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Hierarchies Based on the Number of Cooperating Systems of Finite Automata on Four-Dimensional Input Tapes

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Abstract—In theoretical computer science, the Turing machine has played a number of important roles in understanding and exploiting basic concepts and mechanisms in computing and information processing [20]. It is a simple mathematical model of computers [9]. After that, M.Blum and C.Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing, and investigated their pattern recognition abilities in 1967 [7]. Since then, a lot of researchers in this field have been investigating many properties about automata on a two- or three-dimensional tape. On the other hand, the question of whether processing four-dimensional digital patterns is much more difficult than two- or three-dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the study of four-dimensional automata as a computasional model of four-dimensional pattern processing has been meaningful [8]-[19],[21]. This paper introduces a cooperating system of four-dimensional finite automata as one model of four-dimensional automata. A cooperating system of four-dimensional finite automata consists of a finite number of four-dimensional finite automata and a four-dimensional input tape where these finite automata work independently (in parallel). Those finite automata whose input heads scan the same cell of the input tape can communicate with each other, that is, every finite automaton is allowed to know the internal states of other finite automata on the same cell it is scanning at the moment. In this paper, we mainly investigate some accepting powers of a cooperating system of eight- or seven-way four-dimensional finite automata. The seven-way four-dimensional finite automaton is an eight-way four-dimensional finite automaton whose input head can move east, west, south, north, up, down, or in the fu-ture, but not in the past on a four-dimensional input tape.

Keywords—computational complexity, cooperating system, finite automaton, four-dimension, hierarchy, multihead.

I. Introduction

A Cooperating system of four-dimensional finite automata (CS-4-FA) [2]-[4],[19] consists of a finite number of four-dimensional finite automata and a four-dimensional input tape where these finite automata work independently (in parallel). Those finite automata whose input heads scan the same cell of the input tape can communicate with each other, that is, every finite automaton is allowed to know the internal states of other finite automata on the same cell it is scanning at the moment.

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In this paper, we propose a cooperating system of seven-way four-dimensional finite automata (CS-SV4-FA) which is a restricted version of CS-4-FA's, and mainly investigate its several properties as four-dimensional language acceptors. The seven-way four-dimensional finite automaton [16] is a four-dimensional finite automaton [1] whose input head can move east, west, south, north, up, down, or in the future, but not in the past. The paper has six sections in addition to this Introduction. Section II contains some definitions and notions. Section III investigates a relationship between seven-way four-dimensional simple multihead finite automata (SV4-SPMHFA's) and CS-SV4-FA's. It is shown that SV4-SPMHFA's and CS-SV4-FA's are equivalent in accepting power if each sidelength of each four-dimensional input tape of these automata is equivalent. Section IV investigates the difference between the accepting powers of CS-SV4-FA's and CS-4-FA's, and shows that CS-SV4-FA's are less powerful than CS-4-FA's. Section V investigates the difference between the accepting powers of deterministic and nondeterministic CS-SV4-FA's, and shows that deterministic CS-SV4-FA's are less powerful than nondeterministic CS-SV4-FA's.

Section VI investigates the hierarchies can be obtained by varying the number of finite automata in the system for classes of sets accepted by CS-SV4-FA's and CS-4-FA's. Section VII concludes by giving some open problems.In this paper, we let each sidelength of each input tape of these automata be equivalent in order to increase the theoretical interest.

II. Preliminaries

Definition 2.1. Let $\sum$ be a finite set of symbols. A *four-dimensional tape* over $\sum$ is a four-dimensional rectangular array of elements of $\sum$. The set of all four-dimensional tapes over $\sum$ is denoted by $\sum^{(4)}$. Given a tape $x \in \sum^{(4)}$, for each integer $j (1 \leq j \leq 4)$, we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \sum^{(4)}$ with $l_1(x) = n_1$, $l_2(x) = n_2$, $l_3(x) = n_3$, and $l_4(x) = n_4$ is denoted by $\sum^{(n_1,n_2,n_3,n_4)}$. When $1 \leq i_j \leq l_j(x)$ for each $j (1 \leq j \leq 4)$, let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4), as shown in Fig.1. Furthermore, we define

$$x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)],$$

when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer $j (1 \leq j \leq 4)$, as the four-dimensional input tape y satisfying the follwing conditions:

(i) for each $j(1 \leq j \leq 4)$, $l_j(y) = i'_j - i_j + 1$;

(ii) for each $r_1, r_2, r_3, r_4 (1 \leq r_1 \leq l_1(y), 1 \leq r_2 \leq l_2(y), 1 \leq r_3 \leq l_3(y), 1 \leq r_4 \leq l_4(y)), y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 - 1, r_2 + i_2 - 1, r_3 + i_3 - 1, r_4 + i_4 - 1)$. (We call $x[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$ the $[(i_1, i_2, i_3, i_4), (i'_1, i'_2, i'_3, i'_4)]$-*segment* of x.)

