Implementation of on-line Cutting Stock Problem on NC Machines

Jui P. Hung, Hsia C. Chang, Yuan L. Lai

Abstract—Introduction applicability of high-speed cutting stock problem (CSP) is presented in this paper. Due to the orders continued coming in from various on-line ways for a professional cutting company, to stay competitive, such a business has to focus on sustained production at high levels. In others words, operators have to keep the machine running to stay ahead of the pack. Therefore, the continuous stock cutting problem with setup is proposed to minimize the cutting time and pattern changing time to meet the on-line given demand. In this paper, a novel method is proposed to solve the problem directly by using cutting patterns directly. A major advantage of the proposed method in series on-line production is that the system can adjust the cutting plan according to the floating orders. Examples with multiple items are demonstrated. The results show considerable efficiency and reliability in high-speed cutting of CSP.

Keywords—Cutting stock; Optimization; Heuristics

I. INTRODUCTION

NESTING is a process usually applied in cutting and packing processes, and the balk of the documentations has already recorded the explorations for this rather classical issue. Such nesting problems have always been important in areas involving production. The cutting stock problems (CSP) turn up when the large volume of stock material has to be cut into smaller-sized materials to meet the requirements of customer orders. Therefore, cutting stock problems have numerous applications in a wide variety of manufacturing industries [1], from cutting steel and other metals to cutting paper, textiles, furniture[2], wood [3], glass, fiber, shipbuilding and coastal structures [4-6].

Increased manufacturing speeds, while desirable from economic perspectives, can have intense needs on stock supply arrangement and fast computing results to meet high-speed cutting requirement. Fig. 1 shows a high-speed cutting machine on a production line with a stock conveying speed of 10m/s. For CSP cutting processes, high cutting speeds can induce unpredictable variations in different length, random flaw distribution, holding conditions, and other process bearings. High speed interactions among the machine, cutting blade, and the stock materials may create uncertain conditions that can push the process away from its expected operation, adversely affecting part quality and tool life. It is increasingly realized that to meet such challenges it is necessary to address the process-machine interactions in a comprehensive manner.

Jui P. Hung is with the Department of Mechanical Engineering, National Chin-Yi University of Technology, Taichung, Taiwan, 411,ROC (e-mail: hungjp@ncut.edu.tw).

H.C. Chang is with the Department of Information Management, Hsiuping University of Science and Technology,Taichung, Taiwan, 412, ROC (e-mail: e-Mail：hcchang@mail.hust.edu.tw).

Y.L. Lai is with the Department of Mechanical and Automation Engineering, DaYeh University, Changhua, Taiwan 515, ROC. (corresponding author to provide phone: 886-4-8511888 Ext. 2583;fax: 886-4-8511224;e-mail: lyllaiber@so-net.net.tw, yllai@mail.dyu.edu.tw).

Offering increased manufacturing speeds without compromising quality, our method can significantly improve production capabilities and save costs.

Fig. 1 A production line for high-speed floorboard cutting

The characteristics of a cutting stock problem for different industries may separate into two types. First, there is a variety of criterions such as increasing efficiency of productivity and maximizing yield. A cutting stock problem is followed by an optimal stock selection problem. But most of the previous researches focus on only using the off-line optimum algorithms to find the best arrangement of cutting patterns. Thus, it cannot detect the relationship of whole group fraudsters and the execution process should not be on-line real-time.

A method is developed and applied using both conventional and the heuristic methods. This method gives nearly optimal solutions in real time. It is applied to both batch-solving and on-line solving of one-dimensional cutting of large-scale production or repetitive manufacturing. A major drawback of modern CNC systems is that the machining parameters, such as feedrate, stock size and number of cut, are still programmed off-line.

Wide treatments in combination of optimum problems appear in many application fields, and various algorithms have been developed so far. However, there are numerous combinatorial optimization problems for which no polynomial time algorithm to find an optimal solution, those problems known as NP-hard [7-8]. The Bin Packing Problem and the Cutting Stock Problem are two classes of well-known NP-hard combinatorial optimization problems [9]. In the BPP, the aim is to combine items into bins of a certain capacity so as to minimize the total number of bins, whereas in the CSP, the aim is to cut items from stocks of a certain length, minimizing the total number of stocks. Obviously, these two problem classes are very much related, and the approach proposed in this work will be able to tackle both of them.

This study focuses on one-dimensional nesting problems for multiple lengthen stock material usages. The one-dimensional cutting stock problem (1D-CSP) is one of the most representative combinatorial optimization problems, which arises in many industries. By using exact algorithms for such problems can be a very time consuming and exhausting job.

