How Social Network Structure Affects the Dynamics of Evolution of Cooperation?

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Abstract—The existence of many biological systems, especially human societies, is based on cooperative behavior [1, 2]. If natural selection favors selfish individuals, then what mechanism is at work that we see so many cooperative behaviors? One answer is the effect of network structure. On a graph, cooperators can evolve by forming network bunches [2, 3, 4]. In a research, Ohtsuki et al used the idea of iterated prisoners' dilemma on a graph to model an evolutionary game. They showed that the average number of neighbors plays an important role in determining whether cooperation is the ESS of the system or not [3]. In this paper, we are going to study the dynamics of evolution of cooperation in a social network. We show that during evolution, the ratio of cooperators among individuals with fewer neighbors to cooperators among other individuals is greater than unity. The extent to which the fitness function depends on the payoff of the game determines this ratio.

Keywords—Evolution of cooperation, Iterated prisoner's dilemma, Model dynamics, Social network structure, Intensity of selection.

I. INTRODUCTION

NATURAL selection has no problem in describing phenomena such as sexual lust, hungriness and fear [5]. However, our concern is on how evolution favours cooperation and altruistic behavior in this competitive world? If nature selects *selfish genes* [6], why should we see cooperative behaviors in biological organizations? There are many animals that cooperate in their interactions. Ants and bees are just two examples. Humans, undoubtedly, are the leaders of cooperation. Since thousands of years, social norms have been the main rule of human interactions [7]. Although they are a frequently used concept in social sciences, *we still know little about how social norms are formed, the forces determining their content, and the cognitive and emotional requirements that enable a species to establish and enforce social norms* [8 (Italics in original)].
The problem of altruistic behaviour can be best expressed by the language of game theory. A cooperator is someone who

bears a cost c in order to grant a benefit b to her opponent (where b>c) [2]. Someone who rejects cooperation – as we call her a *defector* – is the one who neither bears a cost, nor grants a benefit. The matrix of the game will be as figure (1) which is the familiar game of prisoners' dilemma. The Nash equilibrium of this game will be the point *(D, D)*. According to this equilibrium, there should be no cooperation in the world. To know why, imagine a population of cooperators and suppose that a mutation makes and individual become a defector. Since she pays no cost and receives benefit from others, her fitness becomes more than others. Hence, natural selection tells us that defection will be the evolutionary stable strategy (ESS) of this system [9]. Nevertheless, not only defection is not the dominant behaviour of human beings, but also the amount of cooperation between them has been increasing during past five thousand years [7].

	C	D
C	b-c, b-c	-c, b
D	b, -c	0, 0

Fig. 1 Prisoners' Dilemma

The huge difference of cooperation level between humans and other species tends us to guess that human cooperation is a result of their *intelligence.* Henrich and Henrich (2005) argued that this intuition is wrong stating that although the level of human intelligence is almost identical between different societies, the level of their cooperation varies dramatically [7]. Richard Dawkins in his book *The God Delusion* stated some theories of evolutionary ethics. He tries to show how natural selection favours altruistic behaviour – which has the same meaning as cooperative behaviour in the terminology of this paper. A very simple theory which describes altruistic behaviour is the Hamilton's Rule. According to this rule, if the donor and receptor of a cooperative behaviour are relatives, natural selection can favour cooperation. You will jump to river to help your brother though you might not jump to help a stranger. However, there is an unanswered question here: why do we see cooperation between strangers? More precisely, neither all relatives are cooperators, nor are all strangers defectors.
The next hypothesis of cooperation is *direct reciprocity* [2, 5]. This hypothesis assumes that there are repeated encounters between individuals. Hence, what you do in a step may affect your payoff in the next step; '*You help me, I will help you later'.* Direct reciprocity works because of asymmetric needs

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between individuals. When I need help, you can help me and I cannot help myself and when you need help, the situation is reversed.

A strategy for such a game is tit – for – tat. I will cooperate if you have cooperated last round. There are also other strategies studied in some researches, which proved that natural selection can favour cooperation if the probabilities of another encounter between individuals satisfy specific conditions [2, 10, 11].

