Table II shows the computational times for the approaches. It is obvious that Krylov MOR is unmatched in the overall computational time. Krylov MOR is much faster than any of the mentioned approaches.

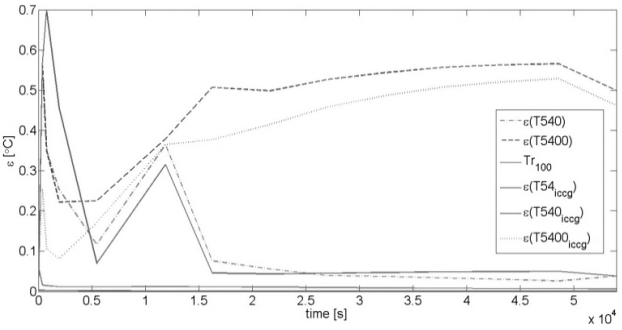

Fig. 7 Comparison of approaches

TABLE I
COMPARISON OF APPROACHES

Time [s]	ϵ(T540) $[\circ C]$	ϵ(T5400) $[\circ C]$	ϵ(T54$_{iccg}$) $[\circ C]$	ϵ(T540$_{iccg}$) $[\circ C]$	ϵ(T5400$_{iccg}$) $[\circ C]$	$\epsilon(Tr_{100})$ $[\circ C]$
1	0,000	0,000	0,003	0,003	0,003	0,004
2	0,000	0,000	0,003	0,003	0,003	0,049
5	0,000	0,000	0,003	0,003	0,018	0,058
14	0,000	0,000	0,003	0,003	0,036	0,062
41	0,000	0,000	0,003	0,003	0,074	0,056
365	0,561	0,561	0,002	0,561	0,254	0,015
743	0,350	0,350	0,002	0,697	0,105	0,014
1931	0,253	0,222	0,002	0,456	0,081	0,011
5441	0,117	0,225	0,001	0,070	0,171	0,011
11867	0,364	0,379	0,001	0,315	0,365	0,012
16241	0,076	0,507	0,000	0,046	0,377	0,011
21641	0,055	0,498	0,000	0,043	0,415	0,011
27041	0,039	0,527	0,000	0,045	0,458	0,010
32441	0,037	0,544	0,000	0,047	0,487	0,009
37841	0,033	0,557	0,000	0,048	0,508	0,008
43241	0,030	0,562	0,000	0,049	0,520	0,007
48641	0,026	0,566	0,000	0,050	0,528	0,006
54000	0,038	0,499	0,000	0,038	0,462	0,006

TABLE II
COMPUTATION TIMES

Approach	Reduction of system [s]	Cool down simulation [s]
T54	~	225240
T540	~	24720
T5400	~	4800
T54_iccg	~	212600
T540_iccg	~	23128
T5400_iccg	~	4210
Tr_100	319	<1

VI. CONCLUSION

The Krylov MOR results exhibits the least error in comparison with the finest time step results of the direct solution (Table I). The maximum temperature error remains below 0.062°C during the entire cool down simulation. Therefore, the requirement on low error of the approximation may be considered fulfilled.

The Krylov MOR method reaches the solution of the cool down model in less than 1 s, whereas the Sparse direct and ICCG solver need hours or days. The requirement of fast computation is thus comfortably met.

Krylov MOR is proved to be superior in terms of solution time and a very accurate way to obtain the solution of the cool down simulation compared to the more conventional approach of coarsening the time step. These abilities are essential for the future development in the areas of large models, robustness analysis, real-time modeling and optimization. Krylov MOR is therefore a method of choice in the case of cool down simulations when shortening of simulation time is required.

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