Exact solution methods for the BPP and the CSP can only be used for very small problem instances. Since the exact solution hard to be found in mass stock material, this lead to some approximating problems. As a fact, the CSP has been studied extensively. Cost functions for the CSP may implement all various objectives, such as minimizing trim loss, minimizing the number of stocks with wastage or maximizing value. It is required that CSP problems must cut in multiple dimensions, incorporate the set-up cost and time associated with changing or adjusting the knife or blade settings.

For an understanding classification of cutting stock problems, it is referred to Dyckhoff[9]and Wäscher et al. [10]. Dyckhoff made a general description and classification of cutting stock problems prior to 1990. Wäscher et al. made another overview in the same field from 1995 to 2004. Gilmore and Gomory[11]were the first who attempted to solve the one-dimensional cutting stock problem by analytical method. Since the linear programming (LP) relaxation often has the property that the rounded up value of the LP lower bound is equal to the optimum value of integer programming problem. They proposed a LP approach which is based on the implicit enumeration of potential cutting patterns. From the beginning, the feasible cutting patterns are determined to describe how many items of each type are cut from stock lengths. The resulting LP relaxation of the integer cutting stock problem is then solved with a column generation method where new cutting patterns are generated when they can improve upon the current solution. Once an optimal solution to the LP relaxation is found, exact methods such as branch-and-bound or heuristic methods, often based on rounding schemes, are used to find integer solutions.

There are two representations have been used in the genetic algorithm (GA) approach, i.e., group-based and order-based representations. The group-based representation uses groups of items as genes, and the number of genes is equal to m which is the total number of stocks used. The order-based representation uses an ordered list to represent all the items to be cut. A decoder, therefore, is needed to organize the cutting points and the stock length of each point in the list. It has been shown that the group-based GA is better than the order-based GA for CSPs. One of the major problems with the order-based GA is that crossover is unable to exploit the ordering information in the chromosome representation. In the group-based GA, the ordering information is encapsulated within genes and less susceptible to the destruction of crossover. The implementation of the crossover operator in GAs is normally a difficult task.

The crossover has to maintain the feasibility of offspring, otherwise some kind of penalty or repair methods must be used in order to evolve a feasible solution. The order-based GA uses uniform order-based crossover [12] that applies a template of randomly generated binary bits to exchange some items and maintain relative order information from both parents. The mutation operator uses both swap and remove and reinsert. The group-based GA uses grouping crossover, where a segment of one parent is inserted into another parent to generate an offspring [13]. Such crossover may not be as straightforward as it first appears because it has to avoid duplicated items being copied into the offspring.

The mutation operator selects a number of genes and rebuilds new genes from the selected genes, which is a much more time-consuming task than a simple swap mutation. Liang et al. [14]proposed a novel evolution algorithm (EP), which is similar to GA but uses mutation as search operation only. The remove and reinsert mutation is also used in the group-based GA. Usually the crossover in the order-based GA could degrade the performance of GA. The crossover operator could not exploit the ordering information and sometimes it even destroyed the ordering information. In this paper, we propose a new evolutionary approach to CSPs with and without contiguity based on an EP algorithm. The EP algorithm is much simpler and less time-consuming than the GA, and only uses swap mutations. No crossover is adopted. The order-based representation will be used in our EP algorithm, which turns out to be much better than the group-based GA.

Holthaus[15] proposed the decomposition approaches based on the classical column-generation technique for solving the integer one-dimensional cutting stock problem with different types of standard lengths. Belov and Scheithauer[16] presented on an algorithm for one-dimensional nesting problem where branching is applied to Gilmore-Gomory formulation. Dikili et al. [17]introduced a successive elimination method to solve one-dimensional cutting-stock problem where cutting plans are achieved directly without the need to establish a mathematical model. The main objective of the method is to reach the optimal integer solution while minimizing the number of different patterns contained in a solution. Valério de Carvalho et al. [18], Degraeve and Peeters[19], Hajizadeh and Lee (2007) as well as Alves and Valério de Carvalho[20]all proposed approaches to finding optimal integer solutions to the one-dimensional cutting stock problem. Poldi and Arenales[21]compared different types of rounding heuristics to convert a fractional solution into a feasible integer solution for a slightly modified problem with limited supply of different lengths of stock material.

