

# Improved asymptotic stability criteria for uncertain neutral systems with time-varying discrete delays

Changchun Shen and Shouming Zhong

**Abstract**—This paper investigates the robust stability of uncertain neutral system with time-varying delay. By using Lyapunov method and linear matrix inequality technology, new delay-dependent stability criteria are obtained and formulated in terms of linear matrix inequalities (LMIs), which can be easy to check the robust stability of the considered systems. Numerical examples are given to indicate significant improvements over some existing results.

**Keywords**—neutral system; Linear matrix inequalities; Lyapunov; Stability.

## I. INTRODUCTION

**D**URING the past several years, considerable attention has been paid to the problem of stability analysis of time-delay systems, for example, biological systems, chemical systems, long-transmission lines in pneumatic, hydraulic systems, metallurgical processing systems, nuclear reactor, electrical networks, and so on. It is well known that the existence of time delays in a system may cause instability or bad system performance. Therefore, stability analysis of time-delay systems has been widely investigated by many researchers, such as [1-5] and references therein.

As well known, neutral system being a special case of time delay system exists in many dynamic systems, a number of stability conditions have been developed for this type of systems in the past. Recently, many researchers have paid a lot of attentions on the problem of robust stability for neutral systems with time-delay [6-8], and many efforts have been made to obtain less conservative delay-dependent conditions in the literature [7-12]. For instance, some robust stability results for neutral systems with different types of time delay were proposed in [7] and [8], respectively. In [9], Lyapunov functional technique combined with matrix inequality technique and a new operator are used to investigate the problem of robust stability for neutral systems with a constant neutral delay. By using descriptor model transformation and decomposition technique, some delay-dependent stability criterions are obtained in [10]. In [11], the stability conditions of uncertain neutral system with time-varying delay are developed by descriptor model transformation technique and the norm-bounded uncertainties is handled by S-procedure. Recently, a free weighting matrices approach combining matrix inequality technique are used in obtain the stability conditions so that less

conservative stability conditions are obtained in [12].

In this paper, the problem of stability analysis for neutral systems with time-varying delays is discussed. Since model transformation and bounding techniques for cross terms appearing in the derivative of corresponding Lyapunov functional may introduce additional conservativeness [13], neither model transformation nor bounding technique for cross terms is applied in analyzing the considered systems which may yield a less conservative stability condition. The free-weighting matrix approach [12] is also employed to further reduce the entailed conservativeness. Numerical examples illustrate the effectiveness and improvement of the obtained results.

Notation:  $\mathbb{R}^n$  is the  $n$ -dimensional Euclidean space,  $\mathbb{R}^{m \times n}$  denotes the set of  $m \times n$  real matrix.  $\|\cdot\|$  refers to the Euclidean vector norm and the induced matrix norm.  $L_2[a, b]$  is the space of the square integral function on the interval  $[a, b]$ .

## II. MODEL DESCRIPTION AND PRELIMINARIES

Consider the uncertain neutral systems with multiple time delays described by following state equation:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t - \tau) &= (A + \Delta A(t))x(t) \\ &\quad + (B + \Delta B(t))x(t - h(t)), \\ x(t_0 + \theta) &= \varphi(\theta), \quad \forall \theta \in [-\tau^*, 0], \end{aligned} \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $\varphi(\cdot) \in L_2[-\max(\tau, h), 0]$  is a differentiable vector-valued initial function,  $\tau > 0$  is a constant neutral delay, the discrete delay  $h(t)$  is a time-varying function that satisfies

$$0 \leq h(t) \leq h, \quad \dot{h}(t) \leq h_D < 1 \quad (2)$$

where  $h, h_D$  are constants,  $\tau^* = \max(\tau, h)$ ,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times n}$ ,  $C \in \mathbb{R}^{n \times n}$  and  $D \in \mathbb{R}^{n \times n}$  are known constant matrices,  $\Delta A(t)$  and  $\Delta B(t)$  are the parametric uncertainties in the system, which are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = LK(t) \begin{bmatrix} E_a & E_b \end{bmatrix} \quad (3)$$

where  $K(t)$  is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$K^T(t)K(t) \leq I \quad (4)$$

and  $L, E_a$  and  $E_b$  are known real constant matrices which characterize how the uncertainty enters the nominal matrices  $A$  and  $B$ .

The purpose of this paper is to formulate a practically computable criterion to check the stability of system described

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by (1-4).

