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Improved asymptotic stability criteria for uncertain neutral systems with time-varying discrete delays

Changchun Shen and Shouming Zhong

Abstract—This paper investigates the robust stability of uncertain neutral system with time-varying delay. By using Lyapunov method and linear matrix inequality technology, new delay-dependent stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs), which can be easy to check the robust stability of the considered systems. Numerical examples are given to indicate significant improvements over some existing results.

Keywords—neutral system; Linear matrix inequalities; Lyapunov; Stability.

I. INTRODUCTION

DURING the past several years, considerable attention has been paid to the problem of stability analysis of time-delay systems, for example, biological systems, chemical systems, long-transmission lines in pneumatic, hydraulic systems, metallurgical processing systems, nuclear reactor, electrical networks, and so on. It is well known that the existence of time delays in a system may cause instability or bad system performance. Therefore, stability analysis of time-delay systems has been widely investigated by many researchers, such as [1-5] and references therein.

As well known, neutral system being a special case of time delay system exists in many dynamic systems, a number of stability conditions have been developed for this type of systems in the past. Recently, many researchers have paid a lot of attentions on the problem of robust stability for neutral systems with time-delay [6-8], and many efforts have been made to obtain less conservative delay-dependent conditions in the literature [7-12]. For instance, some robust stability results for neutral systems with different types of time delay were proposed in [7] and [8], respectively. In [9], Lyapunov functional technique combined with matrix inequality technique and a new operator are used to investigate the problem of robust stability for neutral systems with a constant neutral delay. By using descriptor model transformation and decomposition technique, some delay-dependent stability criterions are obtained in [10]. In [11], the stability conditions of uncertain neutral system with time-varying delay are developed by descriptor model transformation technique and the norm-bounded uncertainties is handled by S-procedure. Recently, a free weighting matrices approach combining matrix inequality technique are used in obtain the stability conditions so that less conservative stability conditions are obtained in [12].

Changchun Shen and Shouming Zhong are with the School of Mathematics Science, University Electronic Science and Technology of China, Chengdu 611731, PR China.
Shouming Zhong is with Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu 611731, PR China.
Email address: sccyjs2008@163.com.

In this paper, the problem of stability analysis for neutral systems with time-varying delays is discussed. Since model transformation and bounding techniques for cross terms appearing in the derivative of corresponding Lyapunov functional may introduce additional conservativeness [13], neither model transformation nor bounding technique for cross terms is applied in analyzing the considered systems which may yield a less conservative stability condition. The free-weighting matrix approach [12] is also employed to further reduce the entailed conservativeness. Numerical examples illustrate the effectiveness and improvement of the obtained results.

Notation: $\Re^n$ is the n-dimensional Euclidean space, $\Re^{m\times n}$ denotes the set of $m\times n$ real matrix. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $L_2[a,b]$ is the space of the square integral function on the interval $[a,b]$.

II. MODEL DESCRIPTION AND PRELIMINARIES

Consider the uncertain neutral systems with multiple time delays described by following state equation:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-\tau) &= (A+\Delta A(t))x(t) \\ &\quad + (B+\Delta B(t))x(t-h(t)), \\ x(t_0+\theta) &= \varphi(\theta), \quad \forall\theta\in[-\tau^*,0], \end{aligned} \tag{1}$$

where $x(t)\in\Re^n$ is the state vector, $\varphi(\cdot)\in L_2[-\max(\tau,h),0]$ is a differentiable vector-valued initial function, $\tau>0$ is a constant neutral delay, the discrete delay $h(t)$ is a time-varying function that satisfies

$$0\le h(t)\le h, \quad \dot{h}(t)\le h_D<1 \tag{2}$$

where h, h_D are constants, $\tau^*=\max(\tau,h)$, $A\in\Re^{n\times n}$, $B\in\Re^{n\times n}$, $C\in\Re^{n\times n}$ and $D\in\Re^{n\times n}$ are known constant matrices, $\Delta A(t)$ and $\Delta B(t)$ are the parametric uncertainties in the system, , which are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = LK(t)\begin{bmatrix} E_a & E_b \end{bmatrix} \tag{3}$$

where $K(t)$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$K^T(t)K(t)\le I \tag{4}$$

and L, E_a and E_b are known real constant matrices which characterize how the uncertainty enters the nominal matrices A and B.