Johnston and Sadinlija[22]developed a new model for one-dimensional nesting problems that does not require prespecification of cutting patterns. The cutting stock problem in its classic form only considers how large items can be cut into smaller ones in order to meet demand. Additional considerations such as order allocation between parallel machines [23]and a multi-period approach where waste material longer than a given threshold can be returned to the warehouse and used later [24]have also been considered. In addition, significant work has been done on cutting stock problems where the number of open stacks during production is a concern. In these problems, the sequencing of cutting patterns is important as it determines the number of partially completed orders at any time during the cutting operation. These problems are therefore much more difficult to solve those cutting stock problems where the sequencing of patterns is arbitrary.

More recently, Yanasse[25]considered sequencing in order to create a cutting plan where the maximum number of open stacks is limited. Dikili et al. [6]proposed an approach to achieve results using cutting patterns directly whereas analytical methods first need to establish a mathematical model. While obtaining ideal solutions of the analytical methods, the new approach limits the wastage to a minimum number of stock materials.

Ragsdale et al. [23]proposed a genetic algorithm for the ordered CSP, and more recently Alves and Valério de Carvalho[7]proposed an exact algorithm for the same problem. Their focus is a reduction of in-process inventory levels and material handling activities and their approach only considers the sequencing of complete orders, as opposed to individual cutting patterns. Erajvee et al. [26] presented a methodology for evaluation CS process renovation benefits.

The rest of this paper is organized as follows. The problem is described in the next section. In section 3, we discuss a solution method for the model and the objectives have to be considered and minimized. Two test cases are introduced to present the new heuristic method with different numerical experiments in section 4. Finally, the conclusions are given in section 5.

II. PROBLEM DEFINITION

The aim of CSP is to cut an object made of material that can be a fast feeding wood, pipe, or wire, etc., to fulfill customer orders. In a cutting problem there are two groups of basic data, whose elements define geometric sizes in one or more dimensions. The material is referred to as the stock of large objects and the list of orders as so-called small items. If there is more than one stock length to be cut to satisfy the demands, the problem is called a multiple stock length CSP. In this paper, we focus on CSPs with multiple stock lengths.

Dyckhoff[9]proposes that cutting problems can be classified using four characteristics. There are dimensionality, kind of assignment, assortment of large objects and assortment of small items. Dyckhoff separates the solution of cutting stock problems into two types, item- and pattern-oriented approach. The item-oriented approach is characterized by individual treatment of all items to be cut. In the pattern-oriented approach, order lengths are combined into cutting patterns at first. Follow a succeeding step, the frequencies that are necessary to satisfy the demands will be determined. In the pattern-oriented approach, the classic LP-based or any other LP-based method (LPM) is mostly used. LPM is possible only when the stock is of the same length or of few groups of standard lengths which means that all objects within the same group are identical and the other when all stock lengths are different, frequencies cannot be determined and therefore only an item-oriented solution can be found. This means that in cases when the stock is of different lengths, there are two different situations which require two different solution approaches. To solve 1D-CSP in this paper, we need the item-oriented method because all stock lengths can be different. There are two choices to be implemented, exact methods (branch and bound, dynamic programming) or approximation algorithms in the form of Sequential Heuristic Procedure (SHP). SHP seems to be better regarding robustness and potential usefulness in a wide range of cases. Time complexity of SHP can be much lower. Creating a cutting plan for an extensive order takes only a few second on a personal computer. If some exact method should be used, this problem would become intractable.

The majority of the existing research papers focus on standard one-dimensional cutting problem where a single order in a single time has to be fulfilled with a fixed amount of material in stock.

The objective of this study is to develop a more feasible method to address one-dimensional nesting problem when multiple sized stock materials are going to be provided during manufacturing process. Once the stock material sizes are known beforehand, the method assists in the choice of the optimum single sized material to be used. A concept of placement factor is introduced which avoids to create each two adjacent items are too small due to the holding equipment for each cutting pattern when different sized stock materials are used. This value realizes the cutting patterns effectively in the nesting problems. The approach presented in this study which incorporates different sized stock materials along with single stock materials, allows the systematic examination of the above-mentioned problem. Moreover, when single stock material is used, it produces more sensitive and economical results. The conventional model for the objective of minimizing the total cost C to satisfy the orders can be formulated as follows:

$$\min C = \sum_{j=1}^{m} f(W_j, z_j) \tag{1}$$

where $f(W_j, z_j)$ is a function of W_j and z_j

$$W_j = L_j - \sum_{i=1}^{n} x_{ij} l_i, \quad j = 1, \dots, m$$

$$z_j = \begin{cases} 1 & if \ w_j > 0, \\ 0 & otherwise, \end{cases} \quad j = 1, \dots, m \tag{2}$$

$$\text{Subject to } \sum_{j=1}^{m} x_{ij} = N_i \quad i = 1, \dots, n \tag{3}$$

where w_j is the wastage of the jth stock, z_j is the jth stockwith wastage, n is the number of different requested items, m is the total number of stocks cut, L_j is the length of the jth stock, x_{ij} is the number of the ith requested item in the jth stock, l_i is the length of the ith requested item, and N_i is the total number of the ith requested item.

