

# Kalman Filter Design in Structural Identification with Unknown Excitation

Z. Masoumi, B. Moaveni

**Abstract**—This article is about first step of structural health monitoring by identifying structural system in the presence of unknown input. In the structural system identification, identification of structural parameters such as stiffness and damping are considered. In this study, the Kalman filter (KF) design for structural systems with unknown excitation is expressed. External excitations, such as earthquakes, wind or any other forces are not measured or not available. The purpose of this filter is its strengths to estimate the state variables of the system in the presence of unknown input. Also least squares estimation (LSE) method with unknown input is studied. Estimates of parameters have been adopted. Finally, using two examples advantages and drawbacks of both methods are studied.

**Keywords**—Structural health monitoring, Kalman filter, Least square estimation, structural system identification.

## I. INTRODUCTION

SYSTEM identification and damage detection for the structural health monitoring of civil infrastructures have recently received considerable attention [1]. Hence, in this paper, structural system identification in order to structural health monitoring is used.

System identification is a process for modeling an unknown system, based on the sampling of inputs and outputs of the system that is used in various fields of engineering. The main purpose of structural system is identification of the structural parameters such as stiffness, damping and mass. The major difference identified in this area is lack of detailed access to excitation inputs, so often only output data are available. In this way, by identifying structural parameters and observing the changes in these parameters, damage in the structural elements can be predicted. Also active control can be used to fix the damage and tried to prevent the destruction of structures [2].

Researchers considered the parameters estimation of structural systems in both frequency-domain and time-domain. The frequency-based model approach is very commonly used to identify a system [3]. It should be noted that measuring the output of the system takes place simultaneously with the application of force to the structures, and the free vibration modes is also important [4]. In the frequency-domain analysis, data frequency response gained from sensors gives some information about the least degrees of freedom, natural

frequency and damping coefficients of structure. One of the methods that can be referred to this area is the Fast Fourier Transform (FFT). In this paper, a system identification method like Eigen system Realization Algorithm (ERA) is considered. ERA can be used as a model analysis technique and it generates a system realization by using the time-domain response input and output data. In this method, by measuring the output in the free vibration mode, the Henkel matrix will be generated. ERA can calculate the natural frequency and damping coefficients of a structural system [4]. Another method to identify the dynamic of structural system is LSE. In this method, the external excitations (inputs) should be available from sensor measurements. Frequently, however, sensors may not be installed in the health monitoring system to measure all the excitations, such as earthquakes [1]. So, LSE to identify structural system is not efficient.

In this paper at first, The Iterative Least Squares procedure with unknown input excitation (ILS-UI) as one of the available methods for the identification of structural systems is expressed. The procedure does not require to any information on the input excitation forces[5]. In the next section Kalman filter (KF) is proposed to identify the parameters for unknown excitation by providing state space model for this type of systems. The main advantage of this filter is its ability to estimate the system state variables with unknown input. Finally, we will compare the methods by presenting and comparing the simulation results.

## II. ILS-UI

### A. LSE

The equation of motion of an  $m$  DOF structure can be written as:

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = f(t) \quad (1)$$

in which  $M$  = mass matrix;  $C$  = damping matrix;  $K$  = stiffness matrix;  $\ddot{X}(t), \dot{X}(t), X(t)$  = acceleration, velocity and displacement vectors;  $f(t)$  = force vector. Assuming  $M$  is a known diagonal matrix, (1) can be rewritten as:

$$A = \begin{bmatrix} \dot{X}(t) & X(t) \end{bmatrix}$$

$$P = \begin{bmatrix} C \\ K \end{bmatrix}$$

$$F(t) = f(t) - M\ddot{X}(t) \quad (2)$$

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$$A_{m \times L} P_{L \times 1} = F(t)_{m \times 1} \quad (3)$$

where  $m$ = dynamic degree of freedom;  $L$ = number of unknown parameters;  $A$ = matrix composed of the system response velocity and displacement vectors  $F(t)$ = vector composed input excitation and inertia forces. If the external excitations are available from sensor measurements, we can compose  $F(t)$  and estimate the unknown parameter with LSE method. Equations (4)-(6) show the equations of LSE method. ( $n$  = the number of samples).