Before proceeding further, system (1) can be written as:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t - \tau) &= Ax(t) + Bx(t - h(t)) + Lu, \\ z &= E_a x(t) + E_b x(t - h(t)), \end{aligned} \quad (5)$$

with the constraint:  $u = K(t)z$ .

We further have:

$$u^T u \leq [E_a x(t) + E_b x(t - h(t))]^T \cdot [E_a x(t) + E_b x(t - h(t))] \quad (6)$$

In this paper, the following Lemma and Assumption are needed:

**Lemma 1.** (Schur Complement). Given constant symmetric matrices  $\Sigma_1, \Sigma_2, \Sigma_3$  where  $\Sigma_1 = \Sigma_1^T$  and  $\Sigma_2 = \Sigma_2^T > 0$ , then  $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$  holds if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

**Assumption 1.** All the eigenvalues of matrix  $C$  are inside the unit circle.

### III. MAIN RESULT

In this section, based on Lyapunov method and linear matrix inequality techniques, following stability criteria are derived.

**Theorem 1.** Under Assumption 1, the system described by (1-4) is robustly asymptotically stable, if there exist real matrices  $P_i (i = 2, 3, \dots, 6)$ ,  $N_i (i = 1, 2, \dots, 5)$ ,  $X_{ij} (i, j = 1, 2, \dots, 5)$ ,  $M_{11}$ ,  $M_{12}$  and  $M_{22}$ , symmetric positive definite matrices  $G$ ,  $R$ ,  $S$ ,  $Q$  and  $P_1$ , scalar  $\epsilon > 0$  satisfying the following matrix inequalities:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} & \epsilon E_a^T \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} & 0 \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} & 0 \\ * & * & * & \Pi_{44} & \Pi_{45} & \epsilon E_b^T \\ * & * & * & * & \Pi_{55} & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix} < 0 \quad (7)$$

$$\Omega = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & N_1 \\ * & X_{22} & X_{23} & X_{24} & X_{25} & N_2 \\ * & * & X_{33} & X_{34} & X_{35} & N_3 \\ * & * & * & X_{44} & X_{45} & N_4 \\ * & * & * & * & X_{55} & N_5 \\ * & * & * & * & * & G \end{bmatrix} \geq 0 \quad (8)$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1 - h_D)S - M_{22} \geq G \quad (9)$$

where

$$\begin{aligned} \Pi_{11} &= P_2 A + A^T P_2 + Q + N_1 + N_1^T + hX_{11}, \\ \Pi_{12} &= P_1 - P_2 + A^T P_3 + N_2^T + hX_{12}, \\ \Pi_{13} &= P_2^T C + A^T P_4 + N_3^T + hX_{13}, \\ \Pi_{14} &= P_2^T B + A^T P_5 + M_1^T + N_4^T - N_1 + hX_{14}, \\ \Pi_{15} &= P_2^T L + A^T P_6 + N_5^T + hX_{15}, \\ \Pi_{22} &= hS + R - P_3^T - P_3 + hX_{22}, \\ \Pi_{23} &= P_3^T C - P_4 + hX_{23}, \\ \Pi_{24} &= P_3^T B - P_5 - N_2 + hX_{24}, \\ \Pi_{25} &= P_3^T L - P_6 + hX_{25}, \\ \Pi_{33} &= P_4^T C + C^T P_4 - R + hX_{33}, \\ \Pi_{34} &= P_4^T B + C^T P_5 - N_3 + hX_{34}, \end{aligned}$$

$$\begin{aligned} \Pi_{35} &= P_4^T L + B^T P_6 + hX_{35}, \\ \Pi_{44} &= P_5^T B + B^T P_5 - (1 - h_D)Q + hM_{11} - M_{12} - M_{12}^T - N_4 - N_4^T + hX_{44}, \\ \Pi_{45} &= P_5^T L + B^T P_6 - N_5^T + hX_{45}, \\ \Pi_{55} &= P_6^T L + L^T P_6 - \epsilon I + hX_{55}. \end{aligned}$$

