

# Ion- Acoustic Solitary Waves in a Self- Gravitating Dusty Plasma Having Two-Temperature Electrons

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**Abstract**—Nonlinear propagation of ion-acoustic waves in a self-gravitating dusty plasma consisting of warm positive ions, isothermal two-temperature electrons and negatively charged dust particles having charge fluctuations is studied using the reductive perturbation method. It is shown that the nonlinear propagation of ion-acoustic waves in such plasma can be described by an uncoupled third order partial differential equation which is a modified form of the usual Korteweg-deVries (KdV) equation. From this nonlinear equation, a new type of solution for the ion-acoustic wave is obtained. The effects of two-temperature electrons, gravity and dust charge fluctuations on the ion-acoustic solitary waves are discussed with possible applications.

**Keywords**—Charge fluctuations, gravitating dusty plasma, Ion-acoustic solitary wave, Two-temperature electrons

## I. INTRODUCTION

IN recent years interest in the field of dusty plasma has been growing rapidly because of its existence in various environments like cometary tails, planetary rings, asteroids, magnetosphere, lower ionosphere, interstellar and circumstellar clouds, laboratory devices etc. Dust grains immersed in ambient plasmas are electrically charged by various processes. The charge on the dust grains influences their motion in electromagnetic fields and also affects the coagulation rate of dust into larger bodies. When the dust intergrain average separation is smaller than plasma Debye length the collective rather than single-particle effects of dust grains become important. It has been found both theoretically and experimentally that the presence of charged dust grains modifies the existing plasma wave spectra. Also in some cases the presence of massive charged dust grains gives rise to new low-frequency eigen modes of the dust-electron-ion plasma. The existence of dust-acoustic wave (DAW) was first predicted theoretically by Rao et al [1], with the dust grains providing the inertia and the pressure of inertialess electrons and ions providing the restoring force. The existence of dust-ion-acoustic wave (DIAW) was predicted by Shukla and Silin [2]. The existence of these new eigen modes has also been confirmed experimentally [3-6]. Rao [7] has also found the Alfvén and magnetosonic modes in dusty plasma. The presence of charged massive dust grains can significantly modify the linear and nonlinear wave propagation through plasma. When the size of the dust grains becomes considerable the gravitational effects of dust grains become

important though the effect is certainly negligible for electrons and ions. In fact a number of authors have considered nonlinear wave propagation in self-gravitating dusty plasma where there is a competition between gravitational self-attraction and electrostatic repulsion between the charged grains, apart from other electromagnetic effects. It has been found that the gravitational effect can also significantly influence the nonlinear wave propagation through dusty plasma. In magnetized dusty plasma with medium-sized grains the gravitational and magnetic effects may become comparable. It has been shown that self-gravitational effect in dusty plasma may lead to macroscopic instability of the Jeans type [8-11]. The magneto-gravitational instability of self-gravitating dusty plasma is relevant to the understanding of star formation. Thus the presence of charged dust grains in plasma influences wave propagation and various other collective properties of the plasma. It is well known that dust particles get negatively charged due to attachment of the background electrons and ions on the surface via collisions [12]. The electrostatic charging of dust grains immersed in plasma is the main feature of dust-plasma interaction in dusty plasma. The charge on a dust particle does not remain fixed but depends on plasma properties, electron and ion currents flowing into or out of dust grains, photoemission etc. In fact the charge on the dust grain can be considered as an extra dynamical variable which controls the motion of dust grains in fluctuating electrostatic fields. It has been shown that the charge fluctuation of the dust grains plays significant role on the dynamical behaviour of dusty plasmas [13, 14]. Most of the earlier works on solitons in dusty plasma are for the dust acoustic waves [15-18], but ion-acoustic solitons can also be excited in dusty plasma. Paul et al [19] have studied the effects of streaming and attachment coefficients of ions and electrons on the formation of soliton in dusty plasma without considering the effects of gravity. They derived an inhomogeneous Korteweg deVries (KdV) equation and predicted the existence of a negative soliton in dusty plasma under some critical conditions and non-existence of solitons in dusty plasma with variable dust charges. Considering a self-gravitating dusty plasma having dust charge fluctuation Paul et al [20] investigated the possibility of the existence of ion-acoustic solitary wave structures in gravitating dusty plasma with warm electrons, warm ions and cold dust particles having charge fluctuations. They showed that the nonlinear excitation of such waves follows a pair of coupled third order partial differential equations which is slightly different from the usual case of coupled KdV system; solitary wave structure is not possible in such self-gravitating dusty plasma with warm electrons and varying charges on the dust particles. However when the variation of charges of the dust particles is neglected the solitary waves may be excited. The amplitude and width of

