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Impulse Response Shortening for Discrete Multitone Transceivers using Convex Optimization Approach

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***Abstract*— In this paper we propose a new criterion for solving the problem of channel shortening in multi-carrier systems. In a discrete multitone receiver, a time-domain equalizer (TEQ) reduces intersymbol interference (ISI) by shortening the effective duration of the channel impulse response. Minimum mean square error (MMSE) method for TEQ does not give satisfactory results. In [1] a new criterion for partially equalizing severe ISI channels to reduce the cyclic prefix overhead of the discrete multitone transceiver (DMT), assuming a fixed transmission bandwidth, is introduced. Due to specific constrained (unit morm constraint on the target impulse response (TIR)) in their method, the freedom to choose optimum vector (TIR) is reduced. Better results can be obtained by avoiding the unit norm constraint on the target impulse response (TIR). In this paper we change the cost function proposed in [1] to the cost function of determining the maximum of a determinant subject to linear matrix inequality (LMI) and quadratic constraint and solve the resulting optimization problem. Usefulness of the proposed method is shown with the help of simulations.

***Keywords*— Equalizer, target impulse response, convex optimization, matrix inequality.

I. Introduction

THE discrete multitone transceiver (DMT) has attracted considerable attention recently as a viable technology for high-speed transmission on spectrally shaped channels [8]. DMT partitions a broadband channel into a large number of virtually independent, narrowband subchannels. Ideally each narrowband subchannel would have a flat frequency response and could be modeled as gain plus additive white Gaussian noise (AWGN). The total number of bits transmitted in a broadband channel would be sum of the bits transmitted in each narrowband subchannel. Modulation by the inverse fast Fourier transform (IFFT) and demodulation by fast Fourier transform (FFT) create orthogonal subchannels. The orthogonality is destroyed by spectrally shaped channel so that they cannot be fully separated at the receiver and causes ISI and inter carrier interference (ICI). The ISI can be avoided by adding long guard period at the beginning of each DMT symbol. When the guard period is a cyclic prefix, i.e, a copy of last ν samples of a DMT symbol, ISI can be reduced. For highly dispersive channels, the length of the cyclic prefix is large resulting in an appreciable bit rate loss, especially for a moderate size FFT.

A channel shortening equalizer commonly known as a time domain equalizer (TEQ) is required to shorten the length of the effective channel to the cyclic prefix length ν. The TEQ is finite impulse response filter (FIR). The equalized channel, which is the cascade of the channel and the TEQ, can be modeled as delay by Δ samples followed by an FIR filter whose impulse response is the target impulse response (TIR) of $\nu + 1$ samples. The TIR would fit into a target window of $\nu + 1$ samples starting at sample index $\Delta + 1$ in the shortened impulse response (SIR). The rest of the SIR would ideally be zero.

The MMSE method for channel shortening is used to reduce the complexity in Maximum likely receivers and has since been extensively utilized by designers as a suboptimal but cost effective solution for TEQ in multitone systems [9]. In this approach, the TEQ is chosen to minimize the MSE between the equalized signal and the signal as seen by communication over the virtual channel with short TIR. The advantage of this method is that the quadratic formulation of the error terms of the TEQ coefficients allows efficient eigensolution for the channel shortener. The MMSE method prevents trivial solution by imposing unity norm tap constraint on the TIR, does not directly maximizes bit error rate (BER) [1]. The second method which is used for channel shortening maximizes shortening signal to noise ratio (SSNR), which also does not maximizes the BER.

In [2], a method is proposed for channel shortening to maximize BER but unfortunately this method does not avoid trivial solution. In this paper, using method proposed in [1], we arrive at another criterion for channel shortening with no constraint on TIR, thus having more freedom to solve the resulting optimization problem.

The rest of the paper is organized as follows:

In section 2, we give overview of the DMT transceiver and give the method proposed in [1] for channel shortening. Section 3 is devoted to the proposed method and explains interior point method. Simulations are given in section 4 and conclusions are drawn in the last section.