The purpose of this paper is to formulate a practically computable criterion to check the stability of system described by (1-4).

Before proceeding further, system (1) can be written as:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-\tau) &= Ax(t) + Bx(t-h(t)) + Lu, \\ z &= E_a x(t) + E_b x(t-h(t)), \end{aligned} \tag{5}$$

with the constraint: $u = K(t)z$.

We further have:

$$\begin{aligned} u^T u \leq [E_a x(t) + E_b x(t-h(t))]^T \\ \cdot [E_a x(t) + E_b x(t-h(t))] \end{aligned} \tag{6}$$

In this paper, the following Lemma and Assumption are needed:

Lemma 1.(Schur Complement). Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $\Sigma_2 = \Sigma_2^T > 0$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ holds if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad or \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Assumption 1. All the eigenvalues of matrix C are inside the unit circle.

III. Main result

In this section, based on Lyapunov method and linear matrix inequality techniques, following stability criteria are derived.

Theorem 1. Under Assumption 1, the system described by (1-4) is robustly asymptotically stable, if there exist real matrices $P_i (i = 2, 3, \cdots, 6)$, $N_i (i = 1, 2, \cdots, 5)$, $X_{ij} (i, j = 1, 2, \cdots, 5)$, M_{11}, M_{12} and M_{22}, symmetric positive definite matrices G, R, S, Q and P_1, scalar $\epsilon > 0$ satisfying the following matrix inequalities:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \varepsilon E_a^T \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} & 0 \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & \varepsilon E_b^T \\ * & * & * & * & \Pi_{55} & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{7}$$

$$\Omega = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & N_1 \\ * & X_{22} & X_{23} & X_{24} & X_{25} & N_2 \\ * & * & X_{33} & X_{34} & X_{35} & N_3 \\ * & * & * & X_{44} & X_{45} & N_4 \\ * & * & * & * & X_{55} & N_5 \\ * & * & * & * & * & G \end{bmatrix} \geq 0 \tag{8}$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1 - h_D)S - M_{22} \geq G \tag{9}$$

where

$\Pi_{11} = P_2 A + A^T P_2 + Q + N_1 + N_1^T + hX_{11}$,
$\Pi_{12} = P_1 - P_2 + A^T P_3 + N_2^T + hX_{12}$,
$\Pi_{13} = P_2^T C + A^T P_4 + N_3^T + hX_{13}$,
$\Pi_{14} = P_2^T B + A^T P_5 + M_{12}^T + N_4^T - N_1 + hX_{14}$,
$\Pi_{15} = P_2^T L + A^T P_6 + N_5^T + hX_{15}$,
$\Pi_{22} = hS + R - P_3^T - P_3 + hX_{22}$,
$\Pi_{23} = P_3^T C - P_4 + hX_{23}$,
$\Pi_{24} = P_3^T B - P_5 - N_2 + hX_{24}$,
$\Pi_{25} = P_3^T L - P_6 + hX_{25}$,
$\Pi_{33} = P_4^T C + C^T P_4 - R + hX_{33}$,
$\Pi_{34} = P_4^T B + C^T P_5 - N_3 + hX_{34}$,
$\Pi_{35} = P_4^T L + B^T P_6 + hX_{35}$,
$\Pi_{44} = P_5^T B + B^T P_5 - (1 - h_D)Q + hM_{11} - M_{12} - M_{12}^T - N_4 - N_4^T + hX_{44}$,
$\Pi_{45} = P_5^T L + B^T P_6 - N_5^T + hX_{45}$,
$\Pi_{55} = P_6^T L + L^T P_6 - \varepsilon I + hX_{55}$.