$$A = [A(t_0), A(t_1), \dots, A(t_n)]^T \quad (4)$$

$$F = [F(t_0), F(t_1), \dots, F(t_n)]^T \quad (5)$$

$$P = (A^T A)^{-1} A^T F \quad (6)$$

**B. ILS-UI**

In structural systems, due to lack of access to all excitation data, at first by taking samples to length  $w$  and composed  $F(t)$  as (7), we can estimate  $P$  vector by using (4)-(6).

$$F(t) = -M\ddot{X}(t) \quad (7)$$

Then,  $i$  with sample  $(w+1)$  and using (2) and (3), we can compose  $F(t)$  into a new vector with next period of samples. Now with this new vector, we can estimate  $P$  vector with LSE again. Fig. 1 provides a flow chart to better understand the procedure.

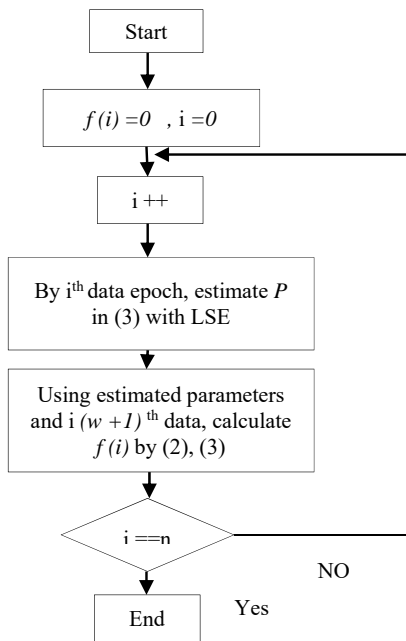


Fig. 1 Flow chart of ILS-UI

In this procedure, the selection of data in each epoch to estimate (the length of  $w$ ) is important. Note that in the LSE,

the number of samples at each epoch can affect the estimation result. If number of data in each epoch is inappropriate, then estimation in each epoch is not fully completed, so at the end of this procedure will be result high error.

In selecting length of  $w$ , the following parameters should be considered:

- 1- Number of samples,
- 2- Speed of dynamic change of displacement variable,
- 3- Sampling time.

According to the mentioned items above,  $w$  can be selected according to (8):

$$w \geq \frac{1}{q \times Ts} \quad (8)$$

**III. KF DESIGN**

KF is a Bayesian filter which can minimize the mean square of estimation error. KF can be used for unstable and time varying systems with non-stationary process and measurement noises. State space formed of structural equations with  $m$  degree of freedom (use (1)) can be used to estimate the parameters of structure with unknown input.

To form the equations in the state space model, output vector and state variables can be defined as:

$$M\ddot{X}(t) = -C\dot{X}(t) - KX(t) - f(t) \quad (9)$$

$$Z = M\ddot{X}(t) \quad (10)$$

$$U_n = \begin{bmatrix} C \\ K \end{bmatrix} \quad (11)$$

Assuming that structure parameters are constant during sampling procedure, state space equations can be expressed as:

$$U(n+1) = U(n)$$

$$Z(n) = \begin{bmatrix} -\dot{X}(n) & -X(n) \end{bmatrix} U(n) + f(n) + v(n) \quad (12)$$

where ( $U \in R^q$ ,  $q$ = number of unknown parameters),  $U$ =state vector; ( $Z \in R^n$ ),  $Z$  = output vector; ( $f \in R^n$ ),  $f$  = force vector;  $v$ = white noise. Now KF equations can be expressed as:

- Measurement Update:

TABLE I  
STRUCTURAL PARAMETERS STUDIED

Symbol	First floor	Second floor
$m$	136 Mg	66 Mg
$c$	307 k N/m <sup>2</sup>	443 k N/m <sup>2</sup>
$k$	30700 k N/m	44300 k N/m

Mg = Mega-gram, k N = kilo-Newton, m = meter.

