Kinematic Parameters for Asa River Routing

A. O. Ogunlela, B. Adelodun

Abstract**—**Flood routing is used in estimating the travel time and attenuation of flood waves as they move downstream a river or channel. The routing procedure is usually classified as hydrologic or hydraulic. Hydraulic methods utilize the equations of continuity and motion. Kinematic routing, a hydraulic technique was used in routing Asa River at Ilorin. The river is of agricultural and industrial importance to Ilorin, the capital of Kwara State, Nigeria. This paper determines the kinematic parameters of kinematic wave velocity, time step, time required to traverse, weighting factor and change in length. Values obtained were 4.67 m/s, 19 secs, 21 secs, 0.75 and 100 m, respectively. These parameters adequately reflect the watershed and flow characteristics essential for the routing. The synthetic unit hydrograph was developed using the Natural Resources Conservation Service (NRCS) method. 24-hr 10yr, 25yr, 50yr and 100yr storm hydrographs were developed from the unit hydrograph using convolution procedures and the outflow hydrographs were obtained for each of 24-hr 10yr, 25yr, 50yr and 100yr indicating 0.11 m^3/s, 0.10 m^3/s, 0.10 m^3/s and 0.10 m^3/s attenuations respectively.

Keywords**—**Asa River, Kinematic parameters, Routing.

I. Introduction

ROUTING is the general name for methods used to estimate the travel time and attenuation of flood waves as they move downstream in a river or channel. It can also be regarded as the technique used in determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. Fread [1] and Linsley et al. [2] defined flood routing as a mathematical method for predicting the changing magnitude, celerity and shape of a flood wave as it propagates through rivers or reservoirs. It is among the most important and common forms of unsteady flow dealt with by engineers.

Flood routing methods are applied to such problems as real-time flood forecasting, dam-breach analyses, modeling of watershed hydrology, peak flow estimation, and floodplain and flood insurance rates studies [3], [4]. Generally, flood routing can be classified into hydrologic and hydraulic routing techniques [5]-[7]. The hydraulic routing technique is considered to be more relatively complex and more adaptable to spatially varied systems [5].

Flood waves can be identified as either of two separate kinds of wave phenomena: the dynamic wave and the kinematic wave [8]. Although both of these kinds of waves are initially present, certain characteristics of a watershed can make kinematic waves the dominant characteristic of a flood event. Kinematic wave theory is based on a one-dimensional approximation of the flow problem whenever a functional relation exists at each point in a medium between the flux and the concentration of a continuously distributed material, the wave motion follows from the equation of continuity [9]. In this case, it is described by the continuity equation and a uniform flow equation plus the imposed initial and boundary conditions [10]. This approach has been applied for channel flow routing [11]-[13] and overland flow [14]-[16].

However, the governing equations of kinematic wave flow can be analytically solved by the characteristic method or numerically solved by the implicit or explicit method [17]. The characteristic solutions for kinematic wave flows are formed by flow travel distance (x) and travel time (t). In order to be able to solve kinematic wave flood routing problems, many reasonable assumptions have to be made to aid in the calculations that equally satisfy the kinematic conditions [18].

Travel time of floods reflects the morphological and hydraulic characteristics of the river and can be measured directly or estimated [19]. In practice, data for long river reaches are rarely available, and it can be time consuming and expensive to collect them. For a given river wave, the choice of a numerical method of resolution, space and time steps to be retained, depend essentially on the form of flood hydrographs and the hydraulic properties of the river [20]. Furthermore the morphological characteristic of the river reaches can change significantly during flooding [21].

Courant et al. [22] developed a numerical criterion for kinematic wave numerical stability in solving kinematic wave differential equations. Both implicit and explicit methods require the selections of incremental distance, Δx in the flow direction and incremental time step, Δt, during the rainfall-runoff simulation. Guo and Hinds [23] stated that a higher ratio of $\frac{\Delta x}{\Delta t}$ is preferred so as to improve the computational efficiency while accuracy demands a smaller ratio of $\frac{\Delta x}{\Delta t}$.