*Proof:* We choose the following Lyapunov-Krasovskii functional candidate as follows:

$$V = V_1 + V_2 + V_3 + V_4 \quad (10)$$

where

$$V_1 = \xi^T(t) E P \xi(t)$$

$$V_2 = \int_{t-\tau}^t \dot{x}^T(s) R \dot{x}(s) ds + \int_{t-h(t)}^t x^T(s) Q x(s) ds$$

$$V_3 = \int_{-h(t)}^0 d\theta \int_{t+\theta}^t \dot{x}^T(\xi) S \dot{x}(\xi) d\xi$$

$$V_4 = \int_0^t d\theta \int_{\theta-h(\theta)}^{\theta} \begin{bmatrix} x^T(\theta - h(\theta)) & \dot{x}^T(s) \end{bmatrix} \cdot \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \begin{bmatrix} x(\theta - h(\theta)) \\ \dot{x}(s) \end{bmatrix} ds$$

where

$$\xi(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & \dot{x}^T(t - \tau) & x^T(t - h(t)) & u^T \end{bmatrix}^T,$$

$$E = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$P = \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 \end{bmatrix}, \quad \text{where } R > 0, S > 0, Q >$$

0 and  $P_1 > 0$  and  $\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0$  are solutions of (7-9).

The derivative of  $V$  along the trajectory of system (1) is given by

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 \quad (11)$$

From (11), we have

$$\begin{aligned} \dot{V}_1 &= 2\xi^T(t) P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} = 2\xi^T(t) P^T \\ &\cdot \begin{bmatrix} \dot{x}(t) \\ \left( \begin{array}{c} Ax(t) - \dot{x}(t) + C\dot{x}(t - \tau(t)) \\ + Bx(t - h(t)) + Lu \end{array} \right) \end{bmatrix} \\ &= 2\xi^T(t) P^T \begin{bmatrix} \dot{x}(t) \\ \left( \begin{array}{c} Ax(t) - \dot{x}(t) + C\dot{x}(t - \tau(t)) \\ + Bx(t - h(t)) + Lu \end{array} \right) \end{bmatrix} \\ &= \xi^T(t) (\Gamma + \Gamma^T) \xi(t) \end{aligned} \quad (12)$$

where

$$\Gamma = \begin{bmatrix} P_1 & P_2^T \\ 0 & P_3^T \\ 0 & P_4^T \\ 0 & P_5^T \\ 0 & P_6^T \end{bmatrix} \begin{bmatrix} 0 & I & 0 & 0 & 0 \\ A & -I & C & B & L \end{bmatrix}$$

$$\begin{aligned} \dot{V}_2 &\leq \dot{x}^T(t) R \dot{x}(t) - \dot{x}^T(t - \tau) R \dot{x}(t - \tau) + x^T(t) Q \\ &\cdot x(t) - (1 - h_D) x^T(t - h(t)) Q x(t - h(t)) \end{aligned} \quad (13)$$

$$\dot{V}_3 \leq h \dot{x}^T(t) S \dot{x}(t) - (1 - h_D) \int_{t-h(t)}^t \dot{x}^T(s) S \dot{x}(s) ds \quad (14)$$

$$\begin{aligned} \dot{V}_4 \leq & h x^T(t-h(t)) M_{11} x(t-h(t)) + 2 x^T(t) M_{12}^T \\ & \cdot x(t-h(t)) - 2 x^T(t-h(t)) M_{12} x(t-h(t)) \\ & + \int_{t-h(t)}^t \dot{x}^T(s) M_{22} \dot{x}(s) ds \end{aligned} \quad (15)$$

Then substituting (12-15) into (11), we further have

$$\begin{aligned} \dot{V} \leq & x^T(t) [P_2 A + A^T P_2 + Q] x(t) + 2 x^T(t) [P_1 - P_2 \\ & + A^T P_3] \dot{x}(t) + 2 x^T(t) [P_2^T C + A^T P_4] \dot{x}(t-\tau) \\ & + 2 x^T(t) [P_2^T B + A^T P_5 + M_{12}^T] x(t-h(t)) + 2 x^T(t) \\ & \cdot [P_2^T L + A^T P_6] L u + \dot{x}^T(t) [h S + R - P_3^T - P_3] \dot{x}(t) \\ & + 2 \dot{x}^T(t) [P_3^T C - P_4] \dot{x}(t-\tau) + 2 \dot{x}^T(t) [P_3^T B - P_5] \\ & \cdot x(t-h(t)) + 2 \dot{x}^T(t) [P_3^T L - P_6] u + \dot{x}^T(t-\tau) \\ & \cdot [P_4^T C + C^T P_4 - R] \dot{x}(t-\tau) + 2 \dot{x}^T(t-\tau) [P_4^T B \\ & + C^T P_5] x(t-h(t)) + 2 \dot{x}^T(t-\tau) [P_4^T L + C^T P_6] u \\ & + x^T(t-h(t)) [P_5^T B + B^T P_5 - (1-h_D)Q + h M_{11} \\ & - M_{12} - M_{12}^T] x(t-h(t)) + 2 x^T(t-h(t)) [P_5^T L \\ & + B^T P_6] u - \int_{t-h(t)}^t \dot{x}^T(s) \dot{x}(s) G ds + u^T [P_6^T L \\ & + L^T P_6] u + 2 [x^T(t) N_1 + \dot{x}^T(t) N_2 + \dot{x}^T(t-\tau) N_3 \\ & + x^T(t-h(t)) N_4 + u^T N_5] \cdot [x(t) - x(t-h(t)) \\ & - \int_{t-h(t)}^t \dot{x}(s) ds]. \end{aligned}$$