III. SOLUTION PROCEDURE

As shown in Fig. 2, the problem we discussed in this paper is known as a cutting stock problem with on-line stock sizes detecting, as shown in Fig. 3. Not only the length could be different, but also there could be three various grades of stock material can be chosen for their appropriate quality, normal-grade, medium-grade and high grade. Usually the stock materials are supplied sufficiently with various demands for each item irregularly. The stock materials are fed into the cutting machine with one cutting blade. The tool cuts the current stock with high cutting speed. In order to cut the stock with premium pattern in a very short cutting period after detecting the current stock size, the computing time must less than 0.1 second with the feeding speed 10m/s.

According to the application's needs, the proposed procedure is implemented by both conventional and heuristic methods described as follows.
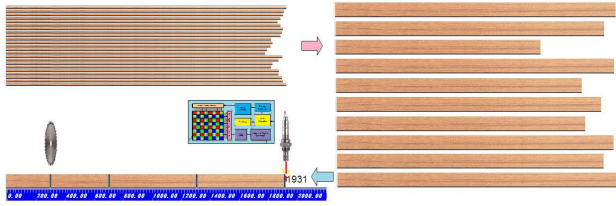

Fig. 2 An illustration of the on-line CSP process

Step 1: n different sized parts demand quantity and grading (L_i, D_i, G_i) are given first.

Step 2: Sort n different sized items according to descending size
,
for $k = 1 \sim (n-1)$, $IL_k \geq IL_{k+1}$

Set of the parts is
$$R = \{(L_1, D_1, G_1), ..., (L_i, D_i, G_i), ..., (L_n D_n, G_n)\} \quad (4)$$

Step 3: Detect the length and grade of the current stock material S_j.

Step 4: Confirm m different sized stock material of length (to the current stock, $m=j$).

Step 5: When the combination of cutting patterns are prepared, the ones that exceed the current stock length are already eliminated.

Step 6: Set up the current cutting pattern by evaluating the cost function C while considering the demand quantity, adjacent sizes and grading.
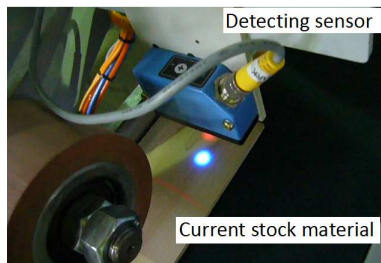

Detecting sensor

Current stock material

Fig. 3 The real time detecting sensor on the cutting machine

Step 7: Record the wastage W_j.

Step 8: Sort the potential cutting patterns with the maximum value.

Step 9: Maximize the placement factor P_i once the maximum value are equal in step 6. For the current cutting, discriminate the placement factor while considering the automatic feeding and transporting handling.

Step 10: Store the best cutting pattern from all the possible cutting patterns prepared for the current stock materials while considering the demand quantities, wastage and priority (if necessary).

Step 11: Terminate the process if the halting criterion is satisfied, otherwise, go to step 5.

In this case, values of wastage W_j, cost function C_j and placement factor P_j can be calculated using the following relationship.

$$W_j = L_j - \sum_{i=1}^{n} x_{ij} l_i \quad (5)$$

$$C_j = \sum_{i=1}^{n} x_{ij} l_i w_i \quad (6)$$

Subject to $\sum_{i=1}^{n} x_{ij} l_i + (Nc_j - 1) w_c \leq L_j \quad (7)$

where Nc_j is the number of cutting times in the jth stock, w_i is the priority weight of the ith requested item, and w_c is the width of cutting blade.

IV. COMPUTATIONAL RESULTS

Two examples are given below and the complete data for these problems is given latter. Typical cases for 1D-CSP are presented here which use four and more different sizes stock materials, respectively.

Example 1

Four types of items will be cut from different sizes stock material with unknown lengths detected on-line. The dimensions and demand quantities of the parts are given as show in Table. I. The initial condition for gain weight of each item is 1. Suppose the stock materials are supplied sufficiently and the current cutting pattern on-line is to be computed.