Proof: We choose the following Lyapunov-Krasovskii functional candidate as follows:

$$V = V_1 + V_2 + V_3 + V_4 \tag{10}$$

where

$$V_1 = \xi^T(t) EP \xi(t)$$

$$V_2 = \int_{t-\tau}^{t} \dot{x}^T(s) R \dot{x}(s) ds + \int_{t-h(t)}^{t} x^T(s) Q x(s) ds$$

$$V_3 = \int_{-h(t)}^{0} d\theta \int_{t+\theta}^{t} \dot{x}^T(\xi) S \dot{x}(\xi) d\xi$$

$$\begin{aligned} V_4 = \textstyle\int_0^t d\theta \int_{\theta-h(\theta)}^{\theta} & \begin{bmatrix} x^T(\theta - h(\theta)) & \dot{x}^T(s) \end{bmatrix} \\ & \cdot \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \begin{bmatrix} x(\theta - h(\theta)) \\ \dot{x}(s) \end{bmatrix} ds \end{aligned}$$

where

$\xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & \dot{x}^T(t-\tau) & x^T(t-h(t)) & u^T \end{bmatrix}^T$,

$E = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$,

$P = \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 \end{bmatrix}$. where $R > 0$, $S > 0$, $Q > 0$ and $P_1 > 0$ and $\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0$ are solutions of (7-9).

The derivative of V along the trajectory of system (1) is given by

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 \tag{11}$$

From (11), we have

$$\begin{aligned} \dot{V}_1 &= 2\xi^T(t) P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} = 2\xi^T(t) P^T \\ & \cdot \begin{bmatrix} \dot{x}(t) \\ \begin{pmatrix} Ax(t) - \dot{x}(t) + C\dot{x}(t - \tau(t)) \\ +Bx(t - h(t)) + Lu \end{pmatrix} \end{bmatrix} \\ &= 2\xi^T(t) P^T \begin{bmatrix} \dot{x}(t) \\ \begin{pmatrix} Ax(t) - \dot{x}(t) + C\dot{x}(t - \tau(t)) \\ +Bx(t - h(t)) + Lu \end{pmatrix} \end{bmatrix} \\ &= \xi^T(t)(\Gamma + \Gamma^T)\xi(t) \end{aligned} \tag{12}$$

where

$$\Gamma = \begin{bmatrix} P_1 & P_2^T \\ 0 & P_3^T \\ 0 & P_4^T \\ 0 & P_5^T \\ 0 & P_6^T \end{bmatrix} \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ A & -I & C & B & L \end{bmatrix}$$

$$\begin{aligned} \dot{V}_2 \leq \dot{x}^T(t) R \dot{x}(t) - \dot{x}^T(t-\tau) R \dot{x}(t-\tau) + x^T(t) Q \\ \cdot x(t) - (1 - h_D) x^T(t - h(t)) Q x(t - h(t)) \end{aligned} \tag{13}$$

$$\dot{V}_3 \leq h\dot{x}^T(t) S \dot{x}(t) - (1 - h_D) \textstyle\int_{t-h(t)}^{t} \dot{x}^T(s) S \dot{x}(s) ds \tag{14}$$

$$\begin{aligned}\dot{V}_4 \leq & hx^T(t-h(t))M_{11}x(t-h(t)) + 2x^T(t)M_{12}^T \\ & \cdot x(t-h(t)) - 2x^T(t-h(t))M_{12}x(t-h(t)) \\ & + \int_{t-h(t)}^{t} \dot{x}^T(s)M_{22}\dot{x}(s)ds\end{aligned} \tag{15}$$

