

# Improved Power Spectrum Estimation for RR-Interval Time Series

B. S. Saini, Dilbag Singh, Moin Uddin, and Vinod Kumar

**Abstract**—The RR interval series is non-stationary and unevenly spaced in time. For estimating its power spectral density (PSD) using traditional techniques like FFT, require resampling at uniform intervals. The researchers have used different interpolation techniques as resampling methods. All these resampling methods introduce the low pass filtering effect in the power spectrum. The lomb transform is a means of obtaining PSD estimates directly from irregularly sampled RR interval series, thus avoiding resampling. In this work, the superiority of Lomb transform method has been established over FFT based approach, after applying linear and cubicspline interpolation as resampling methods, in terms of reproduction of exact frequency locations as well as the relative magnitudes of each spectral component.

**Keywords**—HRV, Lomb Transform, Resampling, RR-intervals.

## I. INTRODUCTION

SINCE the RR-interval series is non-uniformly sampled, for estimating its PSD using Fast Fourier Transform (FFT), requires re-sampling. For re-sampling, most analysts use different interpolation methods [1-4], [6-8] but all these methods alter the frequency contents of the signal due to the nonlinear low pass filtering effect [14]. If the time series contains inappropriate or missing samples i.e. in heart rate time series, PSD estimates can be severely affected and in such cases resampling is further complicated by the need to infer probable values as replacements.

Methods for PSD estimation based directly on irregularly sampled time series have been used, though not in HRV analysis, since at least 1976 [5]. Laguna et al. [4] proposes and recommended the Lomb-Scargle transform [5], [8], [9], [13] in Heart Rate Variability (HRV) studies, thus entirely avoided the problems associated with re-sampling. But after an exhaustive literature survey on ECG spectral estimation, we observed that numbers of studies are silent on the approach of how and in what extent the various interpolation methods affect the PSD estimates [1-3], [14]. Also the adequate justification of the technique adopted is usually lacking.

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Consequently, a need was felt to carry out a detailed study on implications of various re-sampling methods and Lomb transform method on PSD estimates of HRV.

In this paper, uniformly and non-uniformly sampled test signals for known set of frequency components in the autonomic band from 0 to 0.5 Hz are used and sampled (i) uniform sampling time of 250 msec (ii) non-uniform sampling time-base, represented by actual RR-interval time series from a healthy young adult. Firstly, the spectral estimation was done using FFT after applying two different methods of interpolation (i) Linear Interpolation (ii) Cubicspline Interpolation, for re-sampling. Further, Press-Rybicki algorithm [8] for Lomb transform has been used to extract its simplicity and speed. The spectrum estimates obtained using FFT with different interpolation as well as using the Lomb transform are not found to be ideal estimates. However, the Lomb transform has shown superior performance in terms of power spectrum estimation, as compared to the FFT based approach when evaluation was done on synthetic signals. Further it is observed that, the spectral peaks of FFT based PSD estimates get dislocated towards low frequency regions and also their strengths have been attenuated differently as compare to that of Lomb transform method.

## II. CURRENT METHODS IN USE

Traditionally the time and frequency domain measures were widely accepted methods for HRV analysis. For the spectral estimation of RR-interval series in the frequency domain, the FFT and autoregressive (AR) modeling [1], [2], submitted for publication [10], [12] are most commonly used. Before applying FFT or AR technique to a HRV signal, it is necessary to resample the signal at 4 Hz by using interpolation submitted for publication [10], [12]. The FFT based spectrum includes the entire signal variance, irrespective of whether its frequency components appear as specific spectral peaks or as non-peak broadband powers. On the other hand, AR method uses raw data to identify a best-fitting model and from which spectrum is derived. A fixed model order has been proposed as a practical rule for AR spectral estimation [2].

The Lomb Periodogram method of obtaining the power spectrum does not require re-sampling and for this feature it has been used for the analysis for HRV signals [15]. It allows examining frequencies higher than the mean Nyquist frequency, i.e., the Nyquist frequency one would obtain if the same number of data points were evenly sampled at the

average sampling rate. In this paper, the analysis was carried out on re-sampled RR interval series represented as a function of time using FFT and on non-uniformly sampled RR-interval series.