TABLE I
DEFINITION OF THE ITEMS

i	L_i	D_i	w_i
1	300	100	1
2	400	100	1
3	500	100	1
4	600	100	1

In order to obtain the on-line cutting plan for the given problem, the first step should be evaluating the maximum amount of stock materials according to the item specifications and demand quantities to fulfill the continued orders. First, all cutting conditions will be determined for each stock material on-line while taking features of the parts into consideration. Value, wastage and placement factor are calculated for the current cutting pattern. When the current stock size is determined, the demand quantities are considered which reduce the wastage of trim loss. The algorithm exploits the fact that the placement factor canbe chosen independently; it chooses a variable orderingthat often yields very good prioritized solutions. However, there could be several solutions to the problem. The largest placement factor yields the first solution.
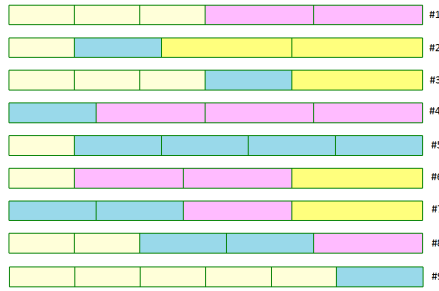
Fig. 4 Pattern layouts of the nine possible solutions in example 1

In this case, the first result obtained from possible cutting patterns formed for the current stock material are the ones with the largest usage rate where stock material with length $L_j = 1931$ is to be used and nine cutting patterns will be considered. Due to the usage rate are equal, placement factor plays a crucial role to choose the best solution. In this case, after the first heuristic combination process, 9 supreme possible solutions have been brought out, as shown in Table II and Fig. 4. The placement factor to be maximized is defined as

$$\frac{\sum_{i=1}^{n}\left(\sum_{1}^{x_{ij}} i^2\right)}{N_c} \qquad (8)$$

where x_{ij} denotes the number of the ith item in the jth stock. The placement factors of the case in Table II are calculated as:

$\textit{Placement Factor } \#1 = (1^2 + 1^2 + 1^2 + 3^2 + 3^2)/4 = 21/4 = 5.25$

$\textit{Placement Factor } \#2 = (1^2 + 2^2 + 4^2 + 4^2)/3 = 37/3 = 12.33$

$\textit{Placement Factor } \#3 = (1^2 + 1^2 + 1^2 + 2^2 + 4^2)/4 = 23/4 = 5.75$

All other possible solutions of placement factor are shown in Table II. The second possible pattern has been chosen based on the rules of the placement factor method. This cutting pattern describes the cutting of one piece of part 1, 2 and two pieces of part 4 from the current stock material, as shown in Fig. 5. The results indicate that the first solution also fit the request of minimum number of cut.

TABLE II
POSSIBLE CUTTING PATTERN FOR CURRENT STOCK MATERIAL WITH LENGTH $L_j = 1931$

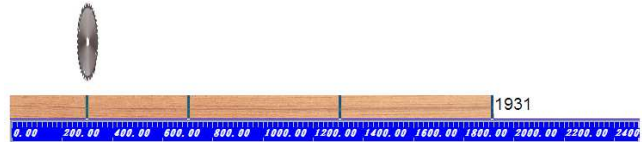| Possible Solutions	Item 1 300	Item 2 400	Item 3 500	Item 4 600	Usage	Usage Rate	No. of Cut	Placement Factor	
#1	$x_{1j} = 3$	$x_{2j} = 0$	$x_{3j} = 2$	$x_{4j} = 0$	1900	0.98	4	5.25	
#2	1	1		2	1900	0.98	3	12.33	(Chosen)
#3	3	1		1	1900	0.98	4	5.75	
#4		1	3		1900	0.98	3	10.33	
#5	1	4			1900	0.98	4	4.25	
#6	1		2	1	1900	0.98	3	11.67	
#7		2	1	1	1900	0.98	3	11	
#8	2	2	1		1900	0.98	4	8.25	
#9	5	1			1900	0.98	5	1.8	