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such solitary waves are shown to be significantly changed by the gravitational effects on the dusty plasma. However, the effects of two-temperature electrons [21] on ion-acoustic solitary waves in gravitating dusty plasma with the fluctuating dust-charges have not yet been studied. It has been found that the presence of two-temperature electrons i.e. the electrons at two different temperatures give rise to fascinating results on solitary waves in plasma [22-27]. So, in this paper, the existence and characteristics of ion-acoustic solitary waves in self-gravitating dusty plasma consisting of warm positive ions, isothermal two-temperature electrons and dust particles with charge variations is theoretically studied. An uncoupled third order partial differential equation which is a modified form of Korteweg-deVries (KdV) equation is derived using the reductive perturbation method. From this uncoupled nonlinear equation, quasi-soliton solution of the ion acoustic wave is obtained. The effects of two-temperature electrons, gravity and dust charge fluctuations on the ion acoustic solitary waves are discussed. The paper is organized in the following way. In section II, the basic equations are presented. In section III the modified KdV equation is derived using reductive perturbation method. Section IV contains results and discussions. The paper ends up with some concluding remarks in section V.

## II. BASIC EQUATIONS

We consider unmagnetized three-component plasma consisting of warm ions, warm micron-sized massive negatively charged dust grains and isothermal two-temperature electrons. The dust particles are mainly charged due attachment of electrons and ions. Once the dust particles get enough negative charge the chance of an electron being attached to a negatively charged dust grains becomes much smaller than the chance of an ion being attached to the dust grain. Under this situation, we may assume that the charge fluctuation of dust grains is due to the attachment of positive ions only. Moreover, we consider the self gravitating effect on the electrostatic dust acoustic wave in dusty plasma which is expected to influence both the linear and nonlinear modes of wave propagation. Under such assumptions, the set of normalized basic equations governing the plasma dynamics are the following:

For ions:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = -\alpha_i n \quad (1)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{\sigma_i}{n_i} \frac{\partial p_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} - \alpha_i (v_i - v_d) \quad (2)$$

$$\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial v_i}{\partial x} = 0 \quad (3)$$

For dust particles:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d v_d) = 0 \quad (4)$$

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} + \frac{\sigma_d}{\mu_d n_d} \frac{\partial p_d}{\partial x} = -\frac{Z_d}{\mu_d} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} - \alpha_i (v_d - v_i) \quad (5)$$

$$\frac{\partial p_d}{\partial t} + v_d \frac{\partial p_d}{\partial x} + 3p_d \frac{\partial v_d}{\partial x} = 0 \quad (6)$$

Poisson's equations:

$$\frac{\partial^2 \phi}{\partial x^2} = n_{ec} + n_{eh} - n_i + Z_d n_d \quad (7)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\omega_{gi}^2}{m_i \omega_i^2} \sum_{s=e,i,d} m_s n_s = X_i \sum_{s=i,d} m_s n_s \quad (8)$$

Charge neutrality condition:

$$n_{io} = n_{eco} + n_{eho} + Z_d n_{do} \quad (9)$$

where the subscripts s = i and d represents ions and dust particles respectively;  $m_s$ ,  $n_s$ ,  $v_s$ ,  $q_s$  and  $p_s$  denote respectively the mass, number density, velocity, charge and scalar pressure of the s-th species of the plasma;  $n_{ec}$ ,  $n_{eh}$  are number density of electrons at low and high temperatures,  $Z_d$  is the number of electronic charge attached on the grains;

$$\mu_d = m_d / m_i, \sigma_i = T_i / T_{ef}, \sigma_d = T_d / T_{ef}$$

$$T_{ef} = T_{ec} T_{eh} / (\mu T_{eh} + \nu T_{ec}), X_i = \omega_{gi}^2 / m_i \omega_i^2, \omega_{gi}^2 = 4\pi G m_i n_i;$$