Then substituting (12-15) into (11), we further have

$$\begin{aligned}\dot{V} \leq & x^T(t)[P_2A + A^TP_2 + Q]x(t) + 2x^T(t)[P_1 - P_2 \\ & + A^TP_3]\dot{x}(t) + 2x^T(t)[P_2^TC + A^TP_4]\dot{x}(t-\tau) \\ & + 2x^T(t)[P_2^TB + A^TP_5 + M_{12}^T]x(t-h(t)) + 2x^T(t) \\ & \cdot [P_2^TL + A^TP_6]Lu + \dot{x}^T(t)[hS + R - P_3^T - P_3]\dot{x}(t) \\ & + 2\dot{x}^T(t)[P_3^TC - P_4]\dot{x}(t-\tau) + 2\dot{x}^T(t)[P_3^TB - P_5] \\ & \cdot x(t-h(t)) + 2\dot{x}^T(t)[P_3^TL - P_6]u + \dot{x}^T(t-\tau) \\ & \cdot [P_4^TC + C^TP_4 - R]\dot{x}(t-\tau) + 2\dot{x}^T(t-\tau)[P_4^TB \\ & + C^TP_5]x(t-h(t)) + 2\dot{x}^T(t-\tau)[P_4^TL + C^TP_6]u \\ & + x^T(t-h(t))[P_5^TB + B^TP_5 - (1-h_D)Q + hM_{11} \\ & - M_{12} - M_{12}^T]x(t-h(t)) + 2x^T(t-h(t))[P_5^TL \\ & + B^TP_6]u - \int_{t-h(t)}^{t} \dot{x}^T(s)\dot{x}(s)Gds + u^T[P_6^TL \\ & + L^TP_6]u + 2[x^T(t)N_1 + \dot{x}^T(t)N_2 + \dot{x}^T(t-\tau)N_3 \\ & + x^T(t-h(t))N_4 + u^TN_5] \cdot [x(t) - x(t-h(t)) \\ & - \int_{t-h(t)}^{t} \dot{x}(s)ds].\end{aligned}$$

We define

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\ * & * & * & \Sigma 44 & \Sigma_{45} \\ * & * & * & * & 0 \end{bmatrix}$$

where $\Sigma_{ij} = \Pi_{ij} - hX_{ij}$ for $i, j = 1, 2, \cdots, 5$, and $\Pi_{ij}(i, j = 1, 2, \cdots, 5)$ are the same as defined in the Theorem 1.

For any matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ & X_{22} & X_{23} & X_{24} & X_{25} \\ & * & X_{33} & X_{34} & X_{35} \\ & * & * & X_{44} & X_{45} \\ & * & * & * & X_{55} \end{bmatrix} \geq 0,$$

We have

$$\xi^T(t)(hX)\xi(t) - \int_{t-h(t)}^{t} \xi^T(t)X\xi(t)ds \geq 0. \tag{16}$$

Then, the following inequality can be obtained:

$$\dot{V} \leq \xi^T(t)\Phi\xi(t) - \int_{t-h(t)}^{t} \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \Omega \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds \tag{17}$$

where $\Phi = \Sigma + hX$ and

$$\Omega = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & N_1 \\ & X_{22} & X_{23} & X_{24} & X_{25} & N_2 \\ & * & X_{33} & X_{34} & X_{35} & N_3 \\ & * & * & X_{44} & X_{45} & N_4 \\ & * & * & * & X_{55} & N_5 \\ & * & * & * & * & G \end{bmatrix}.$$

A sufficient condition for asymptotic stability of system is that there exist real matrices $N_i(i = 1, 2, \cdots 5)$, $R > 0$, $G > 0$, $S > 0$, $Q > 0$ and $P_1 > 0$ and $\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0$ such that

$$\dot{V} \leq \xi^T(t)\Phi\xi(t) - \int_{t-h(t)}^{t} \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \Omega \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds < 0 \tag{18}$$

for all $\xi(t) \neq 0$. Using the S-procedure in [1], one can see that this condition is implied by the existence of a nonnegative scalar $\epsilon > 0$ such that

$$\begin{aligned}\dot{V} \leq & \xi^T(t)\Phi\xi(t) - \int_{t-h(t)}^{t} \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \Omega \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds \\ & + \varepsilon\{[E_ax(t) + E_bx(t-h(t))]^T[E_ax(t) \\ & + E_bx(t-h(t))] - u^Tu\} < 0\end{aligned} \tag{19}$$

for all $\xi(t) \neq 0$. By using Lemma 1, the matrix inequalities (7-9) imply (19). It is well known that Assumption 1 guarantees the stability of different system $x(t) - Cx(t-\tau) = 0$. Therefore, system described by (1-4) is robustly asymptotically stable according to Theorem 8.1 in [2]. ■

Applying the similar method in the proof of Theorem 1, it is easy to obtain the following Theorem for the nominal system of systems (1)-(4), that is the system