$$K(n) = P^-(n)C^T [CP^-(n)C^T + R]^{-1} \quad (13)$$

$$\hat{U}(n) = \hat{U}^-(n) + K(n) [Z(n) - C \hat{U}^-(n)] \quad (14)$$

$$P(n) = P^-(n) - K(n) C P^-(n) \quad (15)$$

- Time Update:

$$\hat{U}^-(n+1) = \phi \hat{U}(n) \quad (16)$$

$$P^-(n+1) = \phi P(n) \phi^T \quad (17)$$

where  $(K(n) \in R^{q \times n})$ ,  $K$ = observer gain;  $\hat{U}(n)$  = the estimated state vector;  $(P(n) \in R^{q \times q})$ ,  $P$ = error covariance matrix;  $R$ = measurement noise. Also to calculate  $K$  need to estimate prior ( $\hat{U}^-$ ) of state variable. So, the updated calculations in each step is on two level; first level: time update, second level: measurement update.

#### IV. THE RESULTS OF THE SIMULATION

In this section, we present two different cases and compare the results of proposed KF with ILS-UI. In [5], the parameters of a two-story structure with real values is expressed.

The structure is assumed to be excited by  $f(t) = 10000 \sin(20t)$ , applied horizontally at the second floor. The two natural frequencies of this building are 11.83 rad/s, 32.89 rad/s.  $M$ ,  $C$  and  $K$  are as:

$$M = \text{diag}(m_1, m_2)$$

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

Now, we can compose matrix  $A$  in (4) by using the results of sampling data of acceleration sensors. Also, we can estimate the unknown parameters of this structure with using ILS-UI procedure that shown in Fig. 1. (Assuming  $n = 1500$  samples)

According to Table II, an increase in  $w$  reduces the estimation error parameters. Note that  $w$  can rise only up to 50% of samples. In Fig. 2, one can see how the convergence parameters in the final epochs. In Fig. 3, one can see estimation error of each parameter by taking 50 samples per epoch ( $w$ ). In the initial epochs, due to lack of information on excitatory input, the estimation error is very high; but, by repeating the steps shown in Fig. 1, error has been reduced in accordance with Fig. 3. So, final estimates have been drawn in Fig. 2.

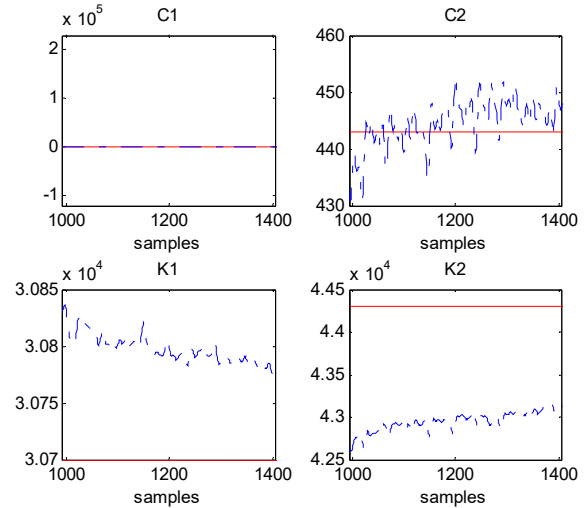


Fig. 2 Convergence parameters in the final instance, ( $w=50$ )