$n_{io}, n_{eco}(n_{eho})$  and  $n_{do}$  are equilibrium number densities of ions, electrons and dust particles;  $G$  is the universal gravitational constant,  $\beta_i$  is the attachment coefficient of the ions to the dust grain;  $\phi$  and  $\psi$  are respectively the electrostatic and gravitational potentials;  $T_{ec}$  and  $T_{eh}$  are the low- and high- temperatures of electrons; other symbols have their usual meanings. In the above equations we have normalized distance  $x$  by Debye length, time  $t$  by the inverse of ion plasma frequency, all pressures by ion plasma pressure  $n_{i0} k_B T_i$ ,  $\phi$  by  $k_B T_e / e$ ,  $\psi$  by  $k_B T_e / m_i$  and all number densities by  $n_{i0}$ . For the isothermal electrons at two-temperatures (cold and hot), the number densities are given by

$$n_{ec} = \mu \exp\left[-\frac{\phi}{\mu + \nu\beta}\right], n_{eh} = \nu \exp\left[\frac{\beta\phi}{\mu + \nu\beta}\right] \quad (10)$$

where,  $\beta = T_{ec} / T_{eh}$ ,  $\mu (= n_{eco})$  and  $\nu (= n_{eho})$  are equilibrium number density of low- and high- temperature electrons.

III. DERIVATION OF MODIFIED KDV EQUATION

For the derivation of the nonlinear equation governing the nonlinear dynamics of the wave, we make the usual stretching of the space co-ordinates and time,

$$\xi = \varepsilon^{1/2}(x - Vt) \text{ and } \tau = \varepsilon^{3/2}t \tag{11a}$$

where,  $V$  is the linear phase velocity and  $\varepsilon$  is a smallness parameter measuring the dispersion and nonlinear effects. Further, we assume the following perturbation expansion for the field variables:

$$Y = Y_0 + \varepsilon Y_1 + \varepsilon^2 Y_2 + \varepsilon^3 Y_3 + \dots \tag{11b}$$

$$\beta_i = \varepsilon^{5/2} \bar{\beta}_i.$$

where  $Y$  stands for  $n_s, v_s, p_s, \phi, \psi$  with  $\phi_0 = 0$

Using (11a) and (11b) in (1) - (8) and equating the coefficients of the lowest order in  $\varepsilon$  we obtain

$$n_{d1} = \frac{Vn_{d0}}{(V - v_{d0})} v_{d1} \tag{12}$$

$$v_{i1} = \frac{1}{(V - v_{i0})} [n_{i0} a_1 p_{i1} + a_2 \phi_1 + a_3 \psi_1],$$

$$v_{d1} = \frac{1}{\mu_d (V - v_{d0})} [n_{d0} b_1 p_{d1} + b_2 \phi_1 + b_3 \psi_1] \tag{13}$$

$$p_{i1} = \frac{3p_{i0}}{(V - v_{i0})} v_{i1}, p_{d1} = \frac{3p_{d0}}{(V - v_{d0})} v_{d1} \tag{14}$$

$$0 = \phi_1 - n_{i1} + Z_d n_{d1} \tag{15}$$

$$0 = X_i (m_i n_{i1} + m_d n_{d1}) \tag{16}$$

where,

$$a_1 = \sigma_i, a_2 = 1, a_3 = 1, b_1 = \sigma_d n_{d0}, b_2 = Z_d, b_3 = \mu_d,$$

$$X_i = \omega_{gi}^2 / m_i \omega^2 \text{ and the mass of electron } m_e \text{ is neglected}$$

in comparison with the masses of ions and dust particles.

The first order density and velocity of the ions and dust particles obtained from (12) - (14) are

$$v_{i1} = \frac{(V - v_{i0})(\phi_1 + \psi_1)}{(V - v_{i0})^2 - 3p_{i0}n_{i0}\sigma_i}, v_{d1} = \frac{(V - v_{d0})(Z_d \phi_1 + \mu_d \psi_1)}{(V - v_{d0})^2 - 3p_{d0}n_{d0}\sigma_d} \tag{17}$$

$$n_{i1} = \frac{(\phi_1 + \psi_1)}{(V - v_{i0})^2 - 3p_{i0}n_{i0}\sigma_i}, n_{d1} = \frac{(Z_d \phi_1 + \mu_d \psi_1)}{(V - v_{d0})^2 - 3p_{d0}n_{d0}\sigma_d} \tag{18}$$

