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Investigations on some operations of soft sets

Xun Ge and Songlin Yang

Abstract—Soft set theory was initiated by Molodtsov in 1999. In the past years, this theory had been applied to many branches of mathematics, information science and computer science. In 2003, Maji et al. introduced some operations of soft sets and gave some operational rules. Recently, some of these operational rules are pointed out to be not true. Furthermore, Ali et al., in their paper, introduced and discussed some new operations of soft sets. In this paper, we further investigate these operational rules given by Maji et al. and Ali et al.. We obtain some sufficient-necessary conditions such that corresponding operational rules hold and give correct forms for some operational rules. These results will be help for us to use rightly operational rules of soft sets in research and application of soft set theory.

Keywords—Soft sets, union, intersection, complement.

I. Introduction

Soft set theory was initiated by Molodtsov in [8]. As a mathematical tool for dealing with uncertainties, soft set theory has a rich potential for applications in several directions. In the past years, this theory had aroused us interesting and concerning, and had been applied to many branch of mathematics, information science and computer science ([1], [3], [4], [5], [6], [9], [10], [11], [13]). In [7], Maji et al. introduced some operations of soft sets and give their operational rules, which makes a theoretical study of the soft set theory in more detail. Recently, Yang and Ali et al. illuminated some operational rules in [7] to be not true by some examples ([12], [2]). Furthermore, Ali et al. introduced some new operations of soft sets and give their operational rules ([2]). In this paper, we further investigate these operational rules in [2], [7] and obtain some interesting results including some different viewpoints with [2]. We obtain some sufficient-necessary conditions such that corresponding operational rules hold and give correct forms for some operational rules. These results will be help for us to use rightly operational rules of soft sets in research and application of soft set theory.

Throughout this paper, U is an initial universe set and E is the set of all possible parameters under consideration with respect to U. Each set of parameters is a subset of E and each soft set is over U.

II. Definitions and Remarks

Definition 2.1 ([8]): Let A be a set of parameters. A pair (F, A) is called a soft set over U if F is a mapping given by $F : A \longrightarrow \mathcal{P}(U)$, where $\mathcal{P}(U)$ is the family of all subsets of U.

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Definition 2.2 ([7]): Let A be a set of parameters. $\{\neg a : a \in A\}$ is called the Not-set of A denoted by $\neg A$, where $\neg a$ means not a for each $a \in A$.

Remark 2.3: It is necessary to give the following assumption.

Assumption: The Not-set of each set of parameters is a subset of E.

Proposition 2.4 ([7]): Let A, B be two sets of parameters, then the following hold.

(1) $\neg(\neg A) = A$.

(2) $\neg(A \bigcup B) = \neg A \bigcup \neg B$.

(2) $\neg(A \bigcap B) = \neg A \bigcap \neg B$.

Definition 2.5 ([7]): Let (F, A) be a soft set.

(1) (F, A) is called to be a null soft set denoted by Φ, if $F(a) = \emptyset$ for each $a \in A$.

(2) (F, A) is called to be an absolute soft set denoted by $\widetilde{A}$, if $F(a) = U$ for each $a \in A$.

Note that several assertions in relation to null soft sets and absolute soft sets in [7] are incorrect, which may be due to the notations of the related definitions in [7]. Ali et al. gave the following definition.

Definition 2.6 ([2]): Let (F, A) be a soft set.

(1) (F, A) is called to be a relative null soft set (with respect to the parameter set A) denoted by Φ_A, if $F(a) = \emptyset$ for each $a \in A$.

(2) (F, A) is called to be a relative whole soft set denoted by $\mathscr{U}_A$, if $F(a) = U$ for each $a \in A$.

Remark 2.7: Except for their notations, null soft set and absolute soft set in the sense of [7] are equivalent to relative null soft set and relative whole soft set in the sense of [2], respectively.

(1) Indeed, the notation Φ of a null soft set will result in some confusions, which results from that the set of parameters for Φ is not clear. For example, let (F, A) and (G, B) be two soft sets, where $A \neq B$, $F(a) = \emptyset$ for each $a \in A$ and $G(b) = \emptyset$ for each $b \in B$. Then $(F, A) \neq (G, B)$ in the sense of [7, Definition 2.4] because $A \neq B$. However, both (F, A) and (G, B) are null soft sets, and so $(F, A) = \Phi = (G, B)$. This is a contradiction. So we only need to give a rational notation for a null soft set.