We define

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\ * & * & * & \Sigma_{44} & \Sigma_{45} \\ * & * & * & * & 0 \end{bmatrix}$$

where  $\Sigma_{ij} = \Pi_{ij} - h X_{ij}$  for  $i, j = 1, 2, \dots, 5$ , and  $\Pi_{ij}(i, j = 1, 2, \dots, 5)$  are the same as defined in the Theorem 1.

For any matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} \\ & X_{22} & X_{23} & X_{24} & X_{25} \\ & * & X_{33} & X_{34} & X_{35} \\ & * & * & X_{44} & X_{45} \\ & * & * & * & X_{55} \end{bmatrix} \geq 0,$$

We have

$$\xi^T(t) (h X) \xi(t) - \int_{t-h(t)}^t \xi^T(t) X \xi(t) ds \geq 0. \quad (16)$$

Then, the following inequality can be obtained:

$$\dot{V} \leq \xi^T(t) \Phi \xi(t) - \int_{t-h(t)}^t \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \Omega \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds \quad (17)$$

where  $\Phi = \Sigma + h X$  and

$$\Omega = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & N_1 \\ & X_{22} & X_{23} & X_{24} & X_{25} & N_2 \\ & * & X_{33} & X_{34} & X_{35} & N_3 \\ & * & * & X_{44} & X_{45} & N_4 \\ & * & * & * & X_{55} & N_5 \\ & * & * & * & * & G \end{bmatrix}.$$

A sufficient condition for asymptotic stability of system is that there exist real matrices  $N_i (i = 1, 2, \dots, 5)$ ,  $R > 0$ ,  $G > 0$ ,

$S > 0$ ,  $Q > 0$  and  $P_1 > 0$  and  $\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0$  such that

$$\dot{V} \leq \xi^T(t) \Phi \xi(t) - \int_{t-h(t)}^t \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \Omega \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds < 0 \quad (18)$$

for all  $\xi(t) \neq 0$ . Using the S-procedure in [1], one can see that this condition is implied by the existence of a nonnegative scalar  $\epsilon > 0$  such that

$$\begin{aligned} \dot{V} \leq & \xi^T(t) \Phi \xi(t) - \int_{t-h(t)}^t \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix}^T \Omega \begin{bmatrix} \xi(t) \\ \dot{x}(s) \end{bmatrix} ds \\ & + \epsilon \{ [E_a x(t) + E_b x(t-h(t))]^T [E_a x(t) \\ & + E_b x(t-h(t))] - u^T u \} < 0 \end{aligned} \quad (19)$$

for all  $\xi(t) \neq 0$ . By using Lemma 1, the matrix inequalities (7-9) imply (19). It is well known that Assumption 1 guarantees the stability of different system  $x(t) - Cx(t-\tau) = 0$ . Therefore, system described by (1-4) is robustly asymptotically stable according to Theorem 8.1 in [2]. ■