Using (18), the values of  $n_{i1}$  and  $n_{d1}$ , equation (15) yields

$$Y_1 \phi_1 = Z_1 \psi_1 \tag{19}$$

Similarly, Eq. (16) gives

$$Y_2 \phi_1 = Z_2 \psi_1 \tag{20}$$

From (19) and (20) the linear dispersion relation for the ion-acoustic wave in self-gravitating dusty plasma is derived as

$$Y_1 Z_2 - Y_2 Z_1 = 0 \tag{21}$$

where,

$$Y_1 = 1 - \frac{a_2 n_{i0}}{\lambda_1^2} + \frac{Z_d n_{d0}}{\lambda_2^2}, Z_1 = \frac{n_{i0}}{\lambda_1^2} - \frac{Z_d n_{d0}}{\lambda_2^2},$$

$$Y_2 = \frac{1}{\omega_i^2} \left( \frac{n_{i0} w_{gi}^2}{\lambda_1^2} + \frac{Z_d n_{d0} w_{gd}^2}{\lambda_2^2} \right),$$

$$Z_2 = -\frac{1}{\omega_i^2} \left( \frac{n_{i0} w_{gi}^2}{\lambda_1^2} + \frac{\mu_d n_{d0} w_{gd}^2}{\lambda_2^2} \right), \lambda_1 = V_1^2 - 3a_1 n_{i0} p_{i0},$$

$$\lambda_2 = \mu_d V_2^2 - 3b_1 n_{d0} p_{d0}$$

$$V_1 = V - v_{i0}, V_2 = V - v_{d0},$$

$$w_{gi}^2 = 4\pi G m_i n_{i0}, w_{gd}^2 = 4\pi G m_d n_{d0}$$

From (19) and (20) a relation between  $\phi_1$  and  $\psi_1$  can be obtained

$$\psi_1 = \left( \frac{Y_1 Y_2}{Z_1 Z_2} \right)^{1/2} \phi_1 \tag{22a}$$

or,

$$\phi_1 = \left( \frac{Z_1 Z_2}{Y_1 Y_2} \right)^{1/2} \psi_1 \tag{22b}$$

Now equating the coefficients of  $\varepsilon^2$  we obtain from (1)-(8)

$$\frac{\partial n_{i1}}{\partial \tau} + \frac{\partial}{\partial \xi} (n_{i1} v_{i1}) + \bar{\alpha}_i n_{i0} = (V - v_{i0}) \frac{\partial n_{i2}}{\partial \xi} - n_{i0} \frac{\partial v_{i2}}{\partial \xi} \tag{23a}$$

$$\frac{\partial v_{i1}}{\partial \tau} + v_{i1} \frac{\partial v_{i1}}{\partial \xi} + \bar{\alpha}_i (v_{i0} - v_{d0}) = (V - v_{i0}) \frac{\partial v_{i2}}{\partial \xi} - a_1 n_{i0} \frac{\partial p_{i2}}{\partial \xi} - a_2 \frac{\partial \phi_2}{\partial \xi} - a_3 \frac{\partial \psi_2}{\partial \xi} \tag{23b}$$

$$\frac{\partial p_{i1}}{\partial \tau} = (V - v_{i0}) \frac{\partial p_{i2}}{\partial \xi} - 3 \frac{\partial v_{i2}}{\partial \xi} \tag{23c}$$

$$\frac{\partial n_{d1}}{\partial \tau} = (V - v_{d0}) \frac{\partial n_{d2}}{\partial \xi} - n_{d0} \frac{\partial v_{d2}}{\partial \xi} \tag{24a}$$

$$\frac{\partial v_{d1}}{\partial \tau} + v_{d1} \frac{\partial v_{d1}}{\partial \xi} + \bar{\alpha}_i (v_{d0} - v_{i0}) = (V - v_{d0}) \frac{\partial v_{d2}}{\partial \xi} - b_1 n_{d0} \frac{\partial p_{d2}}{\partial \xi} - b_2 \frac{\partial \phi_2}{\partial \xi} - b_3 \frac{\partial \psi_2}{\partial \xi} \tag{24b}$$

$$\frac{\partial p_{d1}}{\partial \tau} = (V - v_{i0}) \frac{\partial p_{d2}}{\partial \xi} - 3 \frac{\partial v_{d2}}{\partial \xi} \quad (24c)$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 + \frac{(\mu + \nu \beta^2)}{2(\mu + \nu \beta)} \phi_1^2 - n_{i2} + Z_d n_{d2} \quad (25)$$

and

$$\frac{\partial^2 \psi_1}{\partial \xi^2} = X_i (m_i n_{i2} + m_d n_{d2}) \quad (26)$$