### III. PROCEDURE

We choose to present simulated test signals, instead of experimental, data for this analysis in order to have control over the frequency variations. Two test signals are generated, first test signal  $x_1(n)$  is representing the evenly sampled data and second i.e.  $x_{11}(n)$  and  $x_{12}(n)$  representing the unevenly sampled data sequence, using the two actual RR-interval records as sampling instants for two subjects by using (1), for two different set of frequency components, FIRST SET:  $f_1=0.008$  Hz,  $f_2=0.09$  Hz,  $f_3=0.13$  Hz,  $f_4=0.25$  Hz,  $f_5=0.4$  Hz and SECOND SET:  $f_1=0.01$  Hz,  $f_2=0.12$  Hz,  $f_3=0.20$  Hz,  $f_4=0.27$ ,  $f_5=0.33$  Hz.

The non-uniformly sampled test signals were transformed into evenly sampled signals by different interpolation methods for obtaining the power spectrum using FFT of 1024 windowed data points.

$$x_n = A \begin{bmatrix} \sin(2\pi f_1 n \Delta t) + \sin(2\pi f_2 n \Delta t) \\ + \sin(2\pi f_3 n \Delta t) + \sin(2\pi f_4 n \Delta t) \\ + \sin(2\pi f_5 n \Delta t) \end{bmatrix} \quad (1)$$

where sampling frequency  $f_s=1/\Delta t=4$ Hz for  $s=1,2,\dots,5$ ,  $A$  is the amplitude and  $n$  is the time-index.

Further the PSD estimate is obtained by using Lomb transform method [6]. This method is based on the definition of the discrete Fourier transform (DFT), for unevenly sampled signals  $x(t_n)$ , for  $(n=1, 2, \dots, N)$  using (2):

$$DFT(\omega) = \sum_{n=1}^N x(t_n) e^{-j\omega t_n} \quad (2)$$

where  $\omega=2\pi f$  is the angular frequency (if  $(t_{n+1}-t_n)$  is a constant and equal to 1, the expression for evenly sampled series was obtained).

Equation (2) has been used to define the transform for unevenly sampled series. However, the resulting transform suffers from an important limitation i.e. not invariant to time translations. For this reason Lomb modified the definition of transform and the Lomb transform [15] of a non-uniformly sampled real-valued data sequence  $\{x(t_n)\}$  of length  $N$  as defined using (3)

$$P_x(f) = \frac{1}{2\sigma^2} \left\{ \frac{\left[ \sum_{n=1}^N (x(t_n) - \bar{x}) \cos(2\pi f(t_n - \tau)) \right]^2}{\sum_{n=1}^N \cos^2(2\pi f(t_n - \tau))} + \frac{\left[ \sum_{n=1}^N (x(t_n) - \bar{x}) \sin(2\pi f(t_n - \tau)) \right]^2}{\sum_{n=1}^N \sin^2(2\pi f(t_n - \tau))} \right\} \quad (3)$$

where  $\bar{x}$  and  $\sigma^2$  are the mean and variance of the series  $\{x(t_n)\}$ ,  $\tau(f)$  is a frequency dependent time delay, defined to make the transform insensitive to time shift [4], [8], and is computed using (4)

$$\tan(4\pi f \tau) = \frac{\sum_{n=1}^N \sin(4\pi f t_n)}{\sum_{n=1}^N \cos(4\pi f t_n)} \quad (4)$$

### IV. RESULTS

Generation of test signals corresponding to subject-I and subject -II for first set of frequency components ( $f_1=0.008$ Hz,  $f_2=0.09$ Hz,  $f_3=0.13$ Hz,  $f_4=0.25$  Hz,  $f_5=0.4$ Hz)

Uniformly sampled test signal  $x_1(n)$  is generated for  $f_s=4$  Hz and  $A=2$  using (1) and is shown in Fig. 1 (a). The PSD plot of this simulated signal is shown in Fig. 2 (a). Its power spectrum shows five distinct spectral peaks at  $f_1, f_2, f_3, f_4$ , and  $f_5$  respectively, which represents high resolution and zero spectral leakage. The values of power in the five selected frequency bands corresponding to these frequency components are given in Table I which maintains approximately same magnitude in the entire power spectrum.

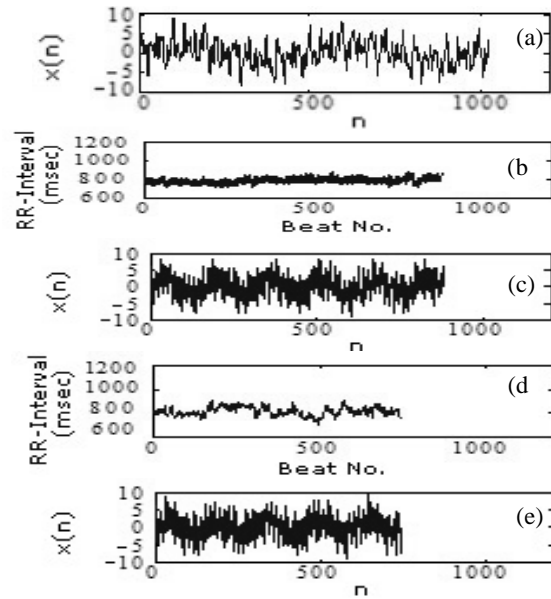


Fig. 1 Test Signals tachograms (a) Uniformly sampled test signal  $x_1(n)$  (b) Time base-I (c) Non-uniformly sampled test signal  $x_{11}(n)$  using time base-I of subject-I (d) Time base-II (e) Non-uniformly sampled test signal  $x_{12}(n)$  using time base-II of subject-II