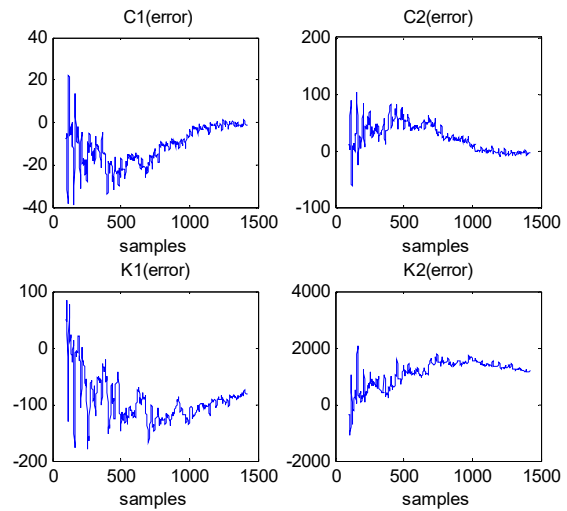


Fig. 3 Parameters estimation errors in ILS-UI procedure

TABLE II  
THE ESTIMATION RESULTS OF THE ILS-UI PROCEDURE

Symbol	Real value	Estimation With $w=10$	Estimation With $w=50$
$c1$	307 k N/m <sup>2</sup>	326.0 k N/m <sup>2</sup>	308.0k N/m <sup>2</sup>
$c2$	443 k N/m <sup>2</sup>	439.0 k N/m <sup>2</sup>	445.0 k N/m <sup>2</sup>
$k1$	30700 k N/m	31504.0 k N/m	30784.0 k N/m
$k2$	44300 k N/m	40016.0 k N/m	43135.0 k N/m

Mg = Mega-gram, k N = kilo-Newton, m = meter,  $w$  = length of samples in each epoch of estimation with LSE.

Now, if KF proposed to be used to estimate the structure parameters, the following results will be achieved.

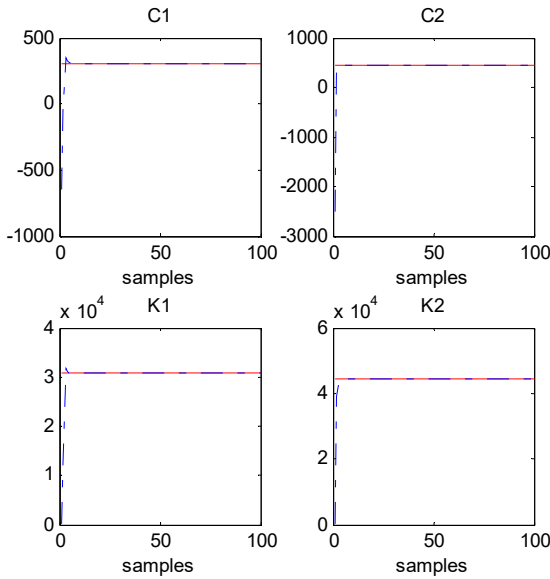


Fig. 4 Convergence parameters in the KF procedure

TABLE III  
THE ESTIMATION RESULTS OF THE KF PROCEDURE

Symbol	Real value	Estimation
c1	307 k N/m <sup>2</sup>	307.0 k N/m <sup>2</sup>
c2	443 k N/m <sup>2</sup>	443.0 k N/m <sup>2</sup>
k1	30700 k N/m	30700.0 k N/m
k2	443000 k N/m	443000.0 k N/m

Mg = Mega-gram, k N = kilo-Newton, m = meter

TABLE IV  
STRUCTURAL PARAMETERS STUDIED

Symbol	First floor	Second floor
m	100 Mg	80 Mg
k	200000 k N/m	200000 k N/m

Mg = Mega-gram, k N = kilo-Newton, m = meter.

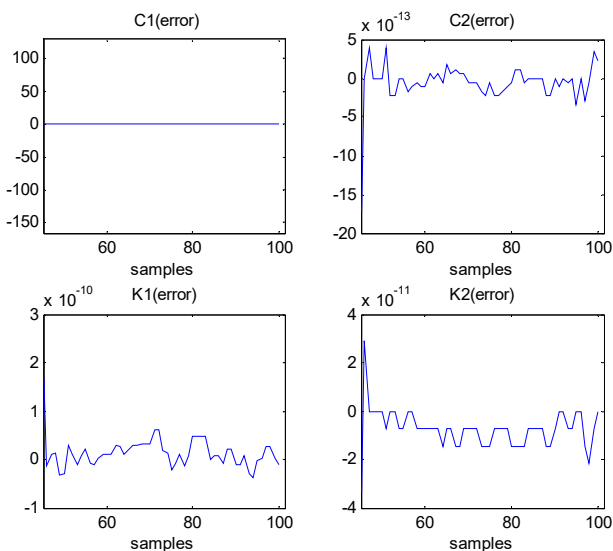


Fig. 5 Parameters estimation errors in KF procedure

As it can be seen in Figs. 4 and 2, in this procedure, convergence of parameters is much faster than ILS-UI procedure. Also according to Figs. 5 and 3, in KF procedure, convergence estimation error to zero is faster than ILS-UI. So, it can be concluded that in this example, KF procedure is more efficient than ILS-UI procedure. As another example, the parameters of a two-story structure with real values are expressed.