Using (23) - (26), we eliminate  $v_{i2}, v_{d2}, P_{i2}$  and  $P_{d2}$  to find the values of  $\partial n_{i2} / \partial \xi$  and  $\partial n_{d2} / \partial \xi$  in terms first order values of the field quantities and obtain the following equations from (25) and (26):

$$\begin{aligned} & \frac{\partial}{\partial \tau} [P_1(\phi_1 + \psi_1) - Z_d P_2(Z_d \phi_1 + \mu_d \psi_1)] + Q_1(\phi_1 + \psi_1) \\ & \frac{\partial}{\partial \xi} (\phi_1 + \psi_1) - Z_d Q_2(Z_d \phi_1 + \mu_d \psi_1) - \frac{\partial}{\partial \xi} (Z_d \phi_1 + \mu_d \psi_1) \\ & + \frac{\partial^3 \phi_1}{\partial \xi^3} - \frac{(\mu + \nu \beta^2)}{(\mu + \nu \beta)} \phi_1 \frac{\partial \phi_1}{\partial \xi} - (R_1 + R_2) \\ & = (1 + \frac{n_{i0}}{\lambda_1 \lambda_3} - \frac{n_{d0}}{\lambda_2 \lambda_4} \frac{Z_d}{\mu_d}) \frac{\partial \phi_2}{\partial \xi} + (\frac{n_{i0}}{\lambda_1 \lambda_3} - \frac{n_{d0}}{\lambda_2 \lambda_4} \frac{Z_d}{\mu_d}) \frac{\partial \psi_2}{\partial \xi} \end{aligned} \quad (27)$$

$$\begin{aligned} & \frac{\partial}{\partial \tau} [X_i m_i P_1(\phi_1 + \psi_1) + X_i m_d P_2(Z_d \phi_1 + \mu_d \psi_1)] \\ & + X_i m_i Q_1(\phi_1 + \psi_1) \frac{\partial}{\partial \xi} (\phi_1 + \psi_1) + X_i m_d Q_2(Z_d \phi_1 + \mu_d \psi_1) \\ & \frac{\partial}{\partial \xi} (Z_d \phi_1 + \mu_d \psi_1) + \frac{\partial^3 \psi_1}{\partial \xi^3} - X_i (m_i R_1 + m_d R_2) \\ & = -(\frac{n_{i0}}{\lambda_1 \lambda_3} + \frac{n_{d0}}{\lambda_2 \lambda_4} \frac{Z_d}{\mu_d}) \frac{\partial \phi_2}{\partial \xi} - (\frac{n_{i0}}{\lambda_1 \lambda_3} + \frac{n_{d0}}{\lambda_2 \lambda_4} \frac{Z_d}{\mu_d}) \frac{\partial \psi_2}{\partial \xi} \end{aligned} \quad (28)$$

These two equations can be thought to be two nonlinear inhomogeneous equation in  $\partial \phi_2 / \partial \xi$  and  $\partial \psi_2 / \partial \xi$ . It is seen that equations (27) and (28) are coupled to each other by  $\phi_1, \psi_1$  and  $\psi_2$ . Using (22a) and (22b) in (27) and (28), the uncoupled nonlinear equations of  $\phi_1$  and  $\psi_1$  can be easily obtained. For non-zero solutions of  $\partial \phi_2 / \partial \xi$  and  $\partial \psi_2 / \partial \xi$ , the right hand side of (27) and (28) must vanish leading to the following nonlinear equations which are the modified form of the Korteweg de Vries(K-dV) equation.