Applying the similar method in the proof of Theorem 1, it is easy to obtain the following Theorem for the nominal system of systems (1)-(4), that is the system

$$\dot{x}(t) - C\dot{x}(t-\tau) = Ax(t) + Bx(t-h(t)) \quad (20)$$

**Theorem 2.** Under Assumption 1, the system (20) is robustly asymptotically stable, if there exist real matrices  $P_i (i = 2, 3, 4, 5)$ ,  $N_i (i = 1, 2, 3, 4)$ ,  $X_{ij} (i, j = 1, 2, 3, 4)$ ,  $M_{11}$ ,  $M_{12}$  and  $M_{22}$ , symmetric positive definite matrices  $G$ ,  $R$ ,  $S$ ,  $Q$  and  $P_1$ , scalar  $\epsilon > 0$  satisfying the following matrix inequalities:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ & * & \Pi_{33} & \Pi_{34} \\ & * & * & \Pi_{44} \end{bmatrix} < 0 \quad (21)$$

$$\Omega = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & N_1 \\ & X_{22} & X_{23} & X_{24} & N_2 \\ & * & X_{33} & X_{34} & N_3 \\ & * & * & X_{44} & N_4 \\ & * & * & * & G \end{bmatrix} \geq 0 \quad (22)$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1-h_D)S - M_{22} \geq G \quad (23)$$

where  $\Pi_{ij} (i, j = 1, 2, 3, 4)$  are the same as defined in the Theorem 1.

**Remark 1.** Choosing a special matrix:

$$X = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix}^T G^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix}$$

we obtain  $X \geq 0$  satisfying  $\Omega \geq 0$ . Applying the similar method in the proof of Theorem 1 and Schur Complement, Corollary 1 is presented as follows.

**Corollary 1.** Under Assumption 1, the system described by (1-4) is robustly asymptotically stable, if there exist real matrices  $P_i (i = 2, 3, \dots, 6)$ ,  $N_i (i = 1, 2, \dots, 5)$ ,  $X_{ij} (i, j = 1, 2, \dots, 5)$ ,  $M_{11}$ ,  $M_{12}$  and  $M_{22}$ , symmetric positive definite

matrices  $G$ ,  $R$ ,  $S$ ,  $Q$  and  $P_1$ , scalar  $\epsilon > 0$  satisfying the following matrix inequalities:

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & \hat{\Pi}_{14} & \hat{\Pi}_{15} & hN_1 & \varepsilon E_a^T \\ & \hat{\Pi}_{22} & \hat{\Pi}_{23} & \hat{\Pi}_{24} & \hat{\Pi}_{25} & hN_2 & 0 \\ & * & \hat{\Pi}_{33} & \hat{\Pi}_{34} & \hat{\Pi}_{35} & hN_3 & 0 \\ & * & * & \hat{\Pi}_{44} & \hat{\Pi}_{45} & hN_4 & \varepsilon E_b^T \\ & * & * & * & \hat{\Pi}_{55} & hN_5 & 0 \\ & * & * & * & * & -hG & 0 \\ & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1 - h_D)S - M_{22} \geq G \quad (25)$$

where  $\hat{\Pi}_{ij} = \Pi_{ij} - hX_{ij}$  for  $i, j = 1, 2, \dots, 5$ , and  $\Pi_{ij}(i, j = 1, 2, \dots, 5)$  are the same as defined in the Theorem 1.

**Remark 2.** Choosing a special matrix:

$$X = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix}^T G^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix}$$

and using the similar method in the proof of Theorem 2 and Schur Complement, we can obtain the Corollary 2.

**Corollary 2.** Under Assumption 1, the system described by (1-4) is robustly asymptotically stable, if there exist real matrices  $P_i (i = 2, 3, 4, 5)$ ,  $N_i (i = 1, 2, 4, 5)$ ,  $X_{ij} (i, j = 1, 2, 3, 4)$ ,  $M_{11}$ ,  $M_{12}$  and  $M_{22}$ , symmetric positive definite matrices  $G$ ,  $R$ ,  $S$ ,  $Q$  and  $P_1$ , scalar  $\epsilon > 0$  satisfying the following matrix inequalities:

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & \hat{\Pi}_{14} & hN_1 \\ & \hat{\Pi}_{22} & \hat{\Pi}_{23} & \hat{\Pi}_{24} & hN_2 \\ & * & \hat{\Pi}_{33} & \hat{\Pi}_{34} & hN_3 \\ & * & * & \hat{\Pi}_{44} & hN_4 \\ & * & * & * & -hG \end{bmatrix} < 0 \quad (26)$$

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^T & M_{22} \end{bmatrix} \geq 0, \quad (1 - h_D)S - M_{22} \geq G \quad (27)$$

where  $\hat{\Pi}_{ij} = \Pi_{ij} - hX_{ij}$  for  $i, j = 1, 2, 3, 4$ , and  $\Pi_{ij}(i, j = 1, 2, 3, 4)$  are the same as defined in the Theorem 1.