(2) Different from notations of null soft set, the notation $\widetilde{A}$ of an absolute soft set does not result in any confusion because it is endowed the set A of parameters.

Remark 2.8: For the sake of conveniences, we use the following notations in this paper for null soft set and absolute soft set.

(1) If (F, A) is a null soft set, then we replace F by Φ. Thus (Φ, A) denotes a null soft set with a set A of parameters, i.e., $\Phi(a) = \emptyset$ for each $a \in A$.

(2) If (F, A) is an absolute soft set, then we replace F by Ω, Thus (Ω, A) denotes an absolute soft set with a set A of

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parameters, i.e., $\Omega(a) = U$ for each $a \in A$.

Definition 2.9 ([7]): Let (F, A) and (G, B) be two soft sets.

(1) The union $(F, A)\widetilde{\bigcup}(G, B)$ of (F, A) and (G, B) is defined as a soft set $(H, A \bigcup B)$, where for each $e \in A \bigcup B$

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ F(e) \bigcup G(e), & if\ e \in A \bigcap B \end{cases}$$

(2) If $A \bigcap B \neq \emptyset$ and $F(e) = G(e)$ for each $e \in A \bigcap B$, then the intersection $(F, A)\widetilde{\bigcap}(G, B)$ of (F, A) and (G, B) is defined as a soft set $(H, A \bigcap B)$, where $H(e) = F(e)$ for each $e \in A \bigcap B$.

Remark 2.10: For the sake of conveniences, $\widetilde{\bigcup}$ and $\widetilde{\bigcap}$ in this paper are replaced by $\bigcup$ and $\bigcap$ respectively, which does not result in any confusion.

Generally, the condition "$F(e) = G(e)$ for each $e \in A \bigcap B$" can not be satisfied for two soft sets (F, A) and (G, B). So Definition 2.9(2) is not sufficient for intersection operation of two soft sets. In order to give preferable intersection operations, Ali et al. gave the following definition.

Definition 2.11 ([2]): Let (F, A) and (G, B) be two soft sets.

(1) The extended intersection $(F, A) \sqcap (G, B)$ of (F, A) and (G, B) is defined as a soft set $(H, A \bigcup B)$, where for each $e \in A \bigcup B$

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ F(e) \bigcap G(e), & if\ e \in A \bigcap B \end{cases}$$

(2) If $A \bigcap B \neq \emptyset$, then the restricted intersection $(F, A) \cap (G, B)$ of (F, A) and (G, B) is defined as a soft set $(H, A \bigcap B)$, where $H(e) = F(e) \bigcap G(e)$ for each $e \in A \bigcap B$.

Remark 2.12: Whenever soft sets (F, A) and (G, A). It is clear that $(F, A) \cap (G, A) = (F, A) \sqcap (G, A)$.

Definition 2.13 ([7]): The complement $(F, A)^c$ of a soft set (F, A) is defined as a soft set $(F^c, \neg A)$, where $F^c(a) = U - F(\neg a)$ for each $a \in \neg A$.

Remark 2.14 ([7]): Let (F, A) be a soft set, then $((F, A)^c)^c = (F, A)$.

III. Investigations On [7, Proposition 2.3]

The following operational rules were given in [7, Definition 2.8] and [7, Proposition 2.3].

Rule 3.1: Let (F, A) be a soft set, then the following hold.

(1) $(\widetilde{A})^c = \Phi$.

(2) $\Phi^c = \widetilde{A}$.

(3) $(F, A) \bigcup \Phi = \Phi$.

(4) $(F, A) \bigcap \Phi = \Phi$.

(5) $(F, A) \bigcup \widetilde{A} = \widetilde{A}$.

(6) $(F, A) \bigcap \widetilde{A} = (F, A)$.

As stated in Remark 2.7, sets of parameters for null soft sets in Rule 3.1(1),(2),(3),(4) are not clear. We investigate Rule 3.1 as follows, where each null soft set is endowed a set of parameters.

Firstly, we investigate Rule 3.1(1),(2).