$$A_1 \frac{\partial \phi_1}{\partial \tau} + B_1 \phi_1 \frac{\partial \phi_1}{\partial \xi} + C_1 \frac{\partial^3 \phi_1}{\partial \xi^3} - \gamma_1 = 0 \quad (29)$$

$$A_2 \frac{\partial \psi_1}{\partial \tau} + B_2 \psi_1 \frac{\partial \psi_1}{\partial \xi} + C_2 \frac{\partial^3 \psi_1}{\partial \xi^3} - \gamma_2 = 0 \quad (30)$$

where,

$$\begin{aligned} A_1 &= P_1 [1 + (\frac{Y_1 Y_2}{Z_1 Z_2})^{1/2}]^2 \\ & - Z_d P_2 [Z_d + \mu_d (\frac{Y_1 Y_2}{Z_1 Z_2})^{1/2}] \end{aligned}$$

$$\begin{aligned} A_2 &= X_i m_i P_1 [1 + (\frac{Z_1 Z_2}{Y_1 Y_2})^{1/2}] \\ & + X_i m_d P_2 [Z_d (\frac{Z_1 Z_2}{Y_1 Y_2})^{1/2} + \mu_d] \end{aligned}$$

$$\begin{aligned} B_1 &= Q_1 [1 + (\frac{Y_1 Y_2}{Z_1 Z_2})^{1/2}]^2 - \frac{(\mu + \nu \beta^2)}{(\mu + \nu \beta)} \\ & - Z_d Q_2 [Z_d + \mu_d (\frac{Y_1 Y_2}{Z_1 Z_2})^{1/2}]^2 \end{aligned}$$

$$\begin{aligned} B_2 &= X_i m_i Q_1 [1 + (\frac{Z_1 Z_2}{Y_1 Y_2})^{1/2}]^2 \\ & + X_i m_d Q_2 [Z_d (\frac{Z_1 Z_2}{Y_1 Y_2})^{1/2} + \mu_d]^2 \end{aligned}$$

$$\begin{aligned} C_1 &= 1, C_2 = 1, \gamma_1 = R_1 + R_2, \\ \gamma_2 &= X_i m_i R_1 + X_i m_d R_2. \end{aligned}$$

$$P_1 = \frac{n_{i0}}{\lambda_1 \lambda_3 \lambda_5} [1 + \lambda_3 + \frac{1}{\lambda_1} (\lambda_1^2 - \lambda_5)],$$

$$Q_1 = \frac{n_{i0}}{\lambda_1 \lambda_3 \lambda_5} [1 + \lambda_3 \lambda_5 + (1 + \frac{1}{\lambda_1}) (\lambda_1^2 - \lambda_5)]$$

$$P_2 = \frac{n_{d0}}{\lambda_2 \lambda_4 \lambda_6} [1 + \lambda_4 + \frac{1}{\lambda_2} (\lambda_2^2 - \lambda_6)],$$

$$Q_2 = \frac{n_{d0}}{\lambda_2 \lambda_4 \lambda_6} [\lambda_2^2 - \lambda_6]$$

$$R_1 = \frac{n_{i0}}{\lambda_1 \lambda_3} (v_{i0} - v_{d0} - 1) \alpha_i, \quad R_2 = \frac{n_{d0}}{\lambda_2 \lambda_4} (v_{d0} - v_{i0}) \alpha_i$$

From (29) and (30) it is observed that the nonlinear terms  $B_1$  and  $B_2$  depend on the gravitational effects of the ions and the dust particles. But these terms are free from the fluctuations of dust charges.

#### IV. RESULTS AND DISCUSSIONS

The modified inhomogeneous K-dV equations (29) and (30) will have the solutions for ion-acoustic solitary waves in a self-gravitating dusty plasma with fluctuating dust charges and two-temperature electrons. It is important to note that the inhomogeneous terms  $\lambda_1$  and  $\lambda_2$  arise due to the charge variations of the dust particles. If we neglect the dust charge variation ( $\alpha_i = 0$ ) i.e.  $\lambda_1 = 0, \lambda_2 = 0$ , then (29) and (30) reduce to standard K.dV equations in isothermal plasma and their solutions are obtained using the transformation  $\theta = \xi - U\tau$

$$\varphi_1 = \Phi_0 \sec h^2 \left[ \frac{1}{2} \sqrt{\frac{UA_1}{C_1}} \theta \right] \quad (31)$$

and

$$\psi_1 = \Psi_0 \sec h^2 \left[ \frac{1}{2} \sqrt{\frac{UA_2}{C_2}} \theta \right] \quad (32)$$

where  $\Phi_0 = \frac{3UA_1}{B_1}$  and  $\Psi_0 = \frac{3UA_2}{B_2}$  are the amplitudes,  $U$  is the velocity of the ion-acoustic solitary waves.