The problem with most of these researches is that they assume populations well-mixed. But real populations are not so and the structure of the network can determine whether natural selection favours cooperation or not. Moreover, sociologists believe that the network structure plays an important role in evolution of social norms [12]. This approach has been studied in some researches and the results show the significant role of graph structure on evolution of cooperation [3].

In a valuable research, Ohtsuki et al stated a very simple rule for evolution of cooperation. They examined different network structures and showed that the ratio of benefit to cost of altruistic behaviour must be more than the average number of neighbours in graph if cooperation is going to be selected [3].

The importance of average neighbours in ESS of a system brings up an idea that whether it can affects *dynamics* of reaching the ESS. Assume that in a specific graph the ratio of benefit to cost exceeds the average number of neighbours (k) and hence, natural selection favours cooperation. Now, if in the graph, some nodes have an average number of neighbours more than k – and some nodes less – does the number of cooperators differ in these two groups *during* the evolution?

In the next section, we briefly explain the model of our paper which is the same as the model of Ohtsuki et al. Finally, in section three, we simulate the rules of their model on a specific graph to see the dynamic of reaching the equilibrium. Interestingly, simulations show that less connections lead to more number of cooperators agents in a social network. Furthermore, we investigated the effect of other model parameters on the dynamic of evolution of cooperation.

II. THE MODEL

In this section we briefly introduce our model, which is exactly the same as the model of Ohtsuki et al. We assume a graph which resembles our social network structure. Each node is a player of our evolutionary game. Each player has two strategies. She can cooperate or defect. A cooperator grants a benefit b to all her neighbours and pays a cost c for each interaction. A defector grants no benefit and pays no cost. If a player has k neighbours and i of them are cooperators, her payoff will be $bk - ci,$ if she is a cooperator and bi if she is a defector.

The fitness of an individual is determined by a constant term plus the effect of a term which is a linear function of their payoff. Mathematically, their fitness is $1 - w + w*p$ where w measures the intensity of selection. If w <<1, then we have weak selection which means that the payoff of an individual is not the main determinant of her fitness. If w = 1, we have strong selection which means that the fitness of our individual

is equal to her payoff and the game plays an important role in determining fitness. It is reasonable to assume weak selection in the model because a specific social norm is not the main determinant of an individual's fitness.

To update our model, we assume that each time step an individual is selected to update her behaviour. She will cooperate with a probability of Fc / (Fc + Fd) where Fc is the total fitness of her cooperator neighbours and Fd is the total fitness of her defector neighbours. Hence, if all her neighbours are cooperator (defector) she certainly becomes cooperator (defector). This entire scenario is shown in figure (2), a figure from Ohtsuki et al paper. The simulations show that natural selection favours cooperation if b/c > k.
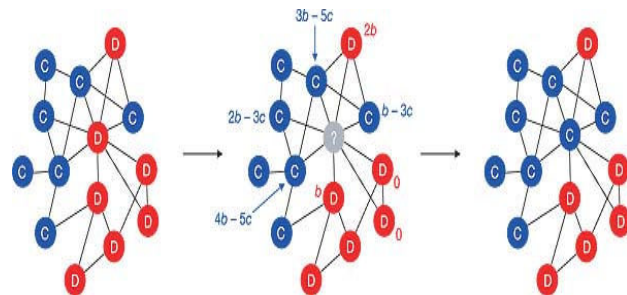

Fig. 2 Each individual occupies the vertex of a graph and derives a payoff, P, from interactions with adjacent individuals. A cooperator (blue) pays a cost, c, for each neighbour to receive a benefit, b. A defector (red) pays no cost and provides no benefit. The fitness of a player is given by $1 - w + wP$, where w measures the intensity of selection. Strong selection means $w = 1$. Weak selection means $w \ll 1$. For 'death–birth' updating, at each time step, a random individual is chosen to die (grey); subsequently the neighbours compete for the empty site in proportion to their fitness. In this example, the central, vacated vertex will change from a defector to a cooperator with a probability $F_C/ (F_C + F_D)$, where the total fitness of all adjacent cooperators and defectors is $F_C = 4(1 - w) + (10b - 16c) w$ and $F_D = 4(1 - w) + 3bw$, respectively [3]