The following proposition gives correct forms for Rule 3.1(1),(2).

Proposition 3.2: Let A be a set of parameters, then the following hold.

(1) $(\Omega, A)^c = (\Phi, \neg A)$.

(2) $(\Phi, A)^c = (\Omega, \neg A)$.

Proof: (1) For each $a \in \neg A$, $\neg a \in A$, and so $\Omega^c(a) = U - \Omega(\neg a) = U - U = \emptyset = \Phi(a)$. Consequently, $(\Omega^c, \neg A) = (\Phi, \neg A)$. It follows that $(\Omega, A)^c = (\Omega^c, \neg A) = (\Phi, \neg A)$.

(2) By the above (1), $(\Phi, \neg(\neg A)) = (\Omega, \neg A)^c$. By Remark 2.4 and Proposition 2.14, $(\Phi, A)^c = (\Phi, \neg(\neg A))^c = ((\Omega, \neg A)^c)^c = (\Omega, \neg A)$. ■

Secondly, we investigate Rule 3.1(3).

Yang pointed out that Rule 3.1(3) is incorrect by an example ([12, Example]). In fact, operation results of $(F, A) \bigcup (\Phi, B)$ are uncertain, which depends on (F, A). Here, we give a sufficient-necessary condition such that Rule 3.1(3) holds.

Proposition 3.3: Let (F, A) be a soft set, then the following are equivalent.

(1) $(F, A) \bigcup (\Phi, B) = (\Phi, A \bigcup B)$.

(2) $(F, A) = (\Phi, A)$.

Proof: Put $(F, A) \bigcup (\Phi, B) = (H, A \bigcup B)$.

(1) $\Longrightarrow$ (2): Assume that $(F, A) \bigcup (\Phi, B) = (\Phi, A \bigcup B)$. Then $(H, A \bigcup B) = (\Phi, A \bigcup B)$. If $a \in A - B$, then $F(a) = H(a) = \Phi(a) = \emptyset$. If $a \in A \bigcap B$, then $F(a) = F(a) \bigcup \emptyset = F(a) \bigcup \Phi(a) = H(a) = \Phi(a) = \emptyset$. Consequently, $F(a) = \emptyset$ for each $a \in A$. It follows that $(F, A) = (\Phi, A)$.

(2) $\Longrightarrow$ (1): Assume that $(F, A) = (\Phi, A)$. Then $F(a) = \Phi(a) = \emptyset$ for each $a \in A$. Let $e \in A \bigcup B$.

$$H(e) = \begin{cases} F(e), & if\ e \in A - B \\ \Phi(e), & if\ e \in B - A \\ F(e) \bigcup \Phi(e), & if\ e \in A \bigcap B \end{cases}$$

$$= \begin{cases} \emptyset, & if\ e \in A - B \\ \emptyset, & if\ e \in B - A \\ \emptyset, & if\ e \in A \bigcap B \end{cases}.$$

So $H(e) = \Phi(e)$. Consequently, $(H, A \bigcup B) = (\Phi, A \bigcup B)$, i.e., $(F, A) \bigcup (\Phi, B) = (\Phi, A \bigcup B)$. ■

In addition, we have another possible result of $(F, A) \bigcup (\Phi, B)$.

Proposition 3.4: Let (F, A) be a soft set, then the following are equivalent.

(1) $(F, A) \bigcup (\Phi, B) = (F, A)$.

(2) $A \supset B$.

Proof: Put $(F, A) \bigcup (\Phi, B) = (H, A \bigcup B)$.

(1) $\Longrightarrow$ (2): Assume that $(F, A) \bigcup (\Phi, B) = (F, A)$. Then $(H, A \bigcup B) = (F, A)$. So $A \bigcup B = A$. It follows that $A \supset B$.

(2) $\Longrightarrow$ (1): Assume that $A \supset B$. Then $(H, A \bigcup B) = (H, A)$. Let $a \in A$.

$$H(a) = \begin{cases} F(a), & if\ a \in A - B \\ F(a) \bigcup \Phi(a), & if\ a \in A \bigcap B \end{cases}$$

$$= \begin{cases} F(a), & if\ a \in A - B \\ F(a), & if\ a \in A \bigcap B \end{cases}.$$

So $H(a) = F(a)$. Consequently, $(H, A) = (F, A)$, i.e., $(F, A) \bigcup (\Phi, B) = (F, A)$. ■

Thirdly, we investigate Rule 3.1(5).