II. System Model

An input bit stream of rate R_{DMT} b/s is buffered into blocks of $b_{DMT} = R_{DMT}T$ bits, where T is the multicarrier symbol period. These b_{DMT} bits are distributed optimally across $\bar{N} \leq \frac{N}{2}$ subchannels. The bits assigned to the ith subchannel, b_i, are mapped by the DMT encoder to the ith complex subsymbol of the kth transmitted symbol, which is denoted by $X_{i,k}$. These $\bar{N}$ complex subsymbols are then transformed by an N-point IFFT into real samples by imposing the Hermitian symmetry condition $X_{i,k} = X^*_{N-i,k}$ $(1 \leq i \ \leq \bar{N})$. The N samples are then converted from parallel to serial format and

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applied, after adding a cyclic prefix and passing them through the digital to analog converter (DAC) and lowpass filter, to the channel $h(t)$. We shall deal with the equivalent discrete-time representation of the channel, which will be assumed an FIR filter. At the receiver , the output signal is first low-pass filtered and sampled and then cyclic prefix is removed. The resultant N real output samples are converted from serial to parallel format and then transformed, through an N-point FFT, to $\bar{N}$ complex subsymbols that are individually decoded.

A. Geometric TEQ Method

In [1], the authors proposed a method to incorporate the optimization of achievable bit rate into TEQ design. The goal is to use the ultimate performance measure as an objective function in the TEQ design procedure. In their derivation they considered the definition of Geometric SNR (GSNR) which is a useful measure related to the bit error rate (BER)

$$GSNR = \Gamma \left(\left[\prod_{i=1}^{\bar{N}} \left(1 + \frac{SNR_i^{EQ}}{\Gamma} \right) \right]^{1/\bar{N}} - 1 \right) \quad (1)$$

where Γ is the SNR gap for achieving Shannon channel capacity and is assumed to be constant over all subchannels, $\bar{N} = N/2$, where N is the number of samples. In [1] it is shown that maximizing the GSNR is equivalent to maximizing the BER. In the above equation, the subchannel SNR is modified to include the effect of equalization [1]

$$SNR_i^{EQ} = \frac{S_{x,i}|B_i|^2}{S_{n,i}|W_i|^2} \quad (2)$$

where
$S_{x,i}$ signal power;

$S_{n,i}$ noise power;

B_i gain of "b" (TIR impulse response) in the ith subchannel;

W_i gain of "w" (TEQ impulse response) in the ith subchannel.

The equalized channel can be modeled as delay by Δ samples followed by an FIR filter whose impulse response is the TIR. The problem statement for optimum TIR problem in [1] is

$$b_{opt} = arg\ max_b\ \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} \ln |B_i|^2\ \ s.t.\ \ ||b|| = 1\ \ and$$

$$b^T R_{\Delta} b \leq MSE_{max} \quad (3)$$

where

$$B_i = \sum_{k=0}^{N_b} b_k^* e^{-j\frac{2\pi}{N}ik} = b^* g_i^{(N_b+1)} \quad (4)$$

where $N_b + 1$ is the length of TIR and where $(.)^*$ denotes Hermitian transpose.

$$g_i^{(N_b+1)} = [1\ \ e^{-j\frac{2\pi}{N}i} \ldots e^{-j\frac{2\pi}{N}iN_b}]^T \quad (5)$$

Now let

$$L(b) = \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} \ln(b^* G_i^{(N_b+1)} b), \quad (6)$$

where

$$G_i^{(N_b+1)} = g_i^{(N_b+1)} g_i^{*(N_b+1)} \quad (7)$$

The above equation can also be written as

$$G_i^{(N_b+1)} =$$

$$\begin{pmatrix} 1 & e^{j\frac{2\pi}{N}i} & \ldots & e^{j\frac{2\pi}{N}iN_b} \\ e^{-j\frac{2\pi}{N}i} & 1 & e^{j\frac{2\pi}{N}i}\ldots & \\ \vdots & & & \\ e^{-j\frac{2\pi}{N}iN_b} & \ldots & e^{-j\frac{2\pi}{N}i} & 1 \end{pmatrix} \quad (8)$$

Here, R_{Δ} is the channel dependent matrix, and MSE_{max} is the channel dependent parameter that limits the MSE. The above equation is nonlinear constrained optimization problem. It does not have closed form solution but it can be solved numerically.
The MGSNR TEQ method is not optimum (in sense of maximizing the BER) due to several approximations. One approximation is the definition of the GSNR itself- the method maximizes an approximation to the actual GSNR. The objective function is derived based on the assumption that the TIR and the TEQ coefficients are independent. However, this is not the case because the optimal TEQ coefficients are calculated from the optimal TIR coefficients by using the following formula

$$w_0^T = b_0^T R_{xy} Ryy^{-1} \quad (9)$$

where
w_0 is the optimal TEQ vector

b_0 is the optimal TIR vector

R_{xy} is the cross correlation between input and output

R_{yy} is the output autocorrelation matrix.