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h(t)) \tag{20}$$

Theorem 2. Under Assumption 1, the system (20) is robustly asymptotically stable, if there exist real matrices $P_i(i = 2, 3, 4, 5)$, $N_i(i = 1, 2, 3, 4)$, $X_{ij}(i, j = 1, 2, 3, 4)$, M_{11}, M_{12} and M_{22}, symmetric positive definite matrices G, R, S, Q and P_1, scalar $\epsilon > 0$ satisfying the following matrix inequalities:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ & * & \Pi_{33} & \Pi_{34} \\ & * & * & \Pi_{44} \end{bmatrix} < 0 \tag{21}$$

$$\Omega = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & N_1 \\ & X_{22} & X_{23} & X_{24} & N_2 \\ & * & X_{33} & X_{34} & N_3 \\ & * & * & X_{44} & N_4 \\ & * & * & * & G \end{bmatrix} \geq 0 \tag{22}$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1-h_D)S - M_{22} \geq G \tag{23}$$

where $\Pi_{ij}(i, j = 1, 2, 3, 4)$ are the same as defined in the Theorem 1.

Remark 1. Choosing a special matrix:

$$X = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix}^T G^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix}$$

we obtain $X \geq 0$ satisfying $\Omega \geq 0$. Applying the similar method in the proof of Theorem 1 and Schur Complement, Corollary 1 is presented as follows.

Corollary 1. Under Assumption 1, the system described by (1-4) is robustly asymptotically stable, if there exist real matrices $P_i(i = 2, 3, \cdots, 6)$, $N_i(i = 1, 2, \cdots, 5)$, $X_{ij}(i, j = 1, 2, \cdots, 5)$, M_{11}, M_{12} and M_{22}, symmetric positive definite

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matrices G, R, S, Q and P_1, scalar $\epsilon > 0$ satisfying the following matrix inequalities:

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & \hat{\Pi}_{14} & \hat{\Pi}_{15} & hN_1 & \varepsilon E_a^T \\ & \hat{\Pi}_{22} & \hat{\Pi}_{23} & \hat{\Pi}_{24} & \hat{\Pi}_{25} & hN_2 & 0 \\ & * & \hat{\Pi}_{33} & \hat{\Pi}_{34} & \hat{\Pi}_{35} & hN_3 & 0 \\ & * & * & \hat{\Pi}_{44} & \hat{\Pi}_{45} & hN_4 & \varepsilon E_b^T \\ & * & * & * & \hat{\Pi}_{55} & hN_5 & 0 \\ & * & * & * & * & -hG & 0 \\ & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1-h_D)S - M_{22} \geq G \tag{25}$$

where $\hat{\Pi}_{ij} = \Pi_{ij} - hX_{ij}$ for $i,j = 1,2,\cdots,5$, and $\Pi_{ij}(i,j = 1,2,\cdots,5)$ are the same as defined in the Theorem 1.

Remark 2. Choosing a special matrix:

$$X = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix}^T G^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix}$$

and using the similar method in the proof of Theorem 2 and Schur Complement, we can obtain the Corollary 2.

Corollary 2. Under Assumption 1, the system described by (1-4) is robustly asymptotically stable, if there exist real matrices $P_i(i = 2,3,4,5)$, $N_i(i = 1,2,4,5)$, $X_{ij}(i,j = 1,2,3,4)$, M_{11}, M_{12} and M_{22}, symmetric positive definite matrices G, R, S, Q and P_1, scalar $\epsilon > 0$ satisfying the following matrix inequalities:

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & \hat{\Pi}_{14} & hN_1 \\ & \hat{\Pi}_{22} & \hat{\Pi}_{23} & \hat{\Pi}_{24} & hN_2 \\ & * & \hat{\Pi}_{33} & \hat{\Pi}_{34} & hN_3 \\ & * & * & \hat{\Pi}_{44} & hN_4 \\ & * & * & * & -hG \end{bmatrix} < 0 \tag{26}$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1-h_D)S - M_{22} \geq G \tag{27}$$

where $\hat{\Pi}_{ij} = \Pi_{ij} - hX_{ij}$ for $i,j = 1,2,3,4$, and $\Pi_{ij}(i,j = 1,2,3,4)$ are the same as defined in the Theorem 1.