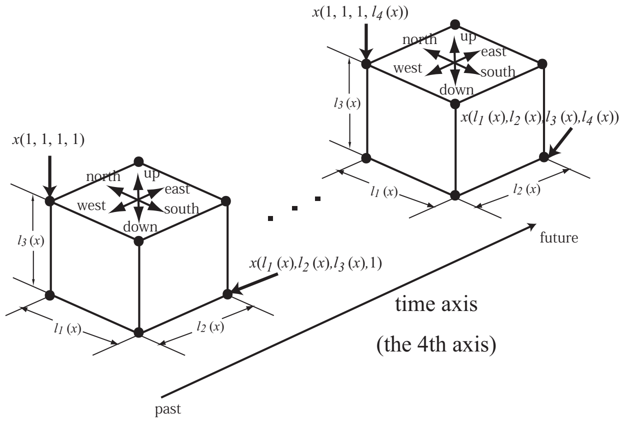

Fig. 1. Four-dimensional Input Tape

We recall a *seven-way four-dimensional simple k-head finite automaton* (SV4-SPk-HFA)[5]-[6]. An SV4-SPk-HFA M is a finite automaton with k read-only input heads operating on a four-dimensional input tape surrounded by boundary symbols $\sharp$'s. The only one head(called the '*reading*' head) of M is capable of distinguishing the symbols in the input alphabet, and the other heads(called *'counting'* heads) of M can only detect whether they are on the boundary symbols or a symbol in the input alphabet. When an input tape x is a presented to M, M determines the next state of the finite control, the next move direction (east, west, south, north, up, down, future, past or no move) of each input head, depending on the present state of the finite contorol, the symbol read by the reading head, and on whether or not the symbol read by each counting head is boundary symbol. We say that M *accepts* x if M, when started in its initial state with all its input heads on $x(1, 1, 1, 1)$, eventually halts in an accepting state with all its heads on the bottom boundary symbols of x. As usual, we define nondeterministic and dterministic SV4-SPk-HFA's.

A *seven-way four-dimensional sensing simple k-head finite automaton*(SV4-SNSPk-HFA) is the same device as a SV4-SPk-HFA except that the former can detect coincidence of the input heads.

We denote a deterministic(nondeterministic) SV4-SPk-HFA by SV4-SPk-HDFA(SV4-SPk-HNFA), and denote a deterministic (nondeterministic)SV4-SNSPk-HFA by SV4-SNSPk-HDFA(SV4-SNSPk-HNFA).

We now give formal definition of a *cooperating system of k four-dimensional deterministic finite automata* (CS-4-DFA(k)) as an acceptor.

Definition 2.2. A CS-4-DFA(k) is a k-tuple $M = (\mathrm{FA}_1, \mathrm{FA}_2, \ldots, \mathrm{FA}_k)$, $k \geq 1$, such that for each $1 \leq i \leq k$,

$$\mathrm{FA}_i = (\sum, Q_i, X_i, \delta_i, q_0 i, F_i, \phi, \sharp),$$

where

1) $\sum$ is a finite set of *input symbols*.
2) Q_i is a finite set of *states*.
3) $X_i = (Q_1 \cup \{\phi\}) \times \cdots \times (Q_{i-1} \cup \{\phi\}) \times (Q_{i+1} \cup \{\phi\}) \times \cdots \times (Q_k \cup \{\phi\})$, where '$\phi$' is a special state not in $(Q_1 \cup Q_2 \cup \cdots \cup Q_k)$.
4) $\delta_i : (\sum \cup \{\sharp\}) \times X_i \times Q_i \to Q_i \times$east$(= (0, +1, 0, 0))$,west$(= (0, -1, 0, 0))$,south$(= (+1, 0, 0, 0))$,north $(= (-1, 0, 0, 0))$,up$(= (0, 0, -1, 0))$,down$(= (0, 0, +1, 0))$,future$(= (0, 0, 0, +1)$,past$(= (0, 0, 0, -1))$,no move$(= (0, 0, 0, 0))$is the *next move function*, where '$\sharp$' is the *boundary symbol* not in $\sum$.
5) $q0_i \in Q_i$ is the *initial state* of FA_i.
6) $F_i \subseteq Q_i$ is the set of *accepting states* of FA_i.