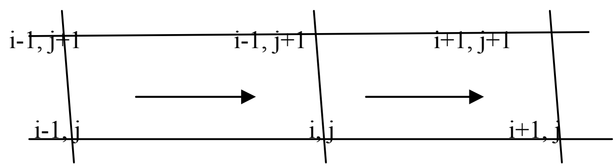

Fig. 1 x – t plane

Thus, the computational time interval depends on the spatial grid spacing, flow velocity and celerity, which are functions of the flow depth. Since the flow depth and the flow velocity may change significantly during the computations, it may be necessary to reduce the size of computational time interval for stability, Courant condition must be satisfied at each grid point

A. O. Ogunlela is a Professor of Soil and Water Engineering in the Department of Agricultural and Biosystems Engineering, University of Ilorin, Ilorin (Phone: +2348035617696, e-mail: aogunlela@yahoo.com).
B. Adelodun is a Masters of Engineering (M. Eng.) student at the Department of Agricultural and Biosystems Engineering, University of Ilorin, Ilorin (Phone: +2347030231398, e-mail: adbash2008@gmail.com).

during each computational interval. The Pressman scheme is unconditionally stable provided $0.5 \leq \Theta \leq 1$ where Θ is the weighting factor [24].

For the development of 24-hr 10yr, 25yr, 50yr and 100yr storm hydrographs from the unit hydrograph using convolution procedures, this paper determines the essential kinematic parameters that adequately reflect the watershed and flow characteristics in routing Asa River.

II. Methods

A. Study Area

The routed river is "Asa" which flows through Ilorin, Kwara State, Nigeria. Its catchment is located between latitudes $8^0$36'N and $8^0$24'N and longitudes $4^0$36'E and $4^0$10'E in Ilorin, with about 56 km long and a maximum of width of about 100 m at the dam site [25], [18]. Asa River is of great domestic, agricultural and industrial importance to Ilorin [26].

B. Determination of Kinematic Parameters

The kinematic wave velocity, U along the upstream and downstream of the flow direction was calculated using "to be published" [23]:

$$U = v \pm C \tag{1}$$

where:

v = average flow velocity (0.588 m/s)

C = wave celerity (m/s)

The "+" sign in (1) represents forward wave speed while the "-" sign represents the backward wave speed. The wave celerity, C, is described as:

$$C = \sqrt{gy} \tag{2}$$

where:

g = acceleration due to gravity (9.81 m/s^2)

y = average depth of flow (1.701 m)

Therefore, the value of kinematic wave velocity, U, was calculated as 4.67 m/s.

For the kinematic wave approximation to be applicable, the flow Froude Number (Fr) must be less than 1.5 [27], [28].

$$Fr = \frac{v}{\sqrt{gy}} \leq 1.5 \tag{3}$$

where:

v = average flow velocity (0.588 m/s)

g = acceleration due to gravity (9.81 m/s^2)

y = average depth of flow (1.701 m)

Fr = 0.144

The time required for a kinematic wave to traverse a distance interval Δx, K, was calculated using [18]:

$$K = \frac{\Delta x}{U} \tag{4}$$

where:

Δx = change in length of the river at any interval (100 m)

U = kinematic wave velocity (4.67 m/s)

K = 21.41 seconds ≈ 21 seconds

In determining the time step, Δt which equals (K – t) the Courant condition Δt ≤ [22], must be satisfied.

$$\Delta t = (K - 2) \tag{5}$$

where:

t = 2 seconds (for Courant condition to be satisfied)

Δt = 19 seconds

Weighting factor, Θ of 0.75 was selected since $\Theta > 0.5$ for the implicit schemes to be unconditionally stable. The four point scheme has a second-order accuracy for $\Theta = 0.5$ and only first-order accuracy for $\Theta = 1.0$.

Also, the area at any given point interval Δx was calculated using [18]:

$$A_{i+1,j+1} = \frac{\Delta t}{K + \Delta t} A_{i+1,j} + \frac{K}{K + \Delta t} A_{i,j+1} + \frac{I}{U\,(K + \Delta t)} \tag{6}$$

where:

$A_{i,j+1}$ is the original area at point where Δx = 0; $A_{i,j+1}$= 61.314 m^2

$A_{i+1,j}$ is the assumed area between $A_{i,j+1}$ and $A_{i+1,j+1}$; $A_{i+1,j} = A_{i+1,j+1} - 0.02$

$A_{i+1,j+1}$ is the new area at every length interval, Δx The outflow was computed using [29]:

$$Q_{i+1,j+1} = \frac{-\Delta x}{\theta \Delta t} A_{i+1,j+1} + \left[\frac{1-\theta}{\theta}(Q_{i,j} - Q_{i+1,j}) + Q_{i,j+1}\right] + \frac{\Delta x}{\theta \Delta t} A_{i+1,j} \tag{7}$$

where:

$Q_{i,j}$= the first inflow

$Q_{i,j+1}$ = second inflow

$Q_{i+1,j}$ = first outflow

$Q_{i+1,j+1}$ = second outflow

Before routing, it was assumed that the initial inflow equaled to the initial outflow. After the outflow $Q_{i+1,j+1}$ was calculated, it was used as I in (6) to get new $A_{i+1,j+1}$. The computed $A_{i+1,j+1}$ was applied in (7) and subsequently used as $A_{i,j+1}$ in (6) for the next iteration to get new $A_{i+1,j+1}$ which was used in (7) to get new $Q_{i+1,j+1}$ and so on.