Considering the charge fluctuation of dust particles,  $\lambda_1 \neq 0$  and  $\lambda_2 \neq 0$ , the solutions of Eqs.(29) and (30) for the solitary waves can be obtained following the works of Waditi [28].

By using the transformation  $\xi = \chi\eta$  and  $\tau = w t$ , we get from (29),

$$\frac{\partial \varphi_1}{\partial \tau} + \frac{B_1 w}{A_1 \chi} \varphi_1 \frac{\partial \varphi_1}{\partial \eta} + \frac{C_1 w}{A_1 \chi^3} \frac{\partial^3 \varphi_1}{\partial \tau^3} = \frac{w \gamma_1}{A_1} \quad (33)$$

But, the exact form of Waditi's equation is

$$\frac{\partial \varphi_1}{\partial \tau} + 6\varphi_1 \frac{\partial \varphi_1}{\partial \eta} + \frac{\partial^3 \varphi_1}{\partial \tau^3} = p_1(\tau) \quad (34)$$

which could be obtained from (33) by assuming,

$$B_1 w / A_1 \chi = -6 \text{ and } w / A_1 \chi^3 = 1.$$

Now, Eq. (34) is connected to the K-dV equation

$$\frac{\partial \bar{\varphi}_1}{\partial \tau} + 6\bar{\varphi}_1 \frac{\partial \bar{\varphi}_1}{\partial \eta} + \frac{\partial^3 \bar{\varphi}_1}{\partial \tau^3} = 0 \quad (35)$$

by the transformation  $\varphi_1 = \bar{\varphi}_1(X, t) + W(t)$ , where

$$X = x + m(t), m(t) = \int W(t') dt' \text{ and } W = p_1(t') dt'.$$

The solution of (35) is

$$\varphi_{1\gamma} = \varphi_0 \sec h^2 \theta_\gamma + \frac{\gamma_1 \tau}{A_1} \quad (36)$$

$$\text{where, } \varphi_0 = \frac{1}{3B_1 \chi^2}, \theta_\gamma = \frac{\zeta}{\chi} + \frac{3\lambda \tau^2}{A_1 w} - \frac{4\tau}{w}$$

The solution (36) shows that the structure of the solitary wave will deviate from its usual form due to the effects of i) dust charge fluctuations, ii) presence of two-temperature electrons, iii) drift velocity of the ions and dust particles and iv) gravitational forces. The role of these parameters on the solitary wave structure and its stability can be studied numerically by considering a model dusty plasma. In natural dusty plasma it would be more appropriate to consider the presence of magnetic field which we have not considered in the present work. We are working on this field and it is expected to be the subject matter of a future paper.

#### IV. SUMMARY AND CONCLUDING REMARKS

To describe the nonlinear propagation of ion-acoustic waves through self-gravitating dusty plasma with fluctuating

dust charges we have derived two uncoupled nonlinear equations which are the modified form of the usual KdV equation and includes the effects of gravity, dust charge fluctuations and isothermal two-temperature electrons. We have obtained the soliton solution of the modified KdV equation in gravitating dusty plasma with fluctuating dust charges. The stability or instability of the soliton depends on the sign of  $\gamma_1$  in the modified KdV equation (29). The attachment coefficient, nonthermal parameter and gravitational effects all enter into the factor  $\gamma_1$  in a complicated way. The stability or instability of the solitary structure will therefore be determined by the relative magnitudes of these effects. Thus, the two-temperature electrons, the attachment coefficients and the gravitational effects involved in  $\gamma_1$  will significantly influence the soliton properties including soliton amplitude, speed, width as well as its stability. In most astrophysical situations the dusts can be considered as cold, but there are some experimental situations where dust temperature may become comparable to ion temperature and in this case our analysis will become important. It is important to point out that starting from Eq. (30) for  $\psi$  and following the same method as adopted above one can examine the possibility of having soliton-like variation of the gravitational potential.

To summarize theoretical infrastructure has been developed for K-dV solitary waves in a complex self-gravitating dusty plasma with two-temperature electrons and dust charge fluctuations. The most important achievement in this paper is that instead of coupled nonlinear equations as obtained by earlier authors uncoupled nonlinear equations have been obtained which is advantageous. Also, the solution obtained here for the ion-acoustic waves in self-gravitating dusty plasma here is new. The beauty of the model considered in the present paper is that it can be easily extended to study the effects of different types of electron distributions on the nonlinear propagation of waves in self-gravitating plasma.

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