IV. Examples

Example 1. Consider the uncertain neutral system with time-varying delay as follow:

$$\dot{x}(t) - \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \dot{x}(t-\tau) = \begin{bmatrix} -2+\delta_1 & 0 \\ 0 & -1+\delta_2 \end{bmatrix} x(t) + \begin{bmatrix} -1+\gamma_1 & 0 \\ -1 & -1+\gamma_2 \end{bmatrix} x(t-h(t)) \tag{28}$$

where δ_1, δ_2, γ_1 and γ_2 are unknown parameters satisfying

$$|\delta_1| \leq 1.6, |\delta_2| \leq 0.05, |\gamma_1| \leq 0.1, |\gamma_2| \leq 0.3.$$

By applying the criteria in [14-17] and Theorem 1 in this paper, Table 1 gives the maximum value h for stability of system (28) for different c with $h_D = 0.1$. Table 2 gives the maximum value h for stability of system (28) for different h_D

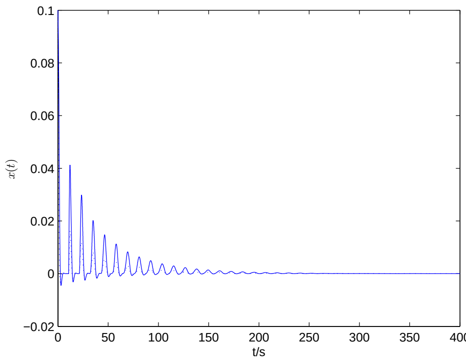

Fig. 1. The time responses of state variable $x(t)$ with $\alpha = 0$.

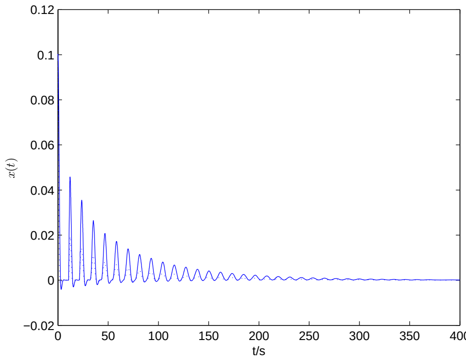

Fig. 2. The time responses of state variable $x(t)$ with $\alpha = 0.1$.

with $c = 0.1$. It is clear to see that the results in this paper are much less conservative than those in [14-17].

Moreover, it is should be pointed out that if we let $c = 0.1$ and $h_D = 0$, from Theorem 1, The maximum bound h guaranteeing robust stability of system (28) is 2.206. However, using the criteria in [15-17], the nominal system is asymptotically stable for any h satisfying $h < 0.87$, $h < 0.998$ and $h < 1.10$, respectively. It shows again that the stability criterion in this paper is much less conservative than these in [15-17].

For $c = 0$ and $h_D = 0$, the upper bounds on time-varying delay obtained from Theorem 2. For comparison, the Table 3 list the upper bounds obtained from the criteria in the literature. According the Table 3, it is clear to see that the criterion in this paper gives much less conservative results than those in the literature.

Example 2. Consider the following uncertain neutral system

TABLE I
Stability bounds of time-delay for different $|c|$ with $h_D = 0.1$

$\|c\|$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70
Han in [14]	0.92	0.73	0.55	0.41	0.29	0.19	0.11	0.04
Zhao in [15]	1.10	0.85	0.64	0.47	0.33	0.22	0.13	0.05
Yu in [16]	1.166	0.962	0.778	0.616	0.472	0.346	0.235	0.130
Kwon in[17]	1.17	1.07	0.95	0.83	0.70	0.51	0.31	0.14
h in this paper	2.05	1.84	1,64	1.43	1.23	1.02	0.82	0.61

TABLE II
STABILITY BOUNDS OF TIME-DELAY FOR DIFFERENT h_D WITH $|c| = 0.1$

h_D	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
h in [14]	0.80	0.73	0.65	0.57	0.49	0.41	0.33	0.24	0.16	0.07
h in this paper	2.20	1.84	1.55	1.30	1.08	0.89	0.76	0.55	0.40	0.24