The most important approximation, however, is the definition of the subchannel SNR, SNR_i^{EQ} in eq(2), which includes the effect of the equalizer but not the effect of the ISI, even though the objective function of the TEQ is to minimize ISI. This issue has been addressed in [3] by modifying the SNR definition to include an ISI term

$$SNR_i^{ISI} = \frac{S_{x,i}|B_i|^2}{S_{x,i}|B_i - W_i H_i|^2 + S_{n,i}|W_i|^2} \quad (10)$$

However, this definition is used to evaluate the performance of the MGSNR TEQ method only, which is still based on the definition given in eq (2). We summarize the drawbacks of MGSNR TEQ method as follows:
- Its derivation is based on a subchannel SNR definition SNR_i^{EQ} that does not include the effect of ISI.
- It depends on the parameter MSE_{max} that has to be tuned for different channels.
- Its objective function assumes that b and w are independent.

However, they are related by eq(11).
- It requires constrained nonlinear optimization solution, which limits its optimality because of the constraint.

In [2], the authors propose new equalizer and present new subchannel SNR definition based on their derivation of equivalent signal, noise, and ISI paths in the DMT system. Based on the subchannel SNR definition, they derived the channel capacity as a nonlinear function of equalizer weights. They developed optimal bit rate solution, which requires a constraint nonlinear optimization and, thus, is not implementable in real time. Their method outperforms MMSE and the method proposed in [1]. The draw back in their method is that it does not avoid trivial solution as it does not pose any constraint on optimal TIR vector, b.

III. Proposed Method

The method proposed by us is variant of the method proposed in [1]. In order to proceed derivation of the our method we write the cost function proposed in [1]

$$L(b) = \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} \ln(b^* G_i^{(N_b+1)} b) \tag{11}$$

where G matrix are defined by eq(7,8). The above equation can also be written as

$$L(b) = \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} \ln Tr(G_i^{(N_b+1)} B) \tag{12}$$

where $B = bb^*$. Now let us evaluate further the $Tr(G_i^{(N_b+1)} B)$ function. The matrix $G_i^{(N_b+1)}$ is positive semidefinite. To make it positive definite we add a very small term on its diagonal elements. For simplicity the positive definite matrix is also represented by $G_i^{(N_b+1)}$. The matrix B is given by , $B = bb^*$. This matrix is of rank-1. We relax this rank-1 constraint by assuming that $B > bb^*$. That is matrix B is positive definite. Since matrix $G_i^{(N_b+1)}$ is diagonalizable, there exists an orthogonal matrix P and a diagonal matrix Σ such that (for the sake of simplicity we denote $N_b + 1 = n$, so the size of matrix B becomes $n \times n$).

$$\Sigma = P^T G_i^{(N_b+1)} P \tag{13}$$

So if eigenvalues of $G_i^{(N_b+1)}$ are

$$\lambda_1, \lambda_2, \ldots, \lambda_n \tag{14}$$

Then

$$\Sigma = diag(\lambda_1, \lambda_2, \ldots, \lambda_n) \tag{15}$$

Let $b_{11}, b_{22}, \ldots, b_{nn}$ denotes the elements of PBP^T. Then

$$\frac{1}{n} Tr(G_i^{(N_b+1)} B) = \frac{1}{n} Tr(P \Sigma P^T B),$$

using cyclic property of trace, we have

$$\frac{1}{n} Tr(G_i^{(N_b+1)} B) = \frac{1}{n} Tr(\Sigma P^T B P)$$

which can be written as

$$\frac{1}{n} Tr(G_i^{(N_b+1)} B) = \frac{1}{n} [\lambda_1 b_{11} + \lambda_2 b_{22} + \ldots + \lambda_n b_{nn}] \tag{16}$$