Proposition 3.5: Let (F, A) be a soft set, then $(F, A) \bigcup (\Omega, A) = (\Omega, A)$.

Proof: Put $(F, A) \bigcup (\Omega, A) = (H, A \bigcup A) = (H, A)$. For each $a \in A$, $H(a) = F(a) \bigcup \Omega(a) = F(a) \bigcup U =$

$U = \Omega(a)$. Consequently, $(H, A) = (\Omega, A)$. It follows that $(F, A) \bigcup (\Omega, A) = (\Omega, A)$. ■

Remark 3.6: Proposition 3.3 shows that Rule 3.1(5) is correct. So illustration for Rule 3.1(5) in [2, Example 2.6] is not true. In [2, Example 2.6], indeed, (G, B) is an absolute soft set and $(F, A) \bigcup (G, B) \neq (G, B)$. Unfortunately, $(G, B) \neq \widetilde{A}$.

Fourthly, we investigate Rule 3.1(4),(6).

Ali et al. deduced that the Rule 3.1(4),(6) are incorrect in general ([2, Example 2.6]). For Rule 3.1(4),(6), we have the following two propositions, which can be obtained immediately by Definition 2.9(2).

Proposition 3.7: Let (F, A) be a soft set, then the following hold.

(1) $(F, A) \bigcap (\Phi, B)$ exists.

(2) $F(\alpha) = \emptyset$ for each $\alpha \in A \bigcap B$.

(3) $(F, A) \bigcap (\Phi, B) = (\Phi, A \bigcap B)$.

Proposition 3.8: Let (F, A) be a soft set, then the following are equivalent.

(1) $(F, A) \bigcap (\Omega, A)$ exists.

(2) $(F, A) = (\Omega, A)$.

(3) $(F, A) \bigcap (\Omega, A) = (\Omega, A)$.

We give some operational rules for extended intersection and restricted intersection of two soft sets, which are similar to Rule 3.1(4),(6).

Proposition 3.9: Let (F, A) be a soft set, then the following hold.

(1) $(F, A) \cap (\Phi, B) = (\Phi, A \bigcap B)$.

(2) $(F, A) \cap (\Omega, A) = (F, A) \sqcap (\Omega, A) = (F, A)$.

Proof: (1) Put $(F, A) \cap (\Phi, B) = (H, A \bigcap B)$. For each $e \in A \bigcap B$, $H(e) = F(e) \bigcap \Phi(e) = F(e) \bigcap \emptyset = \emptyset$. So $(H, A \bigcap B) = (\Phi, A \bigcap B)$, i.e., $(F, A) \cap (\Phi, B) = (\Phi, A \bigcap B)$.

(2) Put $(F, A) \cap (\Omega, A) = (H, A \bigcap A) = (H, A)$. For each $e \in A$, $H(e) = F(e) \bigcap \Omega(e) = F(e) \bigcap U = F(e)$. So $(H, A) = (F, A)$, i.e., $(F, A) \cap (\Omega, A) = (F, A)$. Also, by Remark 2.12, $(F, A) \sqcap (\Omega, A) = (F, A) \cap (\Omega, A)$. So $(F, A) \sqcap (\Omega, A) = (F, A)$. ■

Proposition 3.10: Let (F, A) be a soft set, then the following are equivalent.

(1) $(F, A) \sqcap (\Phi, B) = (\Phi, A \bigcup B)$.

(2) $F(a) = \emptyset$ for each $a \in A - B$.

Proof: Put $(F, A) \sqcap (\Phi, B) = (H, A \bigcup B)$.

(1) $\Longrightarrow$ (2): Assume that $(F, A) \sqcap (\Phi, B) = (\Phi, A \bigcup B)$. Then $(H, A \bigcup B) = (\Phi, A \bigcup B)$. Let $a \in A - B$. Then $F(a) = H(a) = \Phi(a) = \emptyset$.