TABLE III
COMPARATIVE RESULTS OF h FOR DIFFERENT h_D

	[18]	[19]	[14]	[20]	[21]	[15]	[22]	ours
h	0.2412	0.2412	1.030	1.1490	1.1490	1.1490	1.1623	2.4510

with time delays

$$\begin{aligned}\dot{x}(t) - C\dot{x}(t-\tau) = (A + \Delta A(t))x(t) \\ + (B + \Delta B(t))x(t-h)\end{aligned} \tag{29}$$

where $A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}$, $B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}$, $C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}$ and $\Delta A(t)$, $\Delta B(t)$ are unknown matrices satisfying and $\|\Delta A(t)\| \leq \alpha$, $\|\Delta B(t)\| \leq \alpha$, $\alpha \geq 0$, $\forall t$. The system (29) is of the form of (4) with $L = \alpha I$, $E_a = E_b = I$, $E_c = 0$.

For $\Delta A(t) = 0$ and $\Delta B(t) = 0$, system (29) is studied in [15, 23, 24]. Using the criterion in this paper, the maximum value of h for the nominal system to be asymptotically stable is $h = 1.8266$. By the criteria in [15, 23, 24], the nominal system is asymptotically stable for any h satisfying $h \leq 1.3718$, $h \leq 1.7884$ and $h \leq 1.7856$, respectively. Similar to Example 1, this example shows again that the stability criterion in this paper is much less conservative than these in [15, 23, 24].

The effect of the uncertainty bound $alpha$ on the maximum time delay for stability h is also studied. the maximum value h for stability of system (29) is listed in Table 4 different value of α. One can see that as α increases, h decreases.

Example 3. Consider the following uncertain neutral system with time delays

$$\begin{aligned}\dot{x}(t) - C\dot{x}(t-\tau) = (A + \Delta A(t))x(t) \\ + (B + \Delta B(t))x(t-h)\end{aligned} \tag{30}$$

where $A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0.5 & 0 \\ 0.5 & -0.5 \end{bmatrix}$, $C = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}$ and the system (31) is of the form of (19) with $L = \alpha I$, $E_c = 0$, $E_a = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}$, $E_b = \begin{bmatrix} 0 & -0.1 \\ 0.1 & 0.3 \end{bmatrix}$, $F = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$.

For $\alpha = 0$ and $\alpha = 0.1$, this system has been studied in [25]. By the criteria in [25], the maximum value of h for the nominal system to be asymptotically stable is $h \leq 1.0616$ and $h \leq 0.7922$. Using the criterion in this paper, the nominal system is asymptotically stable for any h. It is clear to see that the criterion in this paper gives much less conservative results than those in [25].

By choosing $h(t) = 10.4 + 0.3\sin^2(t)$, system (30) with $\tau = 0.4$ is asymptotically stable, as show in Fig 1-2. In numerical simulation, the two cases are given with the initial state $\varphi(\theta) = [0.1, 0.1]^T$. Fig. 1 depicts the time response of state variable $x(t)$ with no uncertainties. Fig. 2 depicts the time response of state variable $x(t)$ with uncertainties. It is obvious that uncertainties are the important source of neutral system instability.

TABLE IV
STABILITY BOUNDS OF TIME-DELAY FOR DIFFERENT α

$\|c\|$	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
Han in [14]	1.61	1.51	1.41	1.30	1.19	1.08	0.96	0.83
h in this paper	1.82	1.67	1.54	1.43	1.33	1.25	1.17	1.11

V. CONCLUSION

In This paper, the problem of robust stability for a class of uncertain neutral systems with time-varying delays is investigated. Sufficient conditions are given in terms of linear matrix inequalities which can be easily solved by LMI Toolbox in Matlab. Numerical examples are given to indicate significant improvements over some existing results.

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Changchun Shen was born in Sichuan Province,China,in 1982. He received the B.S. degree from Sichuan Normal University, Chengdu, in 2005. and the M.S. degree from University of Electronic Science and Technology of China, Sichuan in 2008, both in applied mathematics.He is currently pursuing the Ph.D.degree with UESTC. His research interests include chaos synchronization, medical image processing and delay dynamic systems.

Shouming Zhong was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yatsen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo, Waterloo, Canada. He is currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations, neural networks, biomathematics and robust control. He has authored more than 80 papers in reputed journals such as the International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics