

Improved Mutual Inductance of Rogowski Coil Using Hexagonal Core

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Abstract—Rogowski coils are increasingly used for measurement of AC and transient electric currents. Mostly used Rogowski coils now are with circular or rectangular cores. In order to increase the sensitivity of the measurement of Rogowski coil and perform smooth wire winding, this paper studies the effect of increasing the mutual inductance in order to increase the coil sensitivity by presenting the calculation and simulation of a Rogowski coil with equilateral hexagonal shaped core and comparing the resulted mutual inductance with commonly used core shapes.

Keywords—Rogowski coil, Mutual inductance, magnetic flux density.

I. INTRODUCTION

ROGOWSKI coils might be considered as the best measurement tool for AC and transient high electric currents [1]. They have many applications related to power systems measurements and instrumentations [2]. They are widely used as a current-to-voltage transducer because of their linearity, wide dynamic bandwidth, little insertion loss, and their simple structure and due to their non-magnetic core they cannot be driven into saturation [3], [4]. Fig. 1 shows the basic structure of Rogowski coil which consists of a non-magnetic core with wire wound around it, the required current to be measured will pass inside through the core. The location of the current cable with respect to the core will affect the measurement [5].

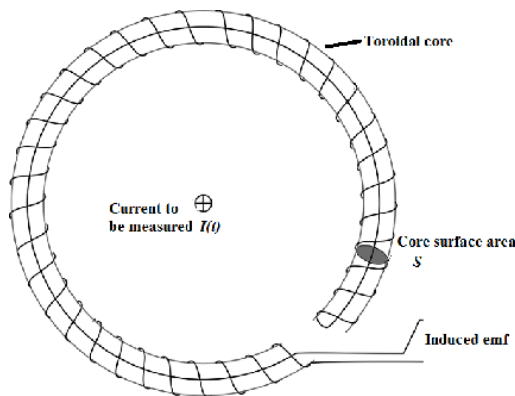


Fig. 1 Basic Structure of Rogowski coil

The aim of this paper is to present an enhanced mutual inductance value of Rogowski by using a hexagonal shaped

core. The calculations and simulation for the hexagonal core mutual inductance will be presented here and comparisons will be made with, the previously presented in literature, circular and square shaped cores. The current is assumed passing at the center of the coil though out this paper.

II. MUTUAL INDUCTANCE AND COIL SENSITIVITY

Rogowski coil operation principle is simple. The coil is homogeneously wound on a toroidal non-magnetic core, this core can have a circular or rectangular shape. The induction electromotive force (volts) over the coil terminals is given by Faraday's law proportional to the time derivative of the current I flowing through the loop according to (1):

$$emf = M \frac{dI}{dt} \quad (1)$$

where M is the mutual inductance of the coil in Henry. It has been presented in [6] a method to relate (1) to equation came from applying ampere's circuit law an amperian path following the edges of the toroidal core which states that the line integral of the magnetic field intensity around that closed amperian path is equal to the enclosed electric current bounded by that closed path, mathematically shown in (2):

$$I_{enc} = \oint_L H \cdot dl \quad (2)$$

where I_{enc} is the total electric current bounded by the closed path. H is the magnetic field intensity in Amperes/m, and is related, in free space, to the magnetic flux density \mathbf{B} (in Tesla) $= \mu_0 H$ where μ_0 is the permeability of the free space which is equal to $4\pi \times 10^{-7}$ Henry/m. The total magnetic flux ψ (in Webers) through the core can be calculated by the surface integral of \mathbf{B} over the core surface, which will lead to (3):

$$N \frac{d\psi}{dt} = M \frac{dI}{dt} \quad (3)$$

where N is the number of turns of the coil. Equation (3) shows the importance of the mutual inductance M in current measurement sensitivity. That is as M increases the induced voltage across the coil increases. It has been shown in the literature [5], [6] that for a circular core Rogowski coil as the one shown in Fig. 2, the mutual inductance is given by (4):

$$M = \mu_0 N \frac{(\sqrt{b} - \sqrt{a})^2}{2} \quad (4)$$

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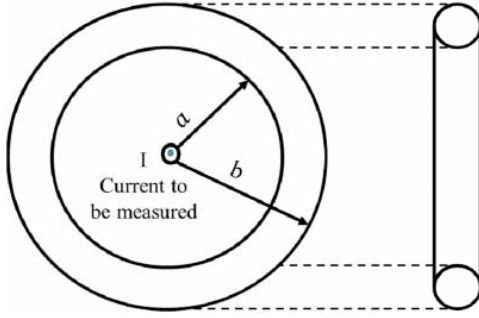


Fig. 2 Rogowski coil with circular core

Typical numbers for the dimensions of a and b are 1 and 2 cm respectively. If the inner radius (a) is to be changed from 1cm to 2cm and the outer radius (b) is $a+1$ cm, the resulting mutual inductance is plotted with respect to inner radius using (4) and shown in Fig. 3. Number of turns was assumed 100.