(2) $\Longrightarrow$ (1): Assume that $F(a) = \emptyset$ for each $a \in A - B$. Let $b \in A \bigcup B$. If $b \in A - B$, then $H(b) = F(b) = \emptyset$. If $b \in B - A$, then $H(b) = \Phi(b) = \emptyset$. If $b \in A \bigcap B$, then $H(b) = F(b) \bigcap \Phi(b) = F(b) \bigcap \emptyset = \emptyset$. Consequently, $(H, A \bigcup B) = (\Phi, A \bigcup B)$. It follows that $(F, A) \sqcap (\Phi, B) = (\Phi, A \bigcup B)$. ■

IV. Investigations On [7, Proposition 2.4]

The following is [7, Proposition 2.4].

Rule 4.1: Let (F, A) and (G, B) be two soft sets, then the following hold.

(1) $((F, A) \bigcup (G, B))^c = (F, A)^c \bigcup (G, B)^c$.

(2) $((F, A) \bigcap (G, B))^c = (F, A)^c \bigcap (G, B)^c$.

Ali et al. illuminated incorrectness of Rule 4.1 ([2, Example 2.3]). We give some sufficient-necessary conditions such that Rule 4.1(1) and Rule 4.1(2) holds respectively.

Proposition 4.2: Let (F, A) and (G, B) be two soft sets, then the following are equivalent.

(1) $((F, A) \bigcup (G, B))^c = (F, A)^c \bigcup (G, B)^c$.

(2) $A \bigcap B = \emptyset$ or $F(e) = G(e)$ for each $e \in A \bigcap B$.

Proof: Put $(F, A) \bigcup (G, B) = (H, A \bigcup B)$. Then $((F, A) \bigcup (G, B))^c = (H^c, \neg A \bigcup \neg B)$. Put $(F^c, \neg A) \bigcup (G^c, \neg B)) = (I, \neg A \bigcup \neg B)$, then $(F, A)^c \bigcup (G, B)^c = (F^c, \neg A) \bigcup (G^c, \neg B) = (I, \neg A \bigcup \neg B)$.

(1) $\Longrightarrow$ (2): Suppose that $((F, A) \bigcup (G, B))^c = (F, A)^c \bigcup (G, B)^c$ and $A \bigcap B \neq \emptyset$. Then $(H^c, \neg A \bigcup \neg B) = (I, \neg A \bigcup \neg B)$.

Let $e \in A \bigcap B$, then $\neg e \in \neg A \bigcap \neg B$, and so $H^c(\neg e) = I(\neg e)$. Note that $H^c(\neg e) = U - H(e) = U - (F(e) \bigcup G(e)) = (U - F(e)) \bigcap (U - G(e))$ and $I(\neg e) = F^c(\neg e) \bigcup G^c(\neg e) = (U - F(e)) \bigcup (U - G(e))$. So $(U - F(e)) \bigcap (U - G(e)) = (U - F(e)) \bigcup (U - G(e))$, and hence $U - F(e) = U - G(e)$. It follows that $F(e) = G(e)$.

(2) $\Longrightarrow$ (1): If $H^c(d) = I(d)$ for each $d \in \neg A \bigcup \neg B$, then $(H^c, \neg A \bigcup \neg B) = (I, \neg A \bigcup \neg B)$. It follows that $((F, A) \bigcup (G, B))^c = (F, A)^c \bigcup (G, B)^c$. Thus, it suffices to prove that $H^c(d) = I(d)$ for each $d \in \neg A \bigcup \neg B$ if (2) holds.

Let $d \in \neg A \bigcup \neg B$, then $\neg d \in A \bigcup B$.

Case 1: Assume that $A \bigcap B = \emptyset$.

Then $\neg A \bigcap \neg B = \emptyset$.

$H^c(d) = U - H(\neg d)$

$$= \begin{cases} U - F(\neg d), & if\ d \in \neg A \\ U - G(\neg d), & if\ d \in \neg B \end{cases}.$$

$$I(d) = \begin{cases} F^c(d), & if\ d \in \neg A \\ G^c(d), & if\ d \in \neg B \end{cases}$$

$$= \begin{cases} U - F(\neg d), & if\ d \in \neg A \\ U - G(\neg d), & if\ d \in \neg B \end{cases}.$$

So $H^c(d) = I(d)$.

Case 2: Assume that $F(e) = G(e)$ for each $e \in A \bigcap B$.