The structure is assumed to be excited by  $f(t) = 10000 \sin(20t)$ , applied horizontally at the second floor.  $M$ ,  $C$  and  $K$  parameters of the structure are as:

$$M = \text{diag}(m_1, m_2)$$

$$C = \alpha M + \beta K = \begin{bmatrix} 595.3403 & -190.9345 \\ -190.9345 & 361.7115 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

In this example, damping matrix ( $C$ ) of linear combination of mass ( $M$ ) and stiffness ( $K$ ) matrix is composed of two symmetric matrices. In Table V, estimated values of the parameters have been evaluated by two procedures.

TABLE V  
THE ESTIMATION RESULTS

Symbol	Real value	Estimation (ILS-UI)	Estimation (KF)
c1	595.3 k N/m <sup>2</sup>	580.0 k N/m <sup>2</sup>	600.0k N/m <sup>2</sup>
-c2	190.9 k N/m <sup>2</sup>	200.0 k N/m <sup>2</sup>	190.9 k N/m <sup>2</sup>
-c3	190.9 k N/m <sup>2</sup>	200.0 k N/m <sup>2</sup>	190.9 k N/m <sup>2</sup>
c4	361.7 k N/m <sup>2</sup>	340.0 k N/m <sup>2</sup>	360.0 k N/m <sup>2</sup>
k1	200000 k N/m	200050 k N/m	200000 k N/m
k2	200000 k N/m	199970 k N/m	200000 k N/m

Mg = Mega-gram, k N = kilo-Newton, m = meter.

According to Fig. 6, in KF procedure, convergence parameters are much faster than ILS-UI procedure. Also according to Figs. 7 and 8, in KF procedure, convergence estimation error to zero is faster than ILS-UI. Such as before example, it can be concluded that KF procedure is more efficient than ILS-UI procedure.

## V. CONCLUSION

In this paper, KF procedure is expressed to identify the parameters for unknown excitation with providing state space model for structural system. According to the simulations, the advantages offered by KF procedure are more than ILS-UI. In KF-based algorithm, the parameters convergence is much faster than ILS-UI. It should be noted that ILS-UI could estimate the parameters without any information of excitation input, while in KF procedure measurement noise covariance matrix for the measurement noise is required.