III. Results and Discussion

The values of the kinematic parameters determined as shown in Table II have great effects on the attenuation of the hydrographs. In determining the kinematic wave velocity, U, along the upstream and downstream of the flow, the average depth of flow of 1.701 m which was calculated from Table I and average velocity of flow of 0.588 m/s also from Table I were used to calculate U as 4.67 m/s. This value of wave velocity has a definite effect on the overall accuracy of the flood routing.

Time and distance steps (Δt and Δx) are of critical importance in routing procedures. Selection of Δx depends on several factors and the time step used in the routing needs to represent the inflow hydrograph shape. The value of 100 m used in this study for Δx was based on wave celerity, top width of the river channel and watershed slope. However time step, Δt was calculated as 19 seconds from the calculated value of time required for the kinematic wave to traverse, K which is 21 seconds.

Furthermore, value of 0.75 was chosen for weighting factor. This value was carefully selected so as to ensure numerical stability of the kinematic routing scheme since the relationship between the attenuation and weighting factor is similar to the relationship between the attenuation and time step.

The design storm hydrographs for 24-hr 10yr, 25yr, 50yr and 100yr produced outflow hydrographs using the kinematic equations (6) and (7) as shown in Figs. 2, 3, 4 and 5 respectively indicating 0.11 m^3/s, 0.10 m^3/s, 0.10 m^3/s and 0.10 m^3/s attenuations respectively.

IV. Conclusion

Kinematic wave flood routing is sensitive to the values of computational parameters, which include the distance between cross sections (Δx), the computational time step (Δt) and the weighting factor (Θ) used in the numerical solution of the kinematic wave equations.

Each of these parameters influences attenuation to varying degrees, and varying their values can lead to more or less numerical damping of routed floods. Numerical damping refers to non-physical attenuation of flood waves. Each parameter must be adjusted such that the trade off between numerical stability and numerical damping is minimized. In order to minimize numerical damping while maintaining stability, weighting factor (Θ) of 0.75, change in length (Δx) of 100 m, time step (Δt) of 19 seconds, time required to traverse (K) of 21 seconds and kinematic wave velocity of 4.67 m/s were estimated.

TABLE I
The Sectional Geometry and Other Hydraulic Properties of Asa River

Distance from Initial Point (m)	Width, B (m)	Hydraulic Depth, Y (m)	Area, A (m^2)	Wetted Perimeter (m)	Hydraulic Radius, R (m)	Discharge (m^3/s)
1.5	1.75	0.73	1.28	3.21	0.52	0.73
3.5	2.25	0.75	1.69	3.75	0.56	0.63
6.0	2.00	0.76	1.71	3.52	0.62	0.51
8.0	2.00	1.28	2.56	4.56	0.78	0.95
10.0	2.00	2.56	3.10	5.10	0.87	1.57
12.0	2.00	2.13	5.00	7.00	1.11	3.08
14.0	2.00	2.80	5.12	7.12	1.12	3.97
18.0	2.00	2.80	4.26	6.26	1.03	3.32
20.0	2.00	2.80	5.60	7.60	1.17	3.11
22.0	2.00	2.80	5.60	7.60	1.17	3.14
24.0	2.00	2.26	4.52	6.52	1.06	2.65
26.0	2.00	1.49	2.98	4.98	0.85	1.92
28.0	2.00	1.34	2.68	4.68	0.80	1.93
30.0	2.00	1.55	3.10	5.10	0.87	2.16
32.0	2.00	1.34	2.68	4.68	0.80	1.74
34.0	2.00	1.43	2.86	4.86	0.83	2.00
36.0	1.90	1.22	2.32	4.34	0.74	1.54

Source: [30]

TABLE II
Summary of Results

Kinematic Parameters	Results
Kinematic wave velocity, U	4.67 m/s
Time step, Δt	19 sec
Time required to traverse, K	21 sec
Weighting factor, θ	0.75
Change in length, Δx	100 m