Using the Geometric-Arithmetic mean equality [6], which states that

Theorem [6]:

If a and w are two positive n-tuples then

$$G_n(a; w) \leq A_n(a; w) \tag{17}$$

with equality iff $a_1 = \ldots = a_n$. Where $G_n(a; w)$ and $A_n(a; w)$ are geometric mean and arithmetic mean respectively. Using this inequality we have

$$\frac{1}{n} Tr(G_i^{(N_b+1)} B) \geq [\lambda_1 \lambda_2 \ldots \lambda_n]^{\frac{1}{n}} [b_{11} b_{22} \ldots b_{nn}]^{\frac{1}{n}} \tag{18}$$

Since

$$\det A \leq a_{11} a_{22} \ldots a_{nn}, \tag{19}$$

for any positive definite matrix A [7], we conclude that

$$\det(P^T B P) \leq b_{11} b_{22} \ldots b_{nn} \tag{20}$$

and

$$\det \Sigma = \lambda_1 \lambda_2 \ldots \lambda_n. \tag{21}$$

Therefore from eq(18) it follows that

$$\frac{1}{n} Tr(G_i^{(N_b+1)} B) \geq [\det \Sigma]^{\frac{1}{n}} . [\det(P^T B P]^{\frac{1}{n}} \tag{22}$$

$$\frac{1}{n} Tr(G_i^{(N_b+1)} B) \geq [\det(P^T G_i^{(N_b+1)} P)]^{\frac{1}{n}} . [\det(P^T B P)]^{\frac{1}{n}} \tag{23}$$

The above inequality gives

$$Tr(G_i^{(N_b+1)} B) \geq n (\det(G_i^{(N_b+1)}). \det B)^{\frac{1}{n}} \tag{24}$$

Plugging the above inequality in eq(14) we get

$$L(b) \geq \frac{1}{\bar{N}} \sum_{i=1}^{\bar{N}} (\ln n + \ln(\det(G_i^{(N_b+1)}). \det B)^{\frac{1}{n}}) \tag{25}$$

$$L(b) \geq \frac{1}{n\bar{N}} \sum_{i=1}^{\bar{N}} (\ln(\det(G_i^{(N_b+1)})) + \ln(\det B)) \tag{26}$$

Now $\frac{1}{n\bar{N}} \sum_{i=1}^{\bar{N}} \ln(\det(G_i^{(N_b+1)}))$ is independent of parameter to be optimized, that is B. Hence our cost function is proportional to $\ln(\det B)$. Hence our overall optimization problem is follows:

$$L(b) \geq K + \ln(\det B)$$

$$subject\ to\ B - bb^* > 0\ and\ b^* R_\Delta b \leq MSE_{max}. \tag{27}$$

In the above optimization problem we are able to avoid unity norm constraint on the TIR. Thus giving more freedom to maximize the cost function. On the other hand our cost function is also relaxed version of the cost function proposed in [1]. The first constraint can be expressed as an linear matrix inequality (LMI) in B and b.

$$\begin{pmatrix} B & b \\ b^* & 1 \end{pmatrix} > 0 \tag{28}$$

So eq(31) is a max-det problem in B and b.

The problem of maximizing the determinant of a matrix subject to linear matrix inequality arises in many fields, including computational geometry, statistics, system identification, information and communication theory [5]. It can also be considered as generalization of the semidefinite programming (SDP). The max-det problem is a convex optimization problem, i.e., the objective function $K + \ln(\det B)$ is convex (on $\{B(b) > 0\}$), and the constraint set is convex. Indeed, LMI constraints can represent many common convex constraints, including linear inequalities, convex quadratic inequalities, and matrix norm and eigen value constraints. The max-det problem can be solved very efficiently using interior point method. Interior point algorithms are efficient in theory and practice. As the name suggests, these algorithms generate a sequence of iterates which moves through the relative interior of the feasible region. Many interior point methods have polynomial complexity. Computationally, interior point methods usually require less time than their worst-case bounds. For more information on interior point method see [4].
Suppose that after solving max det problem we have $\{B, b\}$ as optimal solution set. We generate z from normal distribution, $N(b, B - bb^*)$ and take $b_{opt} = z$ as solution of the max det problem. We can repeat many times and take the best. In our simulations we did it for five times and took the best.