Non-uniformly sampled test signals  $x_{11}(n)$  and  $x_{12}(n)$  are generated, by using the sampling instants of RR-intervals series shown in Fig. 1 (b) and (d), using (1) are shown in Fig. 1 (c) and (e) respectively.

#### A. PSD Estimation using FFT

Prior to FFT spectral estimation it is necessary to resample the signals  $x_{11}(n)$  and  $x_{12}(n)$  at equal intervals using different interpolation techniques [10]. These are as follows:

(i) Linear Interpolation: After applying this method of interpolation on non-uniformly sampled test signal  $x_{11}(n)$  shown in Fig. 1 (c), the power spectrum shown in Fig. 2 (b) is obtained. This plot shows that the spectral peaks are deviated towards the low frequency regions i.e. 0.007 Hz, 0.07 Hz, 0.10 Hz, 0.19 Hz and 0.3 Hz in the plot instead of remain

fixed as specified in the test signal. This leads to the distortion in the power spectrum due to the re-sampling operation. Moreover the values of power in  $P_{f2}$ ,  $P_{f3}$ ,  $P_{f4}$  and  $P_{f5}$  frequency bands as given in Table I are much less than that of uniformly sampled signal.

(ii) Cubicspline Interpolation: The results after applying this method of interpolation are: (A) The spectral leakage is observed in the PSD plot shown in Fig. 2 (c). (B) The spectral peaks are displaced towards the low frequency region and their strength has also been attenuated. (C) The values of power as per Table I, are further reduced from the values obtained using the above method of interpolation. Thus Cubicspline Interpolation leads to the higher attenuation of the spectral components but the deviation in the spectral peaks remains same as compare to Linear Interpolation method.

### B. PSD Estimation using Lomb Transform

The PSD estimates of non-uniformly sampled test signals  $x_{11}(n)$  and  $x_{12}(n)$  are now obtained by using Lomb transform without applying any interpolation method for re-sampling, using (3) and (4). The PSD plots of test signal  $x_{11}(n)$  (Fig. 1 (c)) is shown in Fig. 2 (d). This plot reproduces five distinct spectral peaks  $f_1, f_2, f_3, f_4$ , and  $f_5$  exactly at the same locations as defined in the test signal using (1).

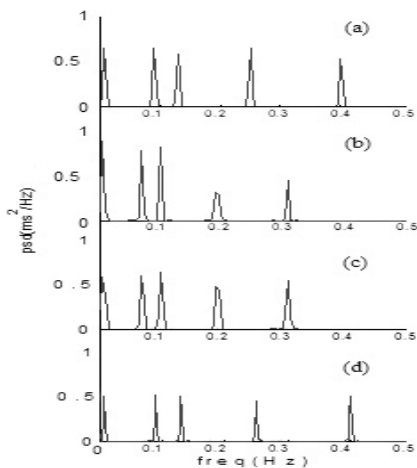


Fig. 2 PSD plots of test signal  $x_1(n)$  &  $x_{11}(n)$  using (a) Uniformly sampled test signal  $x_1(n)$  (b) Test signal  $x_{11}(n)$  after Linear interpolation (c) Test signal  $x_{11}(n)$  after Cubicspline interpolation (d) Test signal  $x_{11}(n)$  after Lomb transform

Hence, no distortion is observed in its PSD plot. The power values for the test signal  $x_{11}(n)$  after applying Lomb transform as shown in Table I are almost similar to that obtained for uniformly sampled test signal. This signifies that PSD estimates which are obtained from this method are free from low pass filtering effects, but which are predominant in all the above mentioned interpolation methods.