#### IV. EXAMPLES

**Example 1.** Consider the uncertain neutral system with time-varying delay as follow:

$$\begin{aligned} \dot{x}(t) - \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \dot{x}(t - \tau) &= \begin{bmatrix} -2 + \delta_1 & 0 \\ 0 & -1 + \delta_2 \end{bmatrix} x(t) \\ &+ \begin{bmatrix} -1 + \gamma_1 & 0 \\ -1 & -1 + \gamma_2 \end{bmatrix} x(t - h(t)) \end{aligned} \quad (28)$$

where  $\delta_1$ ,  $\delta_2$ ,  $\gamma_1$  and  $\gamma_2$  are unknown parameters satisfying

$$|\delta_1| \leq 1.6, |\delta_2| \leq 0.05, |\gamma_1| \leq 0.1, |\gamma_2| \leq 0.3.$$

By applying the criteria in [14-17] and Theorem 1 in this paper, Table 1 gives the maximum value  $h$  for stability of system (28) for different  $c$  with  $h_D = 0.1$ . Table 2 gives the maximum value  $h$  for stability of system (28) for different  $h_D$

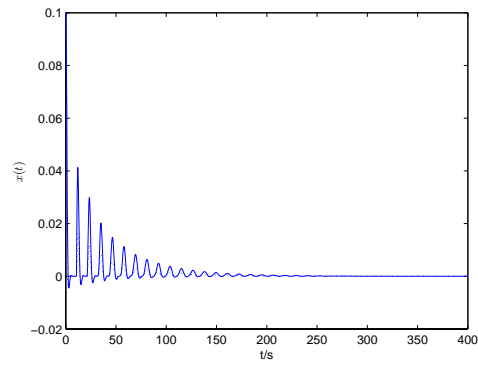


Fig. 1. The time responses of state variable  $x(t)$  with  $\alpha = 0$ .

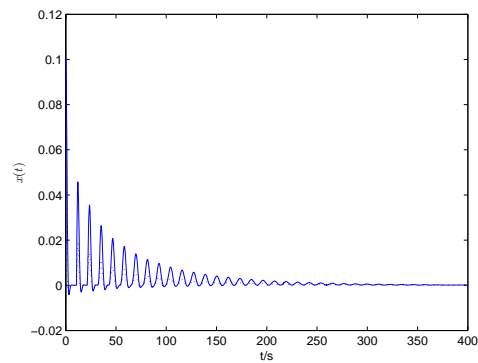


Fig. 2. The time responses of state variable  $x(t)$  with  $\alpha = 0.1$ .

with  $c = 0.1$ . It is clear to see that the results in this paper are much less conservative than those in [14-17].

Moreover, it is should be pointed out that if we let  $c = 0.1$  and  $h_D = 0$ , from Theorem 1, The maximum bound  $h$  guaranteeing robust stability of system (28) is 2.206. However, using the criteria in [15-17], the nominal system is asymptotically stable for any  $h$  satisfying  $h < 0.87$ ,  $h < 0.998$  and  $h < 1.10$ , respectively. It shows again that the stability criterion in this paper is much less conservative than these in [15-17].

For  $c = 0$  and  $h_D = 0$ , the upper bounds on time-varying delay obtained from Theorem 2. For comparison, the Table 3 list the upper bounds obtained from the criteria in the literature. According the Table 3, it is clear to see that the criterion in this paper gives much less conservative results than those in the literature.

**Example 2.** Consider the following uncertain neutral system

TABLE I  
STABILITY BOUNDS OF TIME-DELAY FOR DIFFERENT  $|c|$  WITH  $h_D = 0.1$

$ c $	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70
Han in [14]	0.92	0.73	0.55	0.41	0.29	0.19	0.11	0.04
Zhao in [15]	1.10	0.85	0.64	0.47	0.33	0.22	0.13	0.05
Yu in [16]	1.166	0.962	0.778	0.616	0.472	0.346	0.235	0.130
Kwon in [17]	1.17	1.07	0.95	0.83	0.70	0.51	0.31	0.14
$h$ in this paper	2.05	1.84	1.64	1.43	1.23	1.02	0.82	0.61