III. SIMULATION RESULTS

In the previous section, we briefly introduced our evolutionary model and now, we are going to see the dynamic of reaching the equilibrium in a specific graph. We are going to investigate whether the average number of neighbours can affect density of cooperators during evolution. To do so, we simulated our model on a graph as shown in figure (3). Notice that the graph of figure (3) has just twelve nodes and the graph of our simulations has the same structure, but onehundred nodes.

The graph consists of two types of nodes: Central nodes and marginal nodes. The average number of neighbours for central nodes is approximately two times more than the average number of neighbours for marginal nodes. This is important to notice that central and marginal nodes are connected to each other. We assume that at the beginning, all the population are defectors and by a random mutation, an individual becomes cooperator. Also we assume that b/c > k. To see whether *k* can affect *velocity* of reaching the equilibrium, we compared the
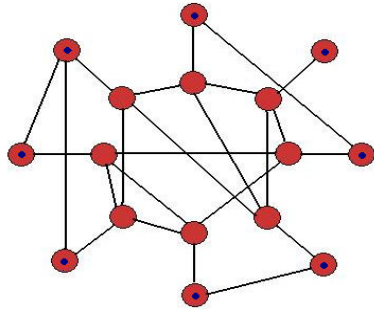
Fig. 3 The graph of our model, which consists of two types of nodes: Central nodes and marginal nodes.

average number of cooperators in central nodes and marginal nodes for a very large number of simulations. As illustrated is figure (4) there are more cooperators in marginal nodes than central nodes *during* evolution; less connections leads to more altruism!
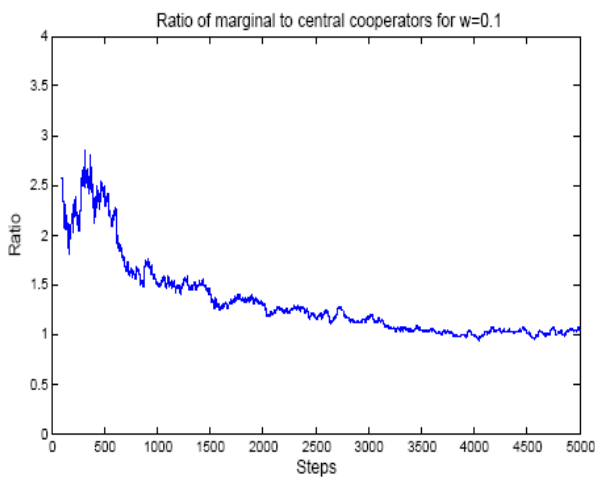

Fig. 4 Ratio of cooperators in marginal nodes to central nodes during evolution. There are more cooperators in marginal nodes than central nodes.

Why this happens? To have an intuition of this phenomenon, imagine a single cooperator that all her neighbours are defectors. Her payoff is –kc, where k is the number of her neighbours. Hence, if she is an individual from central nodes, her payoff is less than a single cooperator in marginal nodes. As a result, if we select a defector to update her behaviour, the likelihood of being cooperator is more in marginal agents than central agents. This means that at the beginning of evolution, there will be more cooperators in marginal agents. As the number of cooperators arises in both groups, the likelihood of being cooperator becomes equal for both groups. This result is shown in figure (4), where the ratio of becomes nearer to one as we go through evolution process.
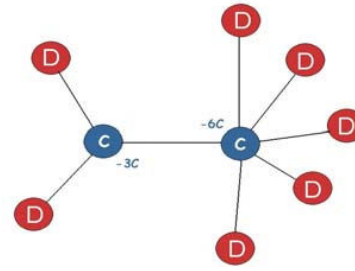

Fig. 5 A simple explanation of this paper's claim: less connections lead to more altruism. At the beginning of the evolution, most of the nodes are defectors. Suppose there is a single cooperator in each group of nodes. In this situation the cooperator in marginal nodes has a higher payoff because she has fewer neighbors. Hence, the likelihood of becoming cooperator is more for the marginal defectors.

