

# Intensity of Singular Stress Field at the Corner of Adhesive Layer in Bonded Plate

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**Abstract**— In this paper the strength of adhesive joint under tension and bending is discussed on the basis of intensity of singular stress by the application of FEM. A useful method is presented with focusing on the stress at the edge of interface between the adhesive and adherent obtained by FEM. After analyzing the adhesive joint strength with all material combinations, it is found that to improve the interface strength, thin adhesive layers are desirable because the intensity of singular stress decreases with decreasing the thickness.

**Keywords**—Adhesive, Adherent, Intensity of singular stress, Bonded strip

## I. INTRODUCTION

WITH the widely used of adhesive joint, the evaluation of adhesive joint strength also be paid more and more

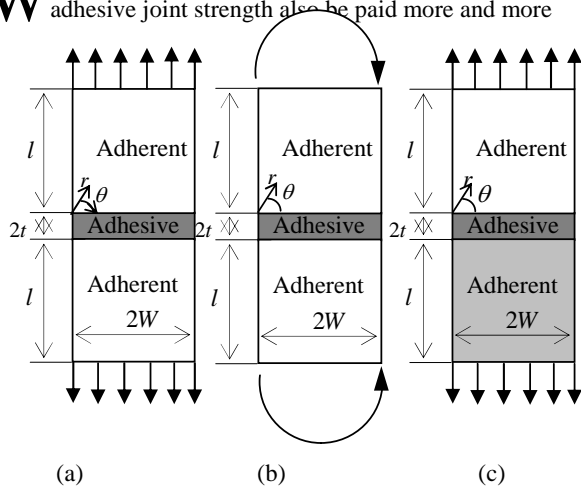


Fig.1 Adhesive joint under (a) tension, (b) bending (c) with different adherents under tension

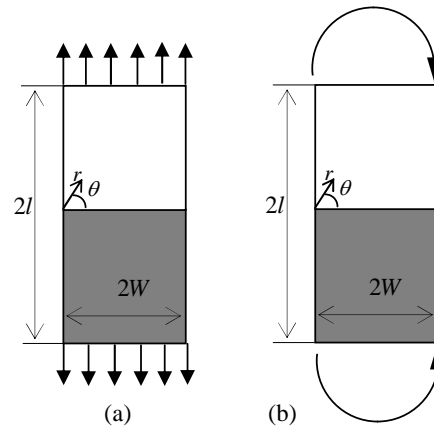


Fig.2 Bonded strip under (a) tension, (b) bending

attention to. About the adhesive joint strength many researches have been done [1],[2], but most of them are limited in the experiments, and the most popular experiments are the adhesive joint subjected to the tension and bending, for example, the mode shown in Fig.1 (a) could describe the micro-tensile bond test, which is a laboratory procedure frequently employed today in an attempt to predict the clinical effectiveness of adhesive used for bonding composite restorations to the dental substrate. In this paper, adhesive joint with the same adherent materials under tension shown Fig.1 (a) and under bending shown in Fig.1(b), and adhesive joint with different adherent materials under tension shown in Fig.1 (c) will be analyzed on the basis of intensity of singular stress using the solutions for the reference problem shown in Fig.2 (a) and (b). Here,  $r, \theta$  are polar coordinates around the interface edge, and  $W, l, t$  are dimensionless of the adhesive joint with the width  $W$ , the height  $l$ , and the adhesive thickness  $t$ .

## II. ANALYSIS METHOD

For the adhesive joint as shown in Fig.1, it is known that the interface stress  $\sigma_{ij}$  ( $ij = rr, \theta\theta, r\theta$ ) goes to infinity at the edge of joint and has singularity of  $\sigma_{ij} \propto 1/r^{1-\lambda}$  when  $\alpha(\alpha-2\beta) > 0$  where  $\alpha, \beta$  are Dunders' parameters; Besides, when  $\theta = \pi/2$ , the order of singularity  $\lambda$  at the joint of interface can be expressed by the following equation[3],[4].