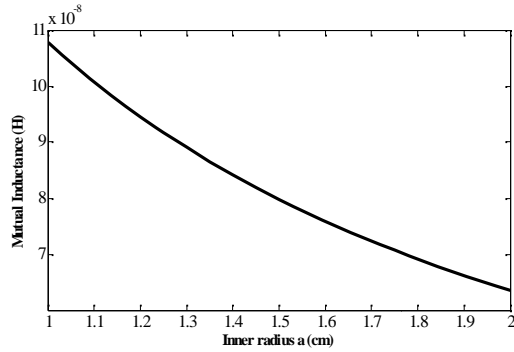
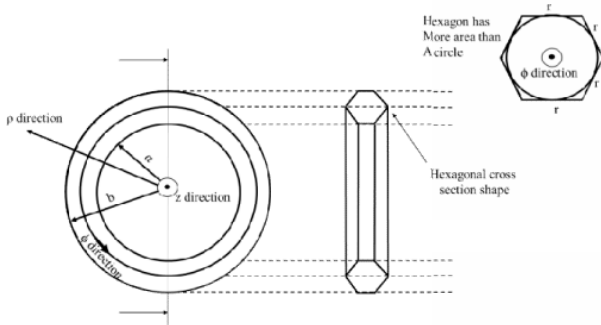
Fig. 3 Mutual inductance for circular core Rogowski coil with respect to inner radius, with $N=100$ turns

Fig. 4 Hexagonal core Rogowski coil

III. HEXAGONAL CORE COIL RESULT AND DISCUSSION

Fig. 4 shows the structure of an equilateral hexagonal core with parameters to be used in the calculation of the mutual inductance: a is the inner radius, b is the outer radius, and r is the side length. Using (3) the mutual inductance of the hexagonal core shape can be calculated. The left hand side of (3) is $N\psi \frac{dI}{dt}$ since the only parameter changing in flux is the current, so solving for flux ψ will lead to the mutual inductance. The inset of Fig. 4 shows that area of hexagon is

larger than the area of a circle which will defiantly makes the magnetic flux ψ more and hence more mutual inductance. Equation (5) is used to calculate the magnetic flux through the core:

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S} \quad (5)$$

where S is the core cross section area, here $dS = \rho d\phi dz$ in the direction of ϕ . The magnetic flux density B , from Biot-Savart's law is nothing but $\frac{\mu_0 I}{2\pi\rho} a_\phi$ also in the direction of ϕ , which means the dot product in (5) will be regular multiplication. ρ is the radial distance. Fig. 5 shows how to find the double integral limits of (5).

$$\Psi = \frac{\mu_0 N I}{2\pi} \iint \frac{1}{\rho} d\rho dz \quad (6)$$

The only parameter in the flux changing with time is the current I , which will lead to $Nd\psi/dt$ equal to $N\psi$ (without I) dI/dt . Comparing this with (3) yields that the mutual inductance M is the flux divided by the current I . The limits of the integral of (6) are clear from the illustration of Fig. 5. If the hexagon is divided into 2 halves and due to symmetry the result will be multiplied by 2. z shall vary from 0 to $r \sin(\alpha/2)$ and ρ shall vary from $a + z/\tan(\alpha/2)$ to $b - z/\tan(\alpha/2)$ that is:

$$M_{hex} = 2 \frac{(\mu_0 N)}{2\pi} \int_0^{r \sin(\alpha/2)} \ln \left(\frac{b - \frac{z}{\tan(\alpha/2)}}{a + \frac{z}{\tan(\alpha/2)}} \right) dz \quad (7)$$

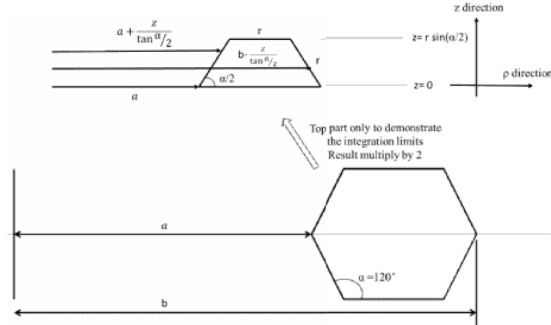


Fig. 5 Integration limits illustration for hexagonal core

It is easy to evaluate the definite integral of (7) to find the mutual inductance. Assuming typical numbers of $r = 1$ cm, $b=1$ cm+ a , and a varies from 1cm to 2cm, Fig. 6 shows the evaluation of mutual inductance of hexagonal core of (7). In addition to hexagonal core coil mutual inductance Fig. 6 shows also the mutual inductance of circular core coil with respect to the inner radius a , this will be different than the one shown in Fig. 3 due to the changes done on the orientation of the geometry of the coil shown in Fig. 7 to unify the comparison, which will make (4) become:

$$M_{cir} = \frac{\mu_0 N}{2} \left(\sqrt{b - \left(\frac{b-a}{2} \right) \left(1 - \frac{\sin \alpha}{2} \right)} - \sqrt{a + \left(\frac{b-a}{2} \right) \left(1 - \frac{\sin \alpha}{2} \right)} \right)^2 \quad (8)$$