Fig. 5 The final cutting pattern with length $L_j = 1931$ in example 1

TABLE III
POSSIBLE CUTTING PATTERNS WITH THE CASE IN FIG. 5

Possible Solutions	Item 1 300	Item 2 400	Item 3 500	Item 4 600	Usage (Virtual)	Usage Rate	No. of Cut	
#1	$x_{1j} = 3$	$x_{2j} = 0$	$x_{3j} = 2$	$x_{4j} = 0$	2350	1.216	4	
#2	1	1		2	2050	1.061	3	
#3	3	1		1	2350	1.216	4	
#4		1	3		1900	0.983	3	
#5	1	4			2050	1.061	4	
#6	1		2	1	2050	1.061	3	
#7		2	1	1	1900	0.983	3	
#8	2	2	1		2200	1.139	4	
#9	5	1			2650	1.372	5	(Chosen)

Fig. 6 Complete data for the first twenty stock materials with length $L_j = 1931$ in example 1

A major advantage of the proposed method in series on-line production is that the system can adjust the cutting plan according to the floating orders. Generally speaking, real-time response for floating orders from different customers anytime, anywhere for some items are found in patterns that are more flexible than those that have only one order for that same production. Take example 1 for instance, if there have been twenty stock materials cut with length $L_j = 1931$.

Part 1 and 2 reach 20% of demand, while part 4 reaches 40% of demand. In order to obtain the on-line cutting plan for the given problem of changing the gain weight from 1 to 1.5 for part 1. Value, wastage and placement factor are calculated for the current cutting pattern. When the current stock size is determined, the demand quantities are considered which reduce the wastage of trim loss. Then, cutting patterns are determined according to the renewed gain weights and similarly the solution with the largest usage rate is chosen. Because the gain weights may larger than one unit, the usage rate may exceed 100% too. The actual usages are subject to equation 7. After the first heuristic combination process, 9 possible solutions have been brought out to meet the maximum usage rate. Further calculation for the virtual usage with different gain weights of the case in Fig.7 are calculated as:

$C \ \#1 = 300 * 1.5 * 3 + 500 * 2 = 2350$

$C \ \#2 = 300 * 1.5 + 400 + 600 * 2 = 2050$

$C \ \#7 = 400 * 2 + 500 + 600 = 1900$

$$C\ \#8 = 300*1.5*2 + 400*2 + 500 = 2200$$

$$C\ \#9 = 300*1.5*5 + 400 = 2650$$

Length	W	QTY	Count	Sorter	%
300	1.5	100	25	3	25.00%
400	1	100	21	0	21.00%
500	1	100	0	0	0.00%
600	1	100	40	0	40.00%

Fig. 7 Complete data for the 21th stock material with length $L_j = 1931$ in example 1

All other possible solutions of usage are shown in Table III. The 9th possible pattern has been chosen based on the rules of the virtual usage method. This cutting pattern describes the cutting of five pieces of part 1 and one piece of part 2 from the current stock material, as shown in Fig. 8. The final results are implemented by gain weights as the best solution without placement factor determination. The manufacturing schedule can vary greatly based on the size and gain weight of each item. The volume of each item will increase by following this cutting pattern until reach the demand of each item or change various parameters.
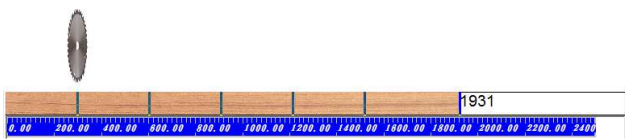

Fig. 8 The final cutting pattern with length $L_j = 1931$ in Fig. 7

V. CONCLUSION

The feature of a cutting stock problem is that the stock material has to be cut to produce smaller pieces of materials, items, in quantities matching received orders. In a modern production the challenge turns from the competition between individual companies to the competition between supply chains. Whenever the JIT concept is adopted, providing easy supply chain integration for larger or smaller quantities of material becomes a simple task to improve productivity. In order to optimize the supply channel, it is necessary to ensure that the least amount of inventory is held or make inventory available at the lowest possible cost.

In this paper we have proposed a novel method to cutting stock problem where we seek an optimum solution in dynamic orders without managing stock materials in advance. The results of the testing cases in practice show that using different sized stock materials without limits yield economical and practicable results.

The present method also helps finding the ideal solution for a single type of stock material. Because the stock materials are fed into the cutting machine with the speed of 10m/sec, the solution is achieved in 1/10 second. Therefore, the solution has to be completed much faster than ever before. The present method which involves heuristic selection at the initial combination is based on the solution for the single sized stock material developed by Gilmore and Gomory.

The solution finding process has been simplified for multiple length stock materials. The creative approximation involves the use of cutting patterns along with placement factor. Whenever the usage rate comes to equal, placement factor is a usage value for any cutting pattern which effectively realizes the cutting patterns in the nesting problem.

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