$$\left[ \sin^2\left(\frac{\pi}{2}\lambda\right) - \lambda^2 \right]^2 \beta^2 + 2\lambda^2 \left[ \sin^2\left(\frac{\pi}{2}\lambda\right) - \lambda^2 \right] \alpha\beta + \lambda^2 (\lambda^2 - 1) \alpha^2 + \frac{\sin^2(\lambda\pi)}{4} = 0 \quad (1)$$

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$$\alpha = \frac{G_1(\kappa_2 + 1) - G_2(\kappa_1 + 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)} \quad \beta = \frac{G_1(\kappa_2 - 1) - G_2(\kappa_1 - 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)}$$

$$\kappa_j = \begin{cases} \frac{3-\nu_j}{1+\nu_j} (\text{plane stress}) \\ 3-4\nu_j (\text{plane strain}) \end{cases}, \kappa_j = (j=1,2) \quad (2)$$

Here,  $G, \nu$  are shear modulus and passion's ratio. The intensity of singular stress  $K_\sigma$  at the adhesive dissimilar joint is expressed as

$$K_\sigma = \lim_{r \rightarrow 0} [r^{1-\lambda} \times \sigma_{\theta=\pi/2}(r)] \quad (3)$$

and the dimensionless of intensity of singular stress  $F_\sigma$  is defined by the following equation.

$$F_\sigma = \frac{K_\sigma}{\sigma(2W)^{1-\lambda}} = \frac{\lim_{r \rightarrow 0} [r^{1-\lambda} \sigma_{\theta=\pi/2}(r)]}{\sigma(2W)^{1-\lambda}} \quad (\text{for tension})$$

$$F_\sigma = \frac{K_\sigma}{(6M/4W^2)(2W)^{1-\lambda}} = \frac{\lim_{r \rightarrow 0} [r^{1-\lambda} \sigma_{\theta=\pi/2}(r)]}{(6M/4W^2)(2W)^{1-\lambda}} \quad (\text{for bending}) \quad (4)$$

Here,  $\sigma$  is the stress applying to the  $y$  direction.

In this paper, the finite element method is used to obtain the stress at the joint of interface, and the software is MSC.MARC 2007. The width of the model is  $W=1000\text{mm}$ , and the length  $l$  is  $2W$ , because it is demonstrated that when  $l \geq 2W$ , the interface stresses are the same. The adhesive thickness  $t/W$  is changed as 0.001, 0.01, 0.1, 0.5, 1, 2, 4.

We will propose the method of calculating the intensity of singular stress from the results of FEM. In this paper, the ratio of intensity of singular stress  $K_\sigma^1/K_\sigma^2$  will be considered. Here, the superscripts 1,2 mean that  $t/W$  is different. As shown in (3),(4), the dimensionless intensity of singular stress is related to the distance  $r$ , singular index  $\lambda$ , and stress  $\sigma$  or  $M$ , width  $W$  and limiting stress  $\lim_{r \rightarrow 0} \sigma_{\theta=\pi/2}$ . Considering different adhesive thicknesses  $t_1, t_2$  as problem 1 and problem 2, both of which have the same stress at infinity  $\sigma$  or  $M$  and material combinations. Therefore, it should be noted that the singular index  $\lambda_1 = \lambda_2$ . As shown in (5), the ratio of intensity of singular stress  $K_\sigma^1/K_\sigma^2$  is only related to the ratio of stress  $\lim_{r \rightarrow 0} (\sigma_{\theta=\pi/2}^1 / \sigma_{\theta=\pi/2}^2)$  depending of adhesive thickness  $t$ , and the ratio of intensity of singular stress is expressed as shown in (5).

Therefore, in this paper, only stress  $\lim_{r \rightarrow 0} \sigma_{\theta=\pi/2}$  is calculated.