Every automaton of M independently (in parallel) works step by step on the same four-dimensional tape x over $\sum$ surrounded by boundary symbols $\sharp$'s. Each step is assumed to require exactly one time for its completion. For each $i (1 \leq i \leq k)$, let q_i be the state of FA_i at time 't'. Then each FA_i, enters the next state 'p_i' at time '$t + 1$' according to the function

$$\delta_i(x(\alpha, \beta, \gamma, \rho), (q'_1, \ldots, q'_{i-1}, q'_{i+1}, \ldots, q'_k), q_i) = (p_i, (d_1, d_2, d_3, d_4)),$$

where $x(\alpha, \beta, \gamma, \rho)$ is the symbol read by the input head of FA_i at time 't' and for each $j \in \{1, \ldots, i-1, i+1, \ldots, k\}$,

$$q'_j = \begin{cases} q_j \in Q_j & \text{if the input heads of } \mathrm{FA}_i \text{ and } \mathrm{FA}_j, \\ & \text{are on the same input position at} \\ & \text{the moment '}t\text{';} \\ \phi & \text{otherwise,} \end{cases}$$

and moves 1st input head to $x(\alpha + d_1, \beta + d_2, \gamma + d_3, \rho + d_4)$ at time '$t + 1$'. We assume that the input head of FA_i never falls off the tape beyond boundary symbols.

When an input tape $x \in \sum^{(4)}$ is presented to M, we say that M *accepts* the tape x if each automaton of M, when started in its initial state with its input head on $x(1, 1, 1, 1)$, eventually enters an accepting state with its input head on one of the bottom boundary symbols.

We next introduce a *cooperating system of k seven-way four-dimensional deteministic finite automata* (CS-SV4-DFA(k)), with which we are mainly concerned in this paper.

Definition 2.3. A CS-SV4-DFA(k) is a CS-4-DFA(k) $M = (\mathrm{FA}_1, \mathrm{FA}_2, \ldots, \mathrm{FA}_k)$ such that the input head of each FA_i can only move east, west, south, north, up, down, or in the future, but not in the past.

To give the formal definition of a *cooperating system of k four-dimensional nondeterministic finite automata* (CS-4-NFA(k)) and a *cooperating system of k seven-way* *four-dimensional nondeterministic finite automata* (CS-SV4-NFA(k)) is left to the reader. For each $X \in$ {SV4-SPk-HDFA,SV4-SPk-HNFA,SV4-SNSPk-HDFA,SV4-SNSPk-HNFA,CS-4-DFA(k),CS-4-NFA(k),CS-SV4-DFA(k),CS-SV4-NFA(k)}, by X^c we denote an X which each sidelength of each input tape is equivalent; by $\mathcal{L}[X](\mathcal{L}([X^c])$ we denote the class of sets of input tapes accepted by X's(X^c's). We will focuse our attention on the acceptors which each sidelength of each input tape is equivalent.

III. SV4-SPMHFA's and CS-SV3-FA's

In this section, we establish a relation between the accepting powers of seven-way four-dimensional simple multihead finite automata and cooperating systems of seven-way four-dimensional finite sutomata over input tapes which each sidelength is equivalent. This result will be used in the latter sections.

Lemma 3.1. *For any* $k \geq 1$ *and* $X \in \{N, D\}$,

$$\mathcal{L}[\text{SV4-SNSP}k\text{-HXFA}^c] \subseteq \mathcal{L}[\text{CS-SV4-XFA}(2k)^c]$$

Proof. Let M be an SV4-SNSPk-HFA^c. We will construct a CS-SV4-XFA$(2k)^c$ M' to simulate M. M' acts as follws:

1) M' simulates the moves of the reading head of M and all the east, west, south, north up, or down moves of counting heads of M by using its $(k+1)$ finite automata.
2) M' simulates all the moves in the future direction of counting heads of M by making the down moves of input heads of its other $(k-1)$ finite automata.
3) During the simulation, if M moves its reading head in the future direction, then M' makes all of input heads of finite automata of M' move in the future direction so that all the automata of M' can keep their input heads on the same three-dimensional rectangular array and can communicate with each other in that three-dimensional rectangular array.