$H^c(d) = U - H(\neg d)$

$$= \begin{cases} U - F(\neg d), & if\ \neg d \in A - B \\ U - G(\neg d), & if\ \neg d \in B - A \\ U - (F(\neg d) \bigcup G(\neg d)), & if\ \neg d \in A \bigcap B \end{cases}$$

$$= \begin{cases} U - F(\neg d), & if\ \neg d \in A - B \\ U - G(\neg d), & if\ \neg d \in B - A \\ U - F(\neg d), & if\ \neg d \in A \bigcap B \end{cases}$$

$$= \begin{cases} U - F(\neg d), & if\ d \in \neg A - \neg B \\ U - G(\neg d), & if\ d \in \neg B - \neg A \\ U - F(\neg d), & if\ d \in \neg A \bigcap \neg B \end{cases}.$$

$$I(d) = \begin{cases} F^c(d), & if\ d \in \neg A - \neg B \\ G^c(d), & if\ d \in \neg B - \neg A \\ F^c(d) \bigcup G^c(d), & if\ d \in \neg A \bigcap \neg B \end{cases}$$

$$= \begin{cases} U - F(\neg d), & if\ d \in \neg A - \neg B \\ U - G(\neg d), & if\ d \in \neg B - \neg A \\ (U - F(\neg d)) \bigcup (U - G(\neg d)), & if\ d \in \neg A \bigcap \neg B \end{cases}$$

$$= \begin{cases} U - F(\neg d), & if\ d \in \neg A - \neg B \\ U - G(\neg d), & if\ d \in \neg B - \neg A \\ U - F(\neg d), & if\ d \in \neg A \bigcap \neg B \end{cases}.$$

So $H^c(d) = I(d)$. ■

Let (F, A) and (G, B) be two soft sets. It is clear that $(F, A) \bigcap (G, B)$ exists if and only if $(F, A) \bigcap (G, B) = (F, A) \cap (G, B)$. So we replace the intersection by the restricted intersection in investigations of Rule 4.1(2).

Proposition 4.3: Let (F, A) and (G, B) be two soft sets and $A \bigcap B \neq \emptyset$. Then the following are equivalent.

(1) $((F, A) \cap (G, B))^c = (F, A)^c \cap (G, B)^c$.

(2) $F(e) = G(e)$ for each $e \in A \bigcap B$.

Proof: Put $(F, A) \cap (G, B) = (H, A \bigcap B)$. Then $((F, A) \cap (G, B))^c = (H^c, \neg A \bigcap \neg B)$. Put $(F^c, \neg A) \cap (G^c, \neg B)) = (I, \neg A \bigcap \neg B)$, then $(F, A)^c \cap (G, B)^c = (F^c, \neg A) \cap (G^c, \neg B)) = (I, \neg A \bigcap \neg B)$.

(1) $\Longrightarrow$ (2): Assume that $((F, A) \cap (G, B))^c = (F, A)^c \cap (G, B)^c$. Let $e \in A \bigcap B$, then $\neg e \in \neg A \bigcap \neg B$. By assumption, $(H^c, \neg A \bigcap \neg B) = (I, \neg A \bigcap \neg B)$, and so $H^c(\neg e) = I^c(\neg e)$. Note that $H^c(\neg e) = U - H(e) = U - (F(e) \bigcap G(e)) = (U - F(e)) \bigcup (U - G(e))$ and $I^c(\neg e) = F^c(\neg e) \bigcap G^c(\neg e) = (U - F(e)) \bigcap (U - G(e))$. So $U - F(e) = U - G(e)$. It follows that $F(e) = G(e)$.

(2) $\Longrightarrow$ (1): Assume that $F(e) = G(e)$ for each $e \in A \bigcap B$. Let $d \in \neg A \bigcap \neg B$, then $\neg d \in A \bigcap B$, and so $F(\neg d) = G(\neg d)$. Note that $H^c(d) = U - H(\neg d) = U - (F(\neg d) \bigcap G(\neg d)) = U - F(\neg d)$ and $I(d) = F^c(d) \bigcap G^c(d) = (U - F(\neg d)) \bigcap (U - G(\neg d)) = U - F(\neg d)$. So $H^c(d) = I(d)$. Consequently, $(H^c, \neg A \bigcup \neg B) = (I, \neg A \bigcup \neg B)$. It follows that $((F, A) \bigcup (G, B))^c = (F, A)^c \bigcup (G, B)^c$. ■

The following proposition investigates Rule 4.1(2) by the extended intersection.

Proposition 4.4: Let (F, A) and (G, B) be two soft sets, then the following are equivalent.

(1) $((F, A) \sqcap (G, B))^c = (F, A)^c \sqcap (G, B)^c$.

(2) $A \bigcap B = \emptyset$ or $F(e) = G(e)$ for each $e \in A \bigcap B$.

Proof: Put $(F, A) \sqcap (G, B) = (H, A \bigcup B)$. Then $((F, A) \bigcup (G, B))^c = (H^c, \neg A \bigcup \neg B)$. Put $(F^c, \neg A) \sqcap (G^c, \neg B)) = (I, \neg A \bigcup \neg B)$, then $(F, A)^c \sqcap (G, B)^c = (F^c, \neg A) \sqcap (G^c, \neg B)) = (I, \neg A \bigcup \neg B)$.

(1) $\Longrightarrow$ (2): Assume that $((F, A) \sqcap (G, B))^c = (F, A)^c \sqcap (G, B)^c$ and $A \bigcap B \neq \emptyset$. Let $e \in A \bigcap B$, then $\neg e \in \neg A \bigcap \neg B$. By assumption, $(H^c, \neg A \bigcup \neg B) = (I, \neg A \bigcup \neg B)$, and so $H^c(\neg e) = I^c(\neg e)$. Note that $H^c(\neg e) = U - H(e) = U - (F(e) \bigcap G(e)) = (U - F(e)) \bigcup (U - G(e))$ and $I^c(\neg e) = F^c(\neg e) \bigcap G^c(\neg e) = (U - F(e)) \bigcap (U - G(e))$. So $U - F(e) = U - G(e)$. It follows that $F(e) = G(e)$.

(2) $\Longrightarrow$ (1): The proof is similar to that of (2) $\Longrightarrow$ (1) in Proposition 4.2 and we omit it. ■

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References

[1] H.Aktas and N.Cagman, Soft sets and soft groups, Information Sciences, 177(2007) 2726-2735.

[2] M.I.Ali, F.Feng, X.Liu, W.K.Minc and M.Shabir, On some new operations in soft set theory, Computers & Mathematics with Applications, 57(2009), 1547-1553.

[3] F.Feng, Y.B.Jun, X.Liu, L.Li, An adjustable approach to fuzzy soft set based decision making, Journal of Computational and Applied Mathematics, 234(2010), 10-20.

[4] X.Ge, Z.Li and Y.Ge, Topological spaces and soft sets, Journal of Computational Analysis and Applications. In Press.

[5] Y.B.Jun and C.Hwan, ParkApplications of soft sets in ideal theory of BCK/BCI-algebras, Information Sciences, 178(2008), 2466-2475.

[6] Z.Kong, L.Gao and L.Wang, Comment on "A fuzzy soft set theoretic approach to decision making problems" Journal of Computational and Applied Mathematics, 223(2009), 540-542.

[7] P.K.Maji, R.Biswas and A.R.Roy, Soft set theory, Computers & Mathematics with Applications, 45(2003), 555-562.

[8] D.Molodtsov, Soft set theory-First results, Computers & Mathematics with Applications, 37(1999), 19-31.

[9] K.Qin and Z.Hong, On soft equality, Journal of Computational and Applied Mathematics. In Press.

[10] A.R.Roy, P.K.Maji, A fuzzy soft set theoretic approach to decision making problems, Journal of Computational and Applied Mathematics, 203(2007), 412-418.

[11] Z.Xiao, K.Gong and Y.Zou, A combined forecasting approach based on fuzzy soft sets, Journal of Computational and Applied Mathematics, 228(2009), 326-333.

[12] C.F.Yang, A note on soft set theory, Computers & Mathematics with Applications, 56(2008) 1899-1900.

[13] Y.Zou and Z.Xiao, Data analysis approaches of soft sets under incomplete information, Knowledge-Based Systems, 21(2008), 941-945.