It is clear from Fig. 6 that there is an increase in the mutual inductance due to using a hexagonal core instead of circular core which consequently will increase the coil sensitivity.

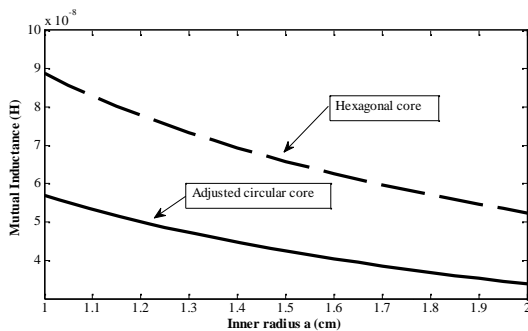


Fig. 6 Mutual inductance of Rogowski coil with hexagonal and adjusted circular core to unify comparison, with $N=100$ turns

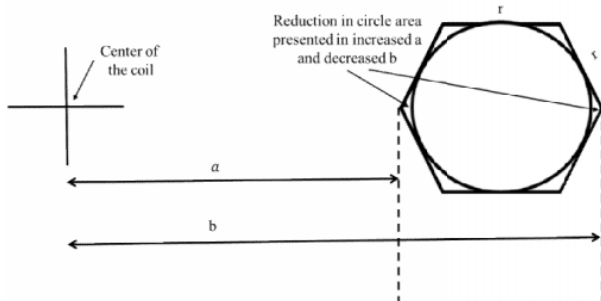


Fig. 7 Adjustments of the calculation borders of the circular core with respect to hexagonal core

It has been introduced in literature the use of rectangular core which is shown with respect to circular and hexagonal cores in Fig. 8, the mutual inductance for the rectangular core has been calculated in literature [6] to be $(Nw\mu_o/2\pi)\ln(b/a)$. Fig. 9 shows the mutual inductance of both hexagonal and rectangular cores varies with respect to the inner radius a , off course the a and b parameters in rectangular mutual inductance equation need to be adjusted the same way as for circular core. It is clear from Fig. 9 that the mutual inductance for both shapes hexagonal and rectangular is comparable, the benefit of using hexagonal core is smooth winding of copper wires around the core compared to rectangular, which will affect the life time of the coil

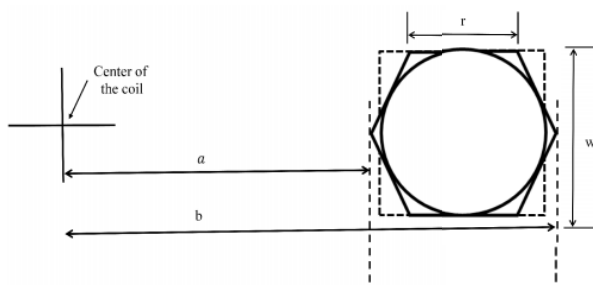


Fig. 8 Hexagonal, circular, and rectangular core coils

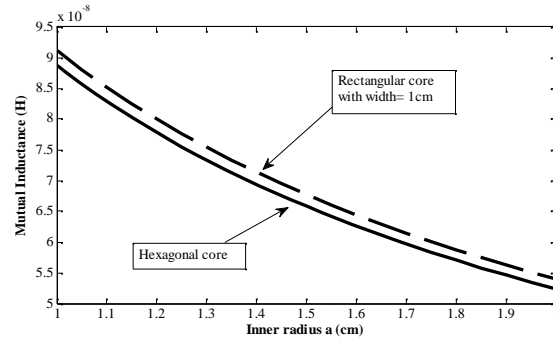


Fig. 9 Mutual inductance of Rogowski coil with hexagonal and rectangular cores, with $N=100$ turns

IV. CONCLUSION

Calculations of Rogowski coil mutual inductance with hexagonal core are presented. It was shown by calculation and simulation that using equilateral hexagonal core for Rogowski coil increases the mutual inductance and hence the sensitivity of the coil current measurements when compared to circular core. Hexagonal core coil mutual inductance comparisons with rectangular core was performed and yielded almost the same mutual inductance which makes hexagonal core better choice due to smoothness of wire winding around the core compared to rectangular.

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