It is easy to see that M' can simulate M. □

Lemma 3.2. *For any* $k \geq 1$ *and any* $X \in \{N, D\}$,

$$\mathcal{L}[\text{CS-SV4-XFA}(k)^c] \subseteq \mathcal{L}[\text{SV4-SNSP}(2k^2-k+1)\text{-HXFA}^c].$$

Proof. Let M = (FA$_1$,FA$_2$,...,FA$_k$)be a CS-SV4-XFA$(k)^c$. We will construct an SV4-SNSP$(2k^2-k+1)$-HXFA^c M' to simulate M. Let R denote the reading head of M', and $h_1, h_2, \ldots, h_{2k^2-k}$ denote the $2k^2-k$ counting heads of M'. M' acts as follws:

1) M' stores the internal states of FA$_1$,FA$_2$,...,FA$_k$ in its finite contorol.
2) For each three-dimensional rectangular array of the input tape:
   (a) M' simulates the east, west, south, north, up, or down moves of input heads of FA$_1$,FA$_2$,...,FA$_k$ by using R and $h_1, h_2, \ldots, h_k$.
   (b) M' stores in its finite control the internal state of each FA$_i$, $1 \leq i \leq k$, when the input head of FA$_i$ leaves the three-dimensional rectangular array and the order, $(d_1, d_2, \ldots, d_k)$, in which the input heads of FA$_1$, FA$_2$, ..., FA$_k$ leave the plane subsequently (i.e., FA$_{d_1}$ firstly moves its input head in the future direction from the three-dimensional rectangular array. FA$_{d_2}$ secondly moves its input head in the future direction from the three-dimensional rectangular array, and so on.), and M' keeps the position where the input head of each FA$_i$, $1 \leq i \leq k$, leaves the three-dimensional rectangular array by the positions of $h_1, h_2, \ldots, h_k$.
   (c) Furthermore, for each $i(1 \leq i \leq k-1)$, the interval between the times at which FA$_{d_i}$ and FA$_{d_{i+1}}$ move their input heads in the future direction from the three-dimensional rectangular array is stored by a counter with $O(n^{6k})$ space bound, which can be realized by using $h_{(2i-1)k-1}, h_{(2i-1)k-2}, \ldots, h_{(2i-1)k}$, where n is the number of rows (or columns or planes or three-dimensional rectangular array) of the input tape.

Note that M works in $O(n^{6k})$ time, that is, if an input tape with n rows (or columns or planes) is accepted by M, then it can be accepted by M in $O(n^{6k})$ time. Thus, it is easy to verify that M' can simulate M. □

From [5],it follows that $\cup_{1 \leq k < \infty}\mathcal{L}$[SV4-SP$k$-HXFA^c] = $\cup_{1 \leq k < \infty}\mathcal{L}$[SV4-SNSP$k$-HXFA^c] for any $X \in \{N, D\}$. Combining this result with Lemmas 3.1 and 3.2, we have the follwing thorem.

Theorem 3.1. $\cup_{1 \leq k < \infty}\mathcal{L}$[SV4-SP$k$-HXFA^c] = $\cup_{1 \leq k < \infty}\mathcal{L}$[CS-SV4-XFA$(k)^c$] for any $X \in \{N, D\}$.

Corollary 3.1. *For any* $k \geq 1$, *there is no* CS-SV4-NFA(k) *that accepts the set of connected patterns.*

Remark 3.1. It is easy to see that for each $k \leq 1$, (1)four-dimensional sensing simple k head finite automata [5] are simulated by cooperating systems of $(k+1)$ four-dimensional finite automata, and (2) cooperating systems of k four-dimensional finite automata are simulated by four-dimensional sensing simple $(k+1)$ head finite automata.