IV. Simulations

For the comparison purposes, we use the same simulation environment as used in [2], namely DMT modulation for the eight standard asymetrical digital subscriber loop (ADSL) carrier serving area (CSA) test configurations [1,2]. We add a fifth-order Chebychev high pass filter with cutoff frequency 5.4 KHz and passband ripple of 0.5 dB to each CSA loop to take into account the effect of the splitter at the transmitter. We consider an N=512 tone system, each tone having the same bandwidth $B_i = 4.3125$ kHz, corresponding to the transmitter signaling at a frequency 2.208 MHz. The input signal power of 23 dBm is distributed equally over all used subchannels. The added white Gaussian noise has spectral density -140 dBm. Near end crosstalk (NEXT) noise is modeled as eight ADSL disturbers. We assume a modulation SNR gap $\gamma_m = 9.8$ dB , coding gain $\gamma_c = 4.2$ dB , and design margin $\gamma_d = 6$ dB , giving over all SNR gap $\Gamma = \gamma_m + \gamma_c + \gamma_d$ dB. The MSE_{max} was fixed to -17 dB as in [1].
We attempt to reproduce the results of [2] for MMSE, and MGSNR, in order to demonstrate relative performances. The percentage of bit rate results relative to the matched filter bound (MFB) for all methods on the eight test loops are listed in table 1 (upper table) for a q=17 tap TEQ, attempting to shorten the channel to length $L = 32$. In table 1 and table 2, the last column shows the bit rate in Mb/s. Table 2 (lower table) gives the percentage of the achievable bit rates for the eight CSA-Loops equalized with MMSE, MGSNR and proposed method as percentage of the matched filter bound R_{MFB} for $q = 3$. Using $h = [h(0)h(1)\ldots h(N-1)]^T$, we define the following :
MFB_i Subchannel matched Filter Bound (maximum SNR)

$$MFB_i = \frac{S_x(\frac{2\pi i}{N})|q_i^* h|^2}{S_n(\frac{2\pi i}{N})} \tag{29}$$

Where q_i is DFT vector. Maximum bit rate assuming SNR gap Γ is

$$R_{MFB} = \sum_{i=0}^{N-1} B_i \log_2(1 + \frac{MFB_i}{\Gamma})\ b/s \tag{30}$$

TABLE I
Achievable bit rates for the eight CSA-Loops equalized with the MMSE, MGSNR and proposed method, in percentage of the matched filter bound. Number of TEQ taps is $q = 17$.

Loops	MMSE	MGSNR	Proposed meth.	MFB
1	59	82	87	8.46
2	72	74	85	9.68
3	79	91	98	8.10
4	63	69	87	8.05
5	70	84	97	8.53
6	80	92	94	7.77
7	76	79	94	7.75
8	70	89	94	6.91

TABLE II
Achievable bit rates for the eight CSA-Loops equalized with the MMSE, MGSNR and proposed method, in percentage of the matched filter bound. Number of TEQ taps is $q = 3$.

Loops	MMSE	MGSNR	Proposed meth.	MFB
1	55	70	85	8.46
2	63	74	88	9.68
3	70	83	93	8.10
4	74	92	95	8.05
5	89	94	95	8.53
6	93	92	96	7.77
7	82	87	93	7.75
8	78	85	90	6.91

V. Conclusion

In this paper we proposed a new criterion for solving the problem of channel shortening in multi-carrier systems. In [1] a criterion for partially equalizing severe ISI channels to reduce the cyclic prefix overhead of the discrete multitone transceiver (DMT), assuming a fixed transmission bandwidth, is introduced. Due to specific constrained (unit norm constraint on the target impulse response) in their method there is limited freedom for the optimum vector (TIR). Better results can be obtained by lifting the unit norm constraint on the target impulse response (TIR). Our criterion is the maximization of the determinant subject to convex constraints. This problem can be solved very efficiently using interior point algorithm. Simulation results shows usefulness of our method. Future research will be focused on devising adaptive schemes for channel shortening.

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