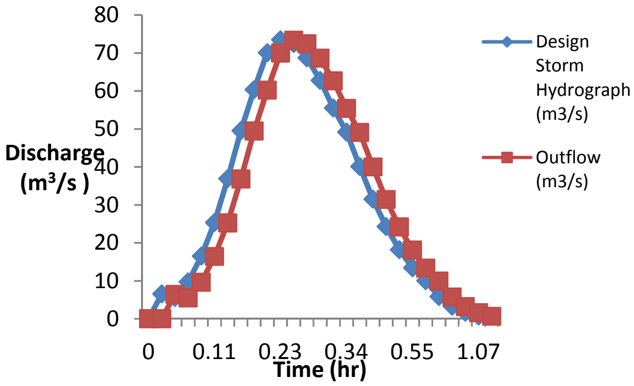

Fig. 2 10 yr, 24 hr Design Storm Hydrograph and its Outflow

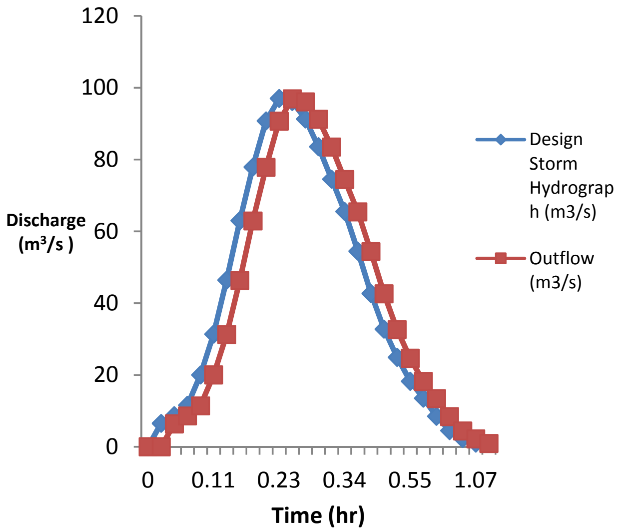

Fig. 3 25 yr, 24 hr Design Storm Hydrograph and its Outflow

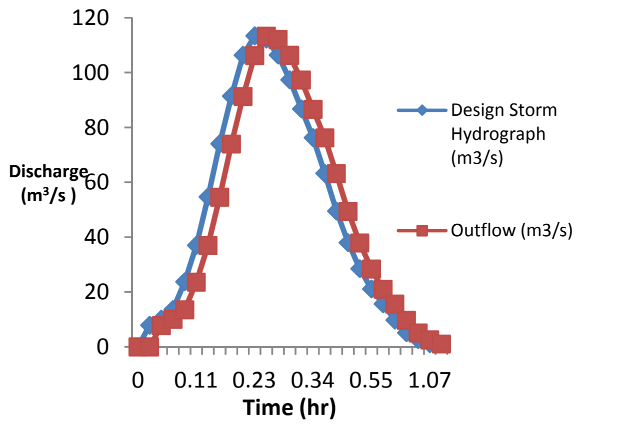

Fig. 4 50 yr, 24 hr Design Storm Hydrograph and its Outflow

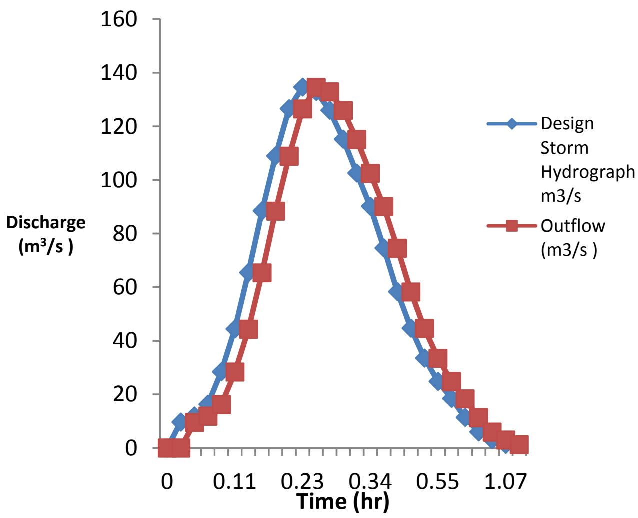

Fig. 5 100 yr, 24 hr Design Storm Hydrograph and its Outflow

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