$$\frac{K_\sigma^1}{K_\sigma^2} = \frac{\sigma^1(2W)^{1-\lambda_1} F_\sigma^1}{\sigma^2(2W)^{1-\lambda_1} F_\sigma^2} = \frac{F_\sigma^1}{F_\sigma^2} = \lim_{r \rightarrow 0} \left[ \frac{r^{1-\lambda_1} \sigma_{\theta=\pi/2}^1(r)}{r^{1-\lambda_1} \sigma_{\theta=\pi/2}^2(r)} \right]$$

$$= \lim_{r \rightarrow 0} \frac{\sigma_{\theta=\pi/2}^1(r)}{\sigma_{\theta=\pi/2}^2(r)} \quad (\text{for tension})$$

$$\frac{K_\sigma^1}{K_\sigma^2} = \frac{(6M/4W^2)^1(2W)^{1-\lambda_1} F_\sigma^1}{(6M/4W^2)^2(2W)^{1-\lambda_1} F_\sigma^2} = \frac{F_\sigma^1}{F_\sigma^2} = \lim_{r \rightarrow 0} \left[ \frac{r^{1-\lambda_1} \sigma_{\theta=\pi/2}^1(r)}{r^{1-\lambda_1} \sigma_{\theta=\pi/2}^2(r)} \right]$$

$$= \lim_{r \rightarrow 0} \frac{\sigma_{\theta=\pi/2}^1(r)}{\sigma_{\theta=\pi/2}^2(r)} \quad (\text{for bending}) \quad (5)$$

In this paper, the ratio of intensity of singular stress is mainly considered in the analysis. To obtain the intensity of singular stress from the ratio, the reference problem as shown in Fig.2 will be used.

### III. INTENSITY OF SINGULAR STRESS FOR BONDED STRIP AS A REFERENCE SOLUTION

However, to obtain the intensity of singular stress, a reference solution is necessary. Chen-Nisitani [5] and Noda et. al [6] have analyzed the intensity of singular stress in a bonded strip under tension and bending in Fig.2 accurately by using the body force method. In the previous studies, only the results for singular stress  $\lambda \leq 1$  are indicated, and the results are showed in Fig.3. However, in this study, all material combinations are newly considered; and, therefore Fig.3 includes new results for  $F_\sigma > 1$  where no singular stress because singular index  $\lambda > 1$ .

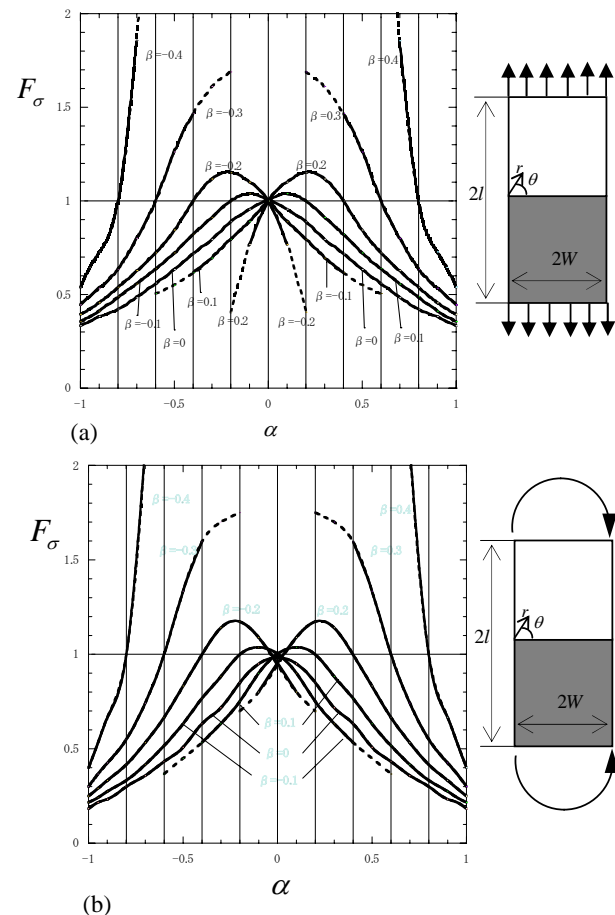
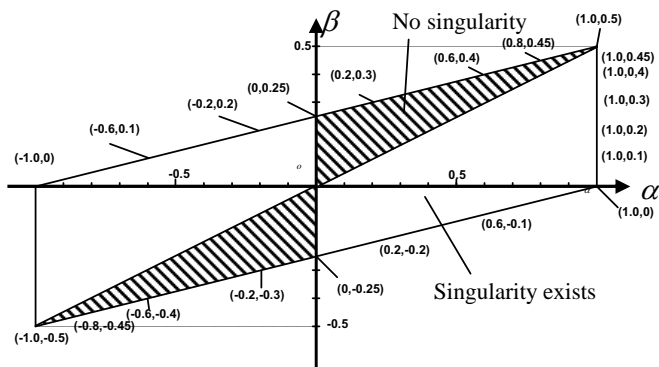