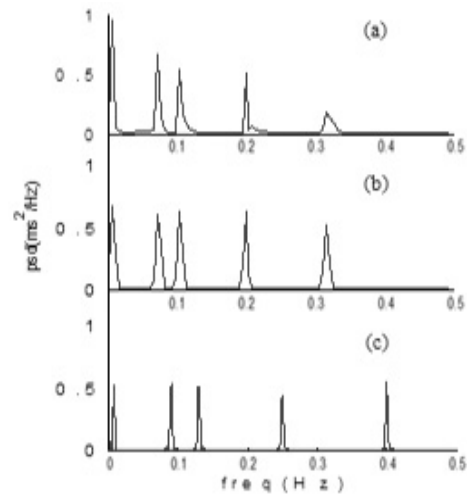


Fig. 3 PSD plots of test signal  $x_{12}(n)$  using (a) Linear Interpolation (b) Cubicspline Interpolation (c) Lomb Transform

Further, this study was extended by using sampling instants of another actual RR-interval time series i.e. for test signal  $x_{12}(n)$ . After applying Linear and Cubicspline methods of interpolation, the PSD plots are shown in Fig. 3(a) and 3(b), indicating higher spectral leakage, deviation and attenuation of the spectral components compared to that of Lomb based PSD plot. The study was extended to several set of frequency components and same observations were repeated.

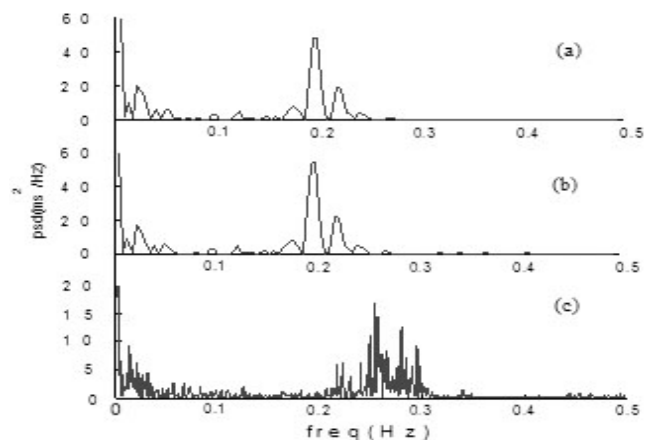


Fig. 4 PSD plots of actual series of data (a) Linear Interpolation (b) Cubicspline Interpolation (c) Lomb Transform

Thus in HRV analysis using FFT, the main source of an error is due to the interpolation, used for re-sampling the unevenly spaced heart rate series. Hence the Lomb transform method which requires no re-sampling is found to be more accurate method for HRV analysis. In addition to the simulated test signals all the above-discussed methods of PSD

TABLE I  
POWER VALUES OF TEST SIGNALS FOR FIRST SET OF FREQUENCY COMPONENTS

Frequency Bands	Subject I				Subject II		
	Uniformly sampled data without any interpolation	Linear Interpolation	Cubicspline Interpolation	Lomb Periodogram	Linear Interpolation	Cubicspline Interpolation	Lomb Periodogram
$P_{f1}$	0.9996	1.0013	1.0766	0.9339	1.0039	1.0037	0.9353
$P_{f2}$	1.0000	0.0201	0.0123	1.0000	0.2341	0.0807	0.9947
$P_{f3}$	0.9963	0.0173	0.0016	0.9835	0.0425	0.0161	1.0000
$P_{f4}$	1.0001	0.0063	0.0003	0.9676	0.0116	0.0062	0.9644
$P_{f5}$	0.9898	0.0020	0.0002	0.9825	0.0003	0.0003	0.9899

$P_{f1}=0.000\text{Hz}-0.01563\text{Hz}$ ,  $P_{f2}=0.08203\text{Hz}-0.09766\text{Hz}$ ,  $P_{f3}=0.1211\text{Hz}-0.1367\text{Hz}$ ,  $P_{f4}=0.2422\text{Hz}-0.2578\text{Hz}$ ,  $P_{f5}=0.3906\text{Hz}-0.4064\text{Hz}$

estimates are also applied on real data recordings. After analyzing the PSD plots shown in Fig. 4 (a), (b) and (c) it is verified that the spectrum obtained using Lomb transform is more authentic. Thus FFT based methods can lead to wrong diagnosis in clinical applications if appropriate re-sampling methods are not chosen.

#### IV. CONCLUSION

In the PSD plots obtained after Linear and Cubicspline interpolations it is noticed that there is a shifting of spectral peaks towards the low frequency region. In this way we proved that the re-sampling operation affects the power spectrum estimation. By using the Lomb transform method we compute the PSD of RR-interval series without any interpolation and found that this method generates better results which closely matches with that of uniformly sampled signal. Further, these observations have been supported by results on actual RR-interval records from volunteers in terms of frequency reproduction as well as their relative magnitudes in the frequency range of interest.

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