Remark 3.2. It is shown in [22] that (one-dimensional) one-way simple multihead finite automata snd cooperating systems of (one-dimensional) one-way deterministic finite automata are incomparable in accepting power. From this fact, it follows that SV4-SPMHFA^c's and CS-SV4-DFA^c's are incomparable in accepting power if the input tapes are restricted to those x such that $l_4(x) > l_1(x) = l_2(x) = l_3(x)$. We can also show that SV4-SPMHFA^c's are more powerful than CS-SV4-DFA^c's if the input tapes are restricted to those x such that $l_4(x) < l_1(x) = l_2(x) = l_3(x)$.

IV. Seven-way versus Eight-Way

In this section, we investigate the difference between the accepting powers of CS-4-DFA$(k)^c$'s [CS-4-NFA$(k)^c$'s] and CS-SV4-DFA$(k)^c$'s [CS-SV4-NFA$(k)^c$'s].

Theorem 4.1. *For each* $X \in \{N, D\}$, $\mathcal{L}$ [CS-4-DFA$(1)^c$]$- \cup_{1\leq k<\infty} \mathcal{L}$[CS-SV4-XFA$(k)^c$] $\neq \emptyset$.

Proof. Let $T_1 = \{x \in \{0,1\}^{(4)} | (\exists m \geq 2)[l_1(x) = l_2(x) = l_3(x) = l_4(x) = m \ \& \ x[(1,1,1,1),(m,m,m,1)] = x[(1,1,1,2),(m,m,m,2)]]\}$. Clearly, $T_1 \in \mathcal{L}$[CS-4-DFA$(1)^c$]. From [5], it is easy to see that T_1 is not in $\cup_{1\leq k<\infty}\mathcal{L}$[SV4-SP$k$-HNFA^c]. From this fact and Theorem 3.1, the theorem follows. □

From Theorem 4.1, we can get the following corollary.

Corollary 4.1. *For each* $k \geq 1$ *and* $X \in \{N, D\}$, (1)$\mathcal{L}$[CS-SV4-XFA$(k)^c$]$\subsetneq$ $\mathcal{L}$[CS-4-XFA$(k)^c$], *and* (2) $\cup_{1\leq k<\infty}\mathcal{L}$[CS-SV4-XFA$(k)^c$] $\subsetneq \cup_{1\leq k<\infty}\mathcal{L}$[CS-4-XFA$(k)^c$].

V. Nondeterminism versus Determinism

In this section, we investigate the difference between the accpting powers of CS-SV4-NFA$(k)^c$'s and CS-SV4-DFA$(k)^c$'s.

Theorem 5.1. $\mathcal{L}$[CS-SV4-NFA$(1)^c$] $- \cup_{1\leq k<\infty}$ $\mathcal{L}$[CS-SV4-DFA$(k)^c$] $\neq \emptyset$.

Proof. Let $T_2 = \{x \in \{0,1\}^{(4)} | (\exists m \geq 2)[l_1(x) = l_2(x) = l_3(x) = l_4(x) = m] \& \exists_i, \exists_j (1 \leq i \leq m, 1 \leq j \leq m, 1 \leq k \leq m)[x(i,j,k,1) = x(i,j,k,2) = 1]$. Clearly, $T_2 \in$ [CS-SV4-NFA$(1)^c$]. From [5], it is easy to see that T_2 is not in $\cup_{1\leq k<\infty}\mathcal{L}$[SV4-SP$k$-HDFA^c]. From this fact and Theorem 3.1, the theorem follows. □

From Theorem 5.1, we get the following corollary.

Corollary 5.1. *For each* $k \geq 1, (1)\mathcal{L}$[CS-SV4-DFA$(k)^c$] $\subsetneq$ $\mathcal{L}$[CS-SV4-NFA$(k)^c$], and (2) $\cup_{1\leq k<\infty}\mathcal{L}$[CS-SV4-DFA$(k)^c$] $\subsetneq \cup_{1\leq k<\infty}\mathcal{L}$[CS-SV4-NFA$(k)^c$].

VI. HIERARCHIES BASAED ON THE NUMBER OF AUTOMATA

In this section, we investigate the hierarchies based on the number of their cooperating systems.

A. Eight-Way Case

We first investigate how the number of automata of *CS*-4-FA^c's affects the accepting power.

Theorem 6.1. *For each* $k \geq 1$ *and each* $X \in \{N, D\}$, $\mathcal{L}$[*CS*-4-$XFA_{\{0\}}(k)^c$]$\subsetneq$ $\mathcal{L}$[*CS*-4-$XFA_{\{0\}}(k+2)^c$], *where* $\mathcal{L}$[*CS*-4-$XFA_{\{0\}}(k)^c$] *denote the class of sets of cubic tapes over a one-letter alphabet which each sidelength of each input tape is equivalent accepted by* CS-4-$XFA(k)^c$'s.

Proof. It is easy to prove that every *CS*-4-*DFA*$(k)^c$[*CS*-4-*NFA*$(k)^c$] can be simulated by an (eight-way) four-dimesional sensing deterministic [nondeterministic] k-head finite automaton, and every (eight-way) four-dimensional sensing deterministic [nondeterministic] k-head finite automaton can be simulated by a *CS*-4-*DFA*$(k{+}1)^c$[*CS*-4-*NFA*$(k{+}1)^c$]. From [8], for sets of four-dimensional tapes over a one-letter alphabet, (eight-way) four-dimensional sensing deterministic [nondeterministic] (k+1)-head finite automata are more powerful than the cooresponding k-head finite automata. From these facts, the theorem follows. □

Unfortunately, it is unknown whether $\mathcal{L}$[*CS*-4-$XFA_{\{0\}}(k)^c$]$\subsetneq$ $\mathcal{L}$[*CS*-4-$XFA_{\{0\}}(k+1)^c$] for any $k \geq 1$ and for any $X \in \{D, N\}$. It is also unknown wtether $\mathcal{L}$[*CS*-4-*XFA*$(k)^c$]$\subsetneq$ $\mathcal{L}$[*CS*-4-*XFA*$(k+1)^c$] for any $k \geq 2$ and for any $X \in \{D, N\}$. (It is easy to show that $\mathcal{L}$[*CS*-4-*XFA*$(1)^c$]$\subsetneq$ $\mathcal{L}$[*CS*-4-*XFA*$(2)^c$].)

B. Seven-Way Case

We next investigate how the number of automata of *CS*-*SV*4-FA^c's affects the accepting power.

For each $n \geq 1$, let $T(n) = \{x \in \{0,1\}^{(4)} | (^\exists m \geq n)$ $[l_1(x) = l_2(x) = l_3(x) = l_4(x) = m \ \& \ x[(1,1,1,1),(m,m,m,1)] = x[(1,1,1,2),(m,m,m,2)]\} \in R_n(m) \ \& \ x[(1,1,1,3),(m,m,m,m)] \in \{0\}^{(3)}]\}$, where $R_n(m) = \{x \in \{0,1\}^{(4)} | \ l_1(x) = m, l_2(x) = m, l_3(x) = m, l_4(x) = m \ \&$ (x has exactly n 1's)$\}$ for each $m \geq n$. It is obvious that for any fixed positive integer n, $T(n)$ can be accepted by a *CS*-*SV*4-*DFA*$(n)^c$.

We first consider the following problem: given a fixed positive integer n, find a *CS*-*SV*4-FA^c which accepts $T(n)$ and uses the minimum number of automata. Unfortunately, we cannot generally solve the problem in the present paper, but we give the lower and upper bounds. Let $f(n)$ denote the minimum number of automata required for deterministic *CS*-*SV*4-FA^c's to accept $T(n)$, and $g(n)$ denote the minimum number of automata required for nondeterministic *CS*-*SV*4-FA^c's to accept $T(n)$. Clearly, $g(n) \leq f(n)$ for any $n \geq 1$.

Theorem 6.2. *For each* $k \geq 1$, (1) $f(k^2{+}k{-}1) \leq 2k{-}1$, (2) $f(k^2{+}2k) \leq 2k$, *and* (3) $f(k(k{-}1)/2{+}1) \geq k$.

Proof. The proofs of (1) and (2) are similar. We only give the proof of (2) here. To prove (2) is equivalent to proving that: for each $k \geq 1$, $T(k^2 + 2k) \in \mathcal{L}[CS\text{-}SV4\text{-}DFA(2k)^c]$.

For each $n \leq 1$, let $T'(n)$=$\{x[(1,1,1,1),(l_1(x),l_2(x), l_3(x),2)] | x \in T(n)\}$. For convenience, we prove by induction on k that $T'(k^2 + 2k) \in \mathcal{L}[CS\text{-}SV4\text{-}DFA(2k)^c]$. It will be obvious that (2) follows from this fact.

We now prove (3). Suppose that there is a CS-$SV4$-$DFA(k-1)^c$ $M(k-1)$= $(FA_1, FA_2, \cdots, FA_{k-1})$ accepting $T(k(k-1)/2+1)$. Let h_i denote the input of FA_i for each $i \in \{1, 2, \cdots, k-1\}$.

For each $m \geq k(k-1)/2+1$, let $V(m) = \{x \in T(k(k-1)/2+1) | l_1(x) = l_2(x) = l_3(x) = l_4(x) = m\}$, and for each permutation $\sigma : \{1, 2, \cdots, k-1\} \to \{1, 2, \cdots, k-1\}$, let $W_\sigma(m)$ be the set of all input tapes $x \in V(m)$ such that during the accepting computation of $M(k-1)$ on x, input heads $h_{\sigma(1)}, h_{\sigma(2)}, \cdots, h_{\sigma(k-1)}$ leave the first cube of x in this order. Then there must exist some permutation τ such that

$$|W_\tau(m)| \geq |V(m)|/(k-1)! = \Omega(m^{3(k(k-1)/2+1)}).$$

For each $x \in W_\tau(m)$ and each $1 \leq i \leq k-1$, let $q_{\tau(i)}(x), p_{\tau(i)}(x)$ and $t_{\tau(i)}(x)$ denote the internal state of $FA_{\tau(i)}$, the position of $h_{\tau(i)}$ and the time, respectively, when $h_{\tau(i)}$ leaves the first cube during the accepting computation of $M(k-1)$ on x.

For each $x \in W_\tau(m)$, let

$$t(x) = (t_{\tau(2)}(x) - t_{\tau(1)}(x), t_{\tau(3)}(x) - t_{\tau(2)}(x), \cdots, t_{\tau(k-1)}(x) - t_{\tau(k-2)}(x)),$$

and

$$u(x) = ((q_{\tau(1)}(x), p_{\tau(1)}(x)), \cdots, (q_{\tau(k-1)}(x), p_{\tau(k-1)}(x)), t(x)).$$

Clearly, for each $2 \leq i \leq k-1$, $t_{\tau(i)}(x) - t_{\tau(i-1)}(x) = O(m^{k-i})$, because otherwise $FA_{\tau(i)}, \cdots, FA_{\tau(k-1)}$ would enter a loop on the first cube, and thus $M(k-1)$ would never accept x. So $|\{u(x)|x \in W_\tau(m)\}|$=$O(m^{k(k-1)})$. Therefore, it follows that for large m

$$|W_\tau(m)| > |\{u(x)|x \in W_\tau(m)\}|,$$

and so there exist two diffirent input tapes $x, y \in W_\tau(m)$ such that $u(x) = u(y)$. Let z be the tape obtained from x by replacing the second cube of x with the second cube of y. It follows that z is also accepted by $M(k-1)$. This is a contradiction, because z in not in $T(k(k-1)/2+1)$. This completes the proof of (3). □

Theorem 6.3. $g(2k^2 - 5k + 4) \geq k$, for $k \geq 1$.

Proof. The proof is very similar to that of (3) of Theorem 6.2. Suppose to the contrary that there is a CS-$SV4$-$NFA(k-1)^c$ $M(k-1) = (FA_1, FA_2, \cdots, FA_{k-1})$ accepting $T(2k^2 - 5k + 4)$. Let h_i denote the input head of FA_i for each $i \in \{1, 2, \cdots, k-1\}$.

For each $m \geq 2k^2 - 5k + 4$, let $V(m) = \{x \in T(2k^2 - 5k + 4)|l_1(x) = l_2(x) = l_3(x) = l_4(x) = m\}$. With each $x \in V(m)$, we associate one fixed accepting computation, $c(x)$, of $M(k-1)$ on x in which $M(k-1)$ operates in $O(m^{4(k-1)})$ time. Furthermore, for each permutation σ:$\{1, 2, \cdots, k-1\} \to \{1, 2, \cdots, k-1\}$, let $W_\sigma(m)$ be the set of all input tapes $x \in V(m)$ such that during $c(x)$, input heads $h_{\sigma(1)}, h_{\sigma(2)}, \cdots, h_{\sigma(k-1)}$ leave the first cube of x in this order. Then there must exist some permutation τ such that

$$|W_\tau(m)| \geq |V(m)|/(k-1)! = \Omega(m^{6k^2-15k+12}).$$

For each $x \in W_\tau(m)$ and each $1 \leq i \leq k-1$, let $q_{\tau(i)}(x), p_{\tau(i)}(x)$ *and* $t_{\tau(i)}(x)$ denote the internal state of $FA_{\tau(i)}$, the position of $h_{\tau(i)}$ and the time, respectively, when $h_{\tau(i)}$ leaves the first cube of x during $c(x)$.

For each $x \in W_\tau(m)$, let

$$t(x) = (t_{\tau(2)}(x) - t_{\tau(1)}(x), t_{\tau(3)}(x) - t_{\tau(2)}(x), \cdots, t_{\tau(k-1)}(x) - t_{\tau(k-2)}(x)),$$

and

$$u(x) = ((q_{\tau(1)}(x), p_{\tau(1)}(x)), \cdots, (q_{\tau(k-1)}(x), p_{\tau(k-1)}(x)), t(x)).$$

Clearly, for each $2 \leq i \leq k-1, t_{\tau(i)}(x) - t_{\tau(i-1)}(x) = O(m^{4(k-1)})$. So $|\{u(x)|x \in W_\tau(m)\}| = O(m^{4k^2-10k+6})$. Therefore, it follows that for large m

$$|W_\tau(m)| > |\{u(x)|x \in W_\tau(m)\}|,$$

and so there exist two different input tapes $x, y \in W_\tau(m)$ such that $u(x) = u(y)$. Let z be the tape obtained from x by replacing the second cube of x with the second cube of y. Clearly, from $c(m)$ and $c(y)$, we can construct an accepting computation of $M(k-1)$ on z. This is a contradiction, because z in not in $T(2k^2 - 5k + 4)$. This completes the proof of the thereom. □

From Theorems 6.2 and 6.3, we can get the following theorem.

Theorem 6.4. *For each* $k \geq 1$ *and each* $X \in \{D, N\}$, $\mathcal{L}$[*CS-SV*4-*XFA*$(k)^c$]$\subsetneq$$\mathcal{L}$[*CS-SV*4-*XFA*$(k+1)^c$].

Proof. For each $k \geq 1$, let $D(k)$=max$\{n|f(n) = k\}$ and $N(k)$=max$\{n|g(n) = k\}$. From Theorem 6.2 (3) and Thorem 6.3, we have

$$D(k) \leq k(k+1)/2 \text{ and } N(k) \leq 2k^2 - k,$$

respectively.

For each $X \in \{D, N\}$, let M be a CS-$SV4$-$XFA(k)^c$ accepting $T(X(k))$. From M, we can easily construct a CS-$SV4$-$XFA(k+1)^c M'$ which accepts $T(X(k)+1)$. Thus $T(X(k)+1) \in \mathcal{L}$[*CS-SV*4-*XFA*$(k+1)^c$]. From this and the fact that $T(X(k)+1) \notin \mathcal{L}$[*CS-SV*4-*XFA*$(k)^c$], it follows that $T(X(k)+1) \in \mathcal{L}$[*CS-SV*4-*XFA*$(k+1)^c$]$-\mathcal{L}$[*CS-SV*4-*XFA*$(k)^c$]. □

VII. Conclusion

We conclude this paper by giving several open problems except the open problem stated in the previous section.

In this paper, we introduced a cooperating system of four-dimensional finite automata, and investigated several basic accepting powers and the hierarchies. We conclude this paper by giving some open problems as follows.

1. For each $k \geq 2$,
   $\mathcal{L}$[CS-4-DFA$(k)^c$] $\subsetneq$ $\mathcal{L}$[CS-4-NFA$(k)^c$] ?
2. For each $k \geq 1$, and each $X$$\in$$\{D, N\}$, $\mathcal{L}$[*CS-SV*4-*XFA*$_{\{0\}}(k)^c$]$\subsetneq$$\mathcal{L}$[*CS-SV*4-*XFA*$_{\{0\}}(k+1)^c$], where $\mathcal{L}$[*CS-SV*4-*XFA*$_{\{0\}}(k)^c$] denote the class of sets of cubic tapes over a one-letter alphabet which each sidelength

of each input tape is equivalent accepted by *CS-SV*4-*XFA*$(k)^c$'s?

3. For n ≥4, $g(n) < f(n)$? (It is easy to show that for 1≤n≤3, $g(n) = f(n)$.)

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