The above description of figure (4) brings up another idea: What is the effect of the intensity of selection – w – on the dynamic of evolution? To find the answer, we simulated our model by different values of w, everything else constant (figure (6)).
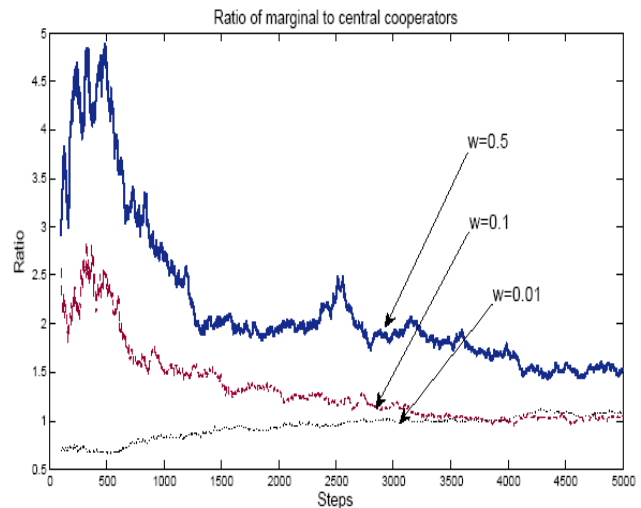

Fig. 6 Ratio of cooperators in marginal nodes to central nodes for different intensity of selections. For w = 0.5 the ratio is much more than one during evolution and for w = 0.1 the ratio is more than one, but not as much as w = 0.5. For w = 0.01 the ratio is near one.

You can see that w plays a significant role in dynamic of evolution. The more the intensity of selection, the more the number of cooperators in marginal nodes. We saw that a single cooperator in marginal nodes has a higher payoff than a single cooperator in central nodes. As a result, the likelihood of becoming cooperator is more in marginal agents than central agents. If w increases, then this effect becomes more effective. Hence, we see more cooperators in marginal individuals when intensity of selection is more.

IV. CONCLUSION

There are many biological organizations that are based on cooperation and altruistic behaviour. Humans are the most cooperative animals and social norms are the fundamental blocks of human societies. Cooperation occurs when an individual bears a cost for another individual to grant a benefit. Both of cost and benefit are related to genetic fitness. According to natural selection, we expect to see no cooperator in the world because evolution selects selfish individuals. But we are seeing these many cooperative and altruistic behaviours among biological systems. Hence, we should answer to this question that what mechanisms are at work that natural selection saved cooperators during evolution? The scientists argued many mechanisms such as kin selection, direct reciprocity, indirect reciprocity and group selection. Another answer to this question is the effect of network or network reciprocity. According to this theory, social networks are not well-mixed and cooperators can evolve by forming *micro social networks* in which they help each other. In their valuable research, Ohtsuki et al showed a simple rule for the evolution of cooperation in social networks. They showed that if the benefit to cost ratio of an altruistic behaviour exceeds the average number of neighbours of the graph, then natural selection favours cooperation.

In this research, we studied the *dynamics* of their model on a specific graph to see how model parameters affect the dynamics of evolution. We divided individuals into two groups: individuals who have a lot of neighbours (central nodes) and individuals who have fewer neighbours (marginal nodes). We showed that during evolution, especially at the beginning of evolution, the density of cooperators is more for the marginal nodes. To know why this happens, suppose there is a single cooperator in each group of nodes. Hence, the cooperator in marginal nodes has a higher payoff because she has fewer neighbors. Thus, the likelihood of becoming cooperator is more for the marginal defectors.

To what extend fitness of individuals depends on the payoff of the game is determined by *w*, intensity of selection. We simulated our model for different values of *w,* and we showed that the more the intensity of selection, the more the ratio of marginal to central cooperators occurs.

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