TABLE II

STABILITY BOUNDS OF TIME-DELAY FOR DIFFERENT  $h_D$  WITH  $|c| = 0.1$ 

$h_D$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$h$ in [14]	0.80	0.73	0.65	0.57	0.49	0.41	0.33	0.24	0.16	0.07
$h$ in this paper	2.20	1.84	1.55	1.30	1.08	0.89	0.76	0.55	0.40	0.24

TABLE III

COMPARATIVE RESULTS OF  $h$  FOR DIFFERENT  $h_D$ 

	[18]	[19]	[14]	[20]	[21]	[15]	[22]	ours
$h$	0.2412	0.2412	1.030	1.1490	1.1490	1.1490	1.1623	2.4510

with time delays

$$\dot{x}(t) - C\dot{x}(t - \tau) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - h) \quad (29)$$

where  $A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}$ ,  $C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}$  and  $\Delta A(t)$ ,  $\Delta B(t)$  are unknown matrices satisfying and  $\|\Delta A(t)\| \leq \alpha$ ,  $\|\Delta B(t)\| \leq \alpha$ ,  $\alpha \geq 0$ ,  $\forall t$ . The system (29) is of the form of (4) with  $L = \alpha I$ ,  $E_a = E_b = I$ ,  $E_c = 0$ .

For  $\Delta A(t) = 0$  and  $\Delta B(t) = 0$ , system (29) is studied in [15, 23, 24]. Using the criterion in this paper, the maximum value of  $h$  for the nominal system to be asymptotically stable is  $h = 1.8266$ . By the criteria in [15, 23, 24], the nominal system is asymptotically stable for any  $h$  satisfying  $h \leq 1.3718$ ,  $h \leq 1.7884$  and  $h \leq 1.7856$ , respectively. Similar to Example 1, this example shows again that the stability criterion in this paper is much less conservative than these in [15, 23, 24].

The effect of the uncertainty bound  $\alpha$  on the maximum time delay for stability  $h$  is also studied. the maximum value  $h$  for stability of system (29) is listed in Table 4 different value of  $\alpha$ . One can see that as  $\alpha$  increases,  $h$  decreases.

**Example 3.** Consider the following uncertain neutral system with time delays

$$\dot{x}(t) - C\dot{x}(t - \tau) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - h) \quad (30)$$

where  $A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0.5 & 0 \\ 0.5 & -0.5 \end{bmatrix}$ ,  $C = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}$  and the system (31) is of the form of (19) with  $L = \alpha I$ ,  $E_c = 0$ ,  $E_a = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}$ ,  $E_b = \begin{bmatrix} 0 & -0.1 \\ 0.1 & 0.3 \end{bmatrix}$ ,  $F = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$ .

For  $\alpha = 0$  and  $\alpha = 0.1$ , this system has been studied in [25]. By the criteria in [25], the maximum value of  $h$  for the nominal system to be asymptotically stable is  $h \leq 1.0616$  and  $h \leq 0.7922$ . Using the criterion in this paper, the nominal system is asymptotically stable for any  $h$ . It is clear to see that the criterion in this paper gives much less conservative results than those in [25].

By choosing  $h(t) = 10.4 + 0.3\sin^2(t)$ , system (30) with  $\tau = 0.4$  is asymptotically stable, as show in Fig 1-2. In numerical simulation, the two cases are given with the initial state  $\varphi(\theta) = [0.1, 0.1]^T$ . Fig. 1 depicts the time response of

TABLE IV

STABILITY BOUNDS OF TIME-DELAY FOR DIFFERENT  $\alpha$ 

$ c $	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
Han in [14]	1.61	1.51	1.41	1.30	1.19	1.08	0.96	0.83
$h$ in this paper	1.82	1.67	1.54	1.43	1.33	1.25	1.17	1.11

state variable  $x(t)$  with no uncertainties. Fig. 2 depicts the time response of state variable  $x(t)$  with uncertainties. It is obvious that uncertainties are the important source of neutral system instability.

## V. CONCLUSION

In This paper, the problem of robust stability for a class of uncertain neutral systems with time-varying delays is investigated. Sufficient conditions are given in terms of linear matrix inequalities which can be easily solved by LMI Toolbox in Matlab. Numerical examples are given to indicate significant improvements over some existing results.

## ACKNOWLEDGMENT

This research was supported by National Basic Research Program of China (2010CB732501) and UESTC Research Fund For Youth Science & Technology (L08010701JX0809).

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