Fig.3  $F_\sigma$  for a bonded strip in Fig.2 (a) tension, (b) bending

Fig.4 The map of  $\alpha$  and  $\beta$  Units

#### IV. RESULTS FOR THE ADHESIVE JOINT

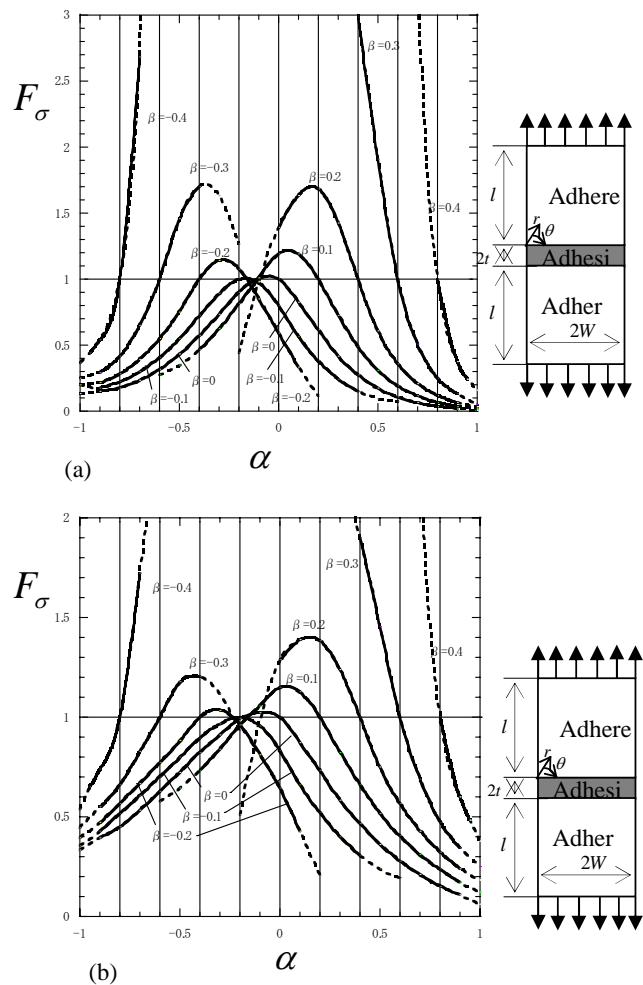
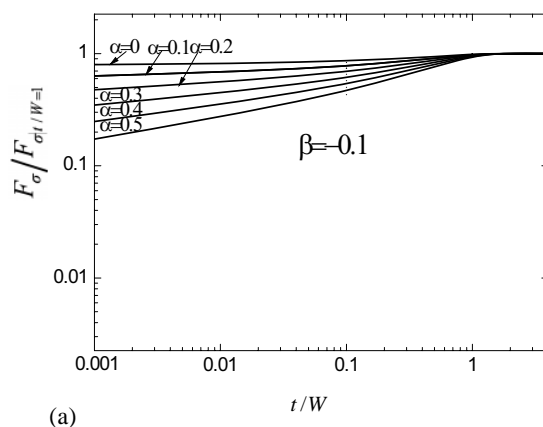
From the analysis method, it is found that the ratio of intensity of singular stress can be given very accurately independent of mesh size, and only the values for the first element are need as the ratio of stresses keeps the constant along the interface when  $r \rightarrow 0$  [7]. However, because of the limited space of the paper, that will not be illustrated here.

As discussed before, it is found that the ratio  $F_{\sigma}^1/F_{\sigma}^2$  is equivalent to the ratio  $\sigma_y^1/\sigma_y^2$  along interface  $r$ . By calculating the ratio  $\sigma_y/\sigma_{y|t/W=1}$  around the edge of interface, the ratio  $F_{\sigma}/F_{\sigma|t/W=1}$  has been obtained for all material combinations. Fig.4 shows the map of  $\alpha$  and  $\beta$  used for this calculations.

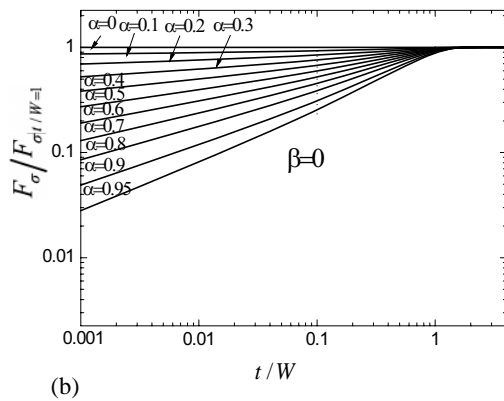
##### A. Results for the adhesive joint under tension

Fig.5 shows  $F_{\sigma}$  with varying  $\alpha$  and  $\beta$  when (a)  $t/W = 0.001$ ; (b)  $t/W = 0.1$ . It can be seen that  $F_{\sigma}$  increases with increasing of  $\alpha$  when  $\alpha$  is small. On the other hand,  $F_{\sigma}$  decreases with increasing of  $\alpha$  when  $\alpha$  is large. Comparing the results of  $t/W = 0.001$  and  $t/W = 0.1$ , it is found that the increase and decrease of  $F_{\sigma}$  for  $t/W = 0.001$  is quick, while the increase and decrease of  $F_{\sigma}$  for  $t/W = 0.1$  is slower. Moreover, the variation range for  $t/W = 0.001$  is large, while the variation range for  $t/W = 0.1$  is smaller.

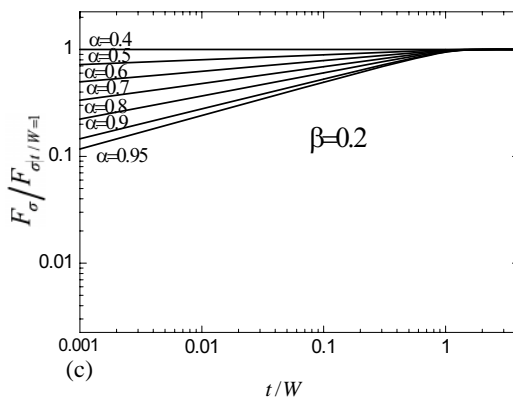
Generally, the Young's modulus  $E_2$  of adhesive is smaller than the Young's modulus  $E_1$  of adherent:  $E_2 \leq E_1$ , and the Poisson's ratio  $\nu_2$  of adhesive is larger than the Poisson's ratio  $\nu_1$  of adherent:  $\nu_2 \geq \nu_1$ . In this case, it is found that  $\alpha \geq 0$  and  $\alpha - 2\beta \geq 0$  and therefore singularity stress exists around the edge of interface. Fig.6 shows examples of the variation of logarithmic  $F_{\sigma}/F_{\sigma|t/W=1}$  and  $t/W$  for  $\beta = -0.1$  and  $\beta = 0$ ,  $\beta = 0.2$  satisfying  $\alpha \geq 0$  and  $\alpha - 2\beta > 0$ .

Fig.5  $F_{\sigma}$  with varying material combination  $\beta$  in Fig.1 (a) when (a)  $t/W = 0.001$ ; (b)  $t/W = 0.1$ 

(a)



(b)



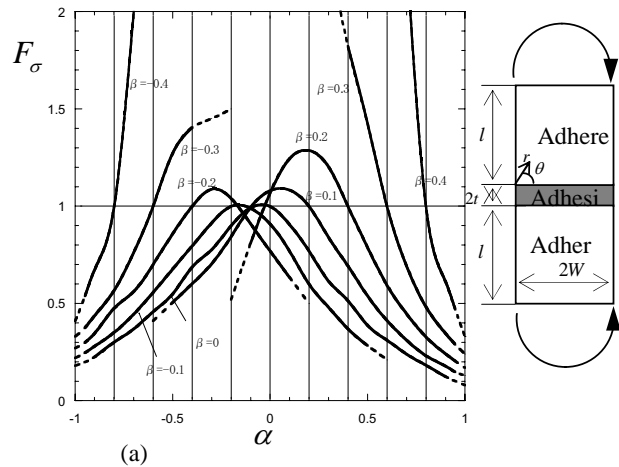
(c)

Fig.6  $F_{\sigma}/F_{\sigma|t/W=1}$  with varying adhesive thickness  $t/W$  in Fig.1 (a) for (a)  $\beta = -0.1$  (b)  $\beta = 0$ , (c)  $\beta = 0.2$

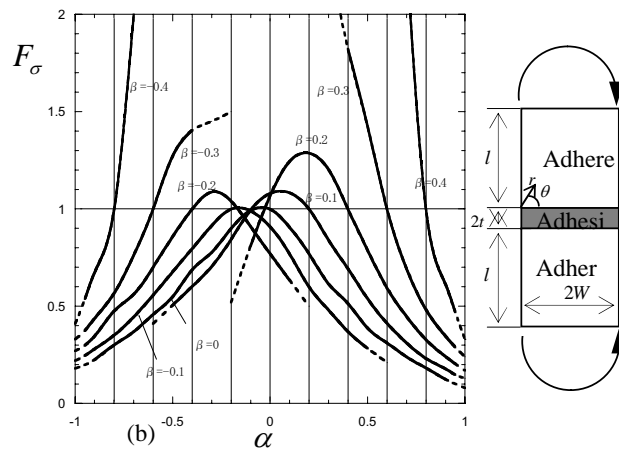
It is found that  $F_{\sigma}/F_{\sigma|t/W=1}$  increases with increasing  $t/W$  until  $t/W=1$  for all the material combinations. To improve the interface strength, thin adhesive layers are desirable because the intensity of singular stress decreases with decreasing the thickness.

#### B. Results for the adhesive joint under bending

When the adhesive joint is under bending, results of  $F_{\sigma}$  and obtained the effect of adhesive thickness and material combinations are discussed using the same analysis method. Fig.7 shows variations  $F_{\sigma}$  with all material combinations when (a)  $t/W = 0.001$ ; (b)  $t/W = 0.1$  and Fig.8 shows examples of the variation of logarithmic  $F_{\sigma}/F_{\sigma|t/W=1}$  and  $t/W$  for  $\beta = -0.1$ ,  $\beta = 0$  and  $\beta = 0.2$  satisfying  $\alpha \geq 0$  and  $\alpha - 2\beta > 0$ . Comparing the results of tension and bending, it is should be noticed that for  $t/W = 0.001$  and  $0.1$   $F_{\sigma}$  for bending sometimes is larger than  $F_{\sigma}$  for tension when  $t/W = 0.001$  and  $t/W = 0.1$ , although  $F_{\sigma}$  for tension is always larger than  $F_{\sigma}$  for bending for the boned strip.

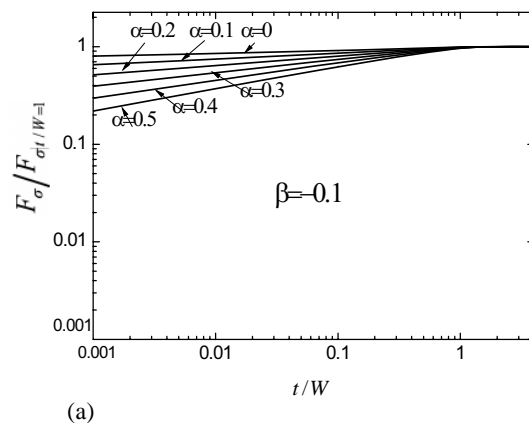


(a)

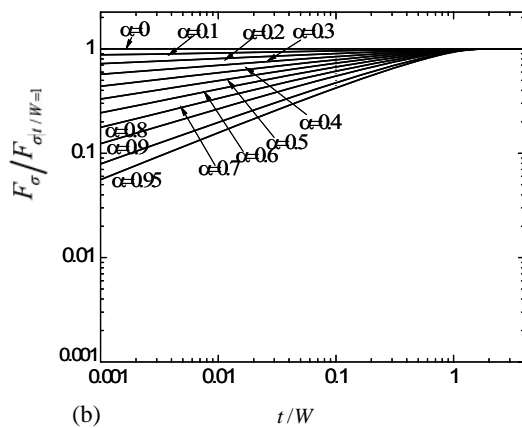


(b)

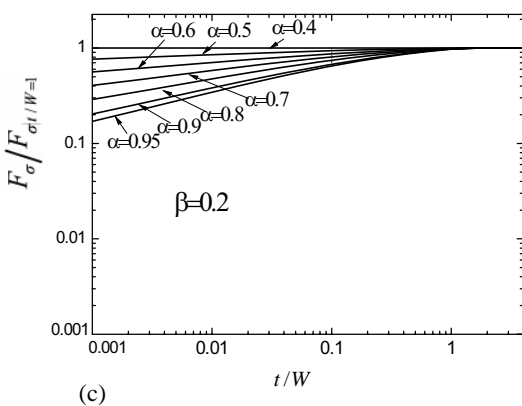
Fig.7  $F_{\sigma}$  with varying material combination  $\beta$  in Fig.1(b) when (a)  $t/W = 0.001$ ; (b)  $t/W = 0.1$



(a)



(b)



(c)

Fig.8  $F_{\sigma}/F_{\sigma t/W=1}$  with varying adhesive thickness  $t/W$  in Fig.1 (a) for (a)  $\beta = -0.1$  (b)  $\beta = 0$ , (c)  $\beta = 0.2$

### C. Results for the adhesive joint with different adherents

In fact, the adhesive joint is often used to bond different adherent materials. In our research, the intensity of singular stress of the adhesive joint with different adherent materials shown in Fig.1(c) is also analyzed.

Here, only results of two material combinations are presented as examples. For the first example, SUS 304 and YH75 are considered as adherents, and Epoxy is considered as the adhesive. For the second example, Silicon and FR-4.5 are considered as adherents and resin is considered as the adhesive. For the two material combinations, Elastic modulus of the up-adherents is larger than the Elastic modulus of the down-adherent. Table 1 shows material properties of these material combinations.

Using the analysis method explained in the above and the intensity of singular stress for the model shown in Fig.2 (a) as the reference problem, intensities of singular stress for the model shown in Fig.1 (c) have been obtained. Effects of adhesive thickness on intensity of singular stress also be studied, results are shown in Fig.9 and Fig.10. Since the singular stress fields at the up end of interface is different from

TABLE I  
MATERIAL PROPERTIES

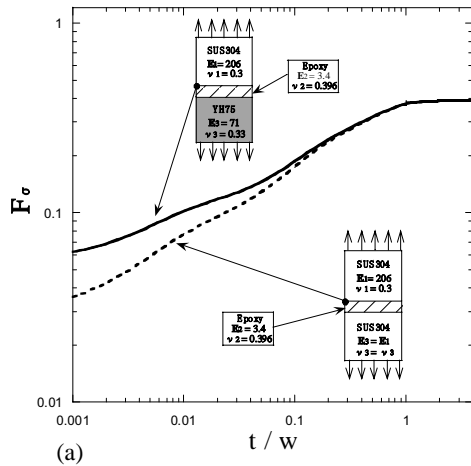
		Material	Elastic modulus/G Pa	Poisson's ratio
1	Adherent(up)	SUS304	206	0.3
	Adhesive	Epoxy	3.4	0.396
	Adherent(down)	YH75	71	0.33
2	Adherent(up)	Silicon	166	0.26
	Adhesive	Resin	2.74	0.38
	Adherent(down)	FR-4.5	15.34	0.15

those at the down end of interface, the results are shown in different figures.

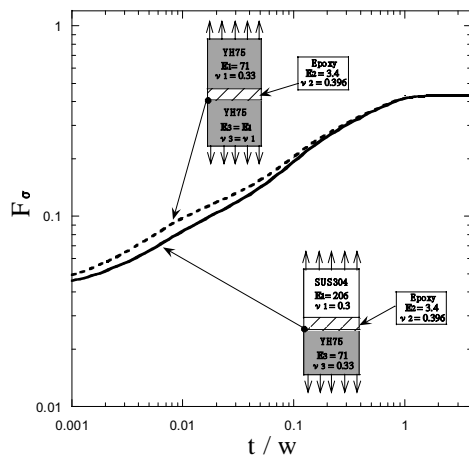
The results of intensity of singular stress for the adhesive joint with different adherent materials are shown using solid lines, and results of up interface for example 1 material combination, down interface for example 1 material combination, up interface for example 2 material combination, down interface for example 2 material combination are shown in Fig.9 (a), (b) and Fig.10 (a), (a) respectively. It is found that intensity of singular stress also decreases with decreasing adhesive thickness until the adhesive thickness equaling to the width of the model, and then keeps constant, which is the same with the results for the adhesive joint with the same adherent materials.

To comparing the results between adhesive joint with different and the same adherent materials, intensities of singular stress for the same adherent material are also shown in Fig.9 with dashed lines. In Fig.9 (a), (b), adherents for SUS304 and YH-75 are considered. At the upper end of interface the results are compared with the ones of bonded SUS304, and it is found that the results are larger than the ones of bonded SUS304 as shown in Fig.9 (a). At the lower end of interface the results are compared with the ones of bonded YH-75, and it is seen that the results are smaller than the ones of bonded YH-75 as shown in Fig.9 (b). In Fig.10 (a), (b), adherents for Silicon and FR-4.5 are considered. At the upper end of interface the results are compared with the ones of bonded Silicon, and it is found that the results are larger than the ones of bonded Silicon as shown in Fig.10 (a). At the lower end of interface the results are compared with the ones of bonded FR-4.5, and it is seen that the results sometimes smaller and sometimes larger than the ones of bonded FR-4.5 as shown in Fig.10 (b).

From those results, it may be concluded that the intensity of singular stress decreases with decreasing of adhesive thickness owing to the interaction at the upper and lower bonded ends. The effect of interaction may be changed slightly by the material combinations.

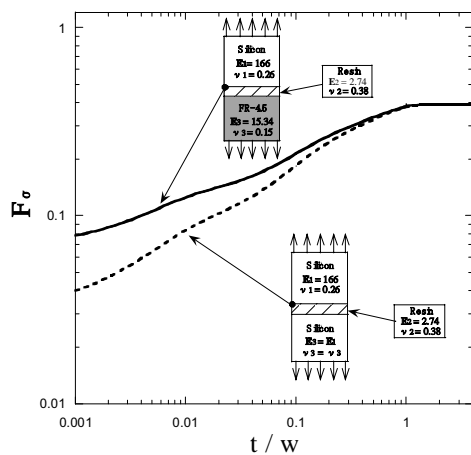


(a)

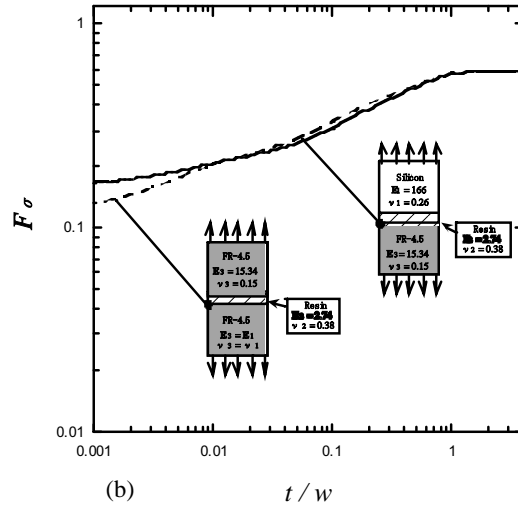


(b)

Fig.9 Comparison between the effects of adhesive thickness on the results for different adherents and the same adherents for example 1 material combination (a) the upper corner (b) the lower corner



(a)



(b)

Fig.10 Fig.9 Comparison between the effects of adhesive thickness on the results for different adherents and the same adherents for example 2 material combination (a) the upper corner (b) the lower corner

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