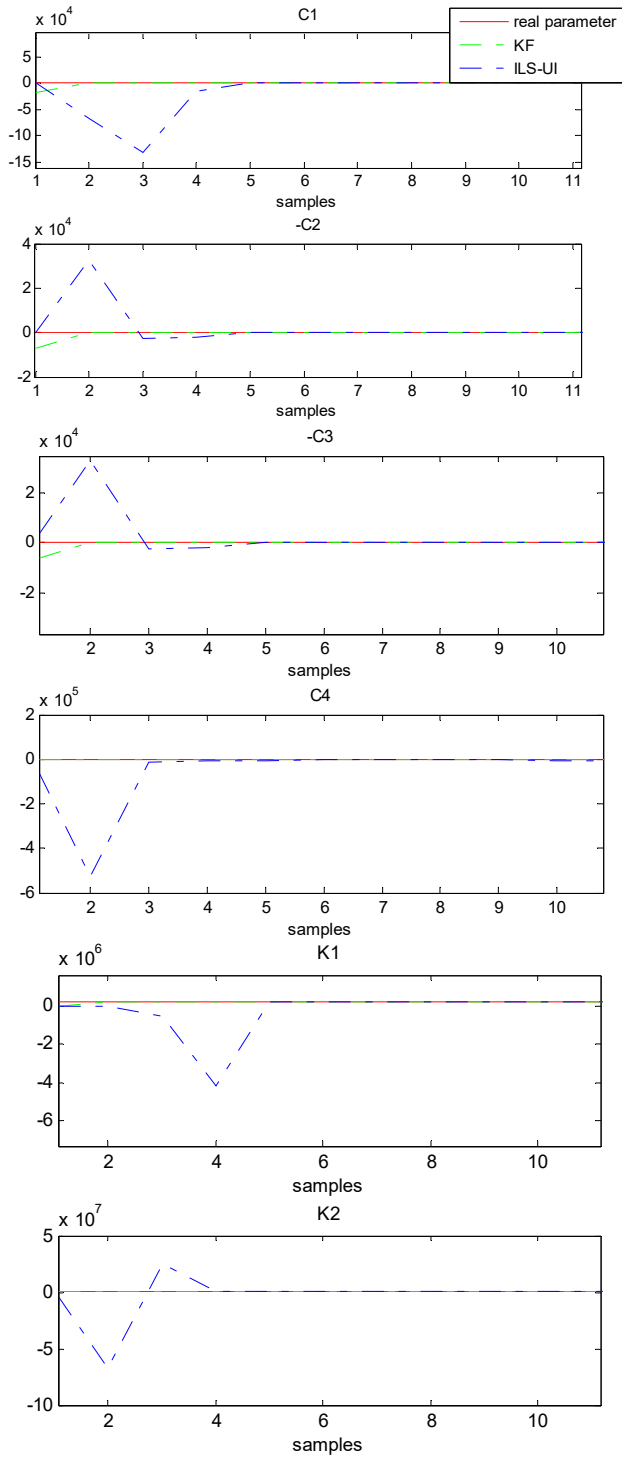


Fig. 6 Convergence parameters in the KF and ILS-UI procedure

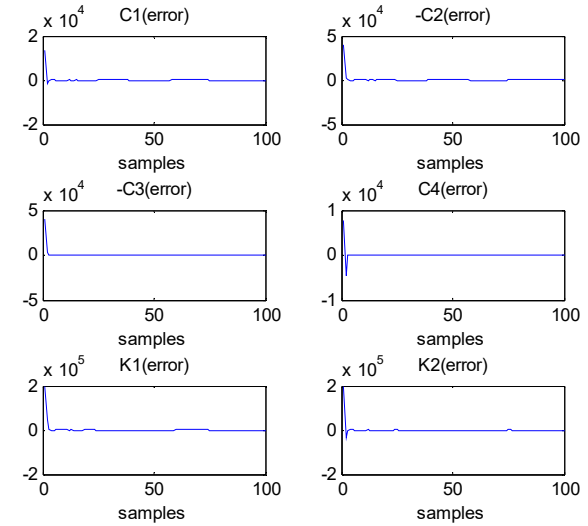


Fig. 7 Parameters estimation errors in ILS-UI procedure

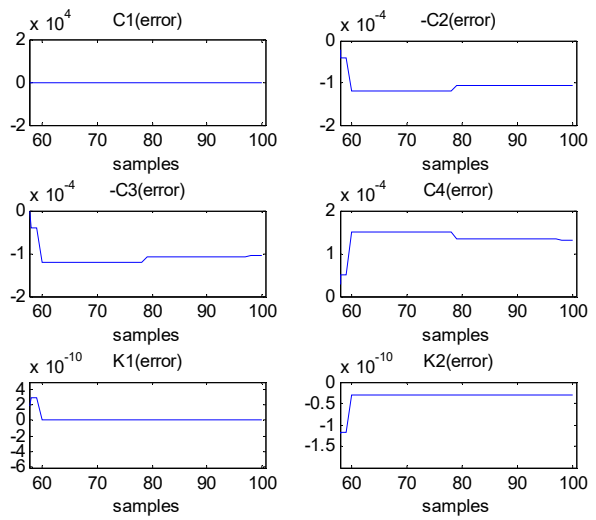


Fig. 8 Parameters estimation errors in KF procedure

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