

Heat and Mass Transfer over an unsteady Stretching Surface embedded in a porous medium in the presence of variable chemical reaction

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Abstract—The effect of variable chemical reaction on heat and mass transfer characteristics over unsteady stretching surface embedded in a porous medium is studied. The governing time dependent boundary layer equations are transformed into ordinary differential equations containing chemical reaction parameter, unsteadiness parameter, Prandtl number and Schmidt number. These equations have been transformed into a system of first order differential equations. *MATHEMATICA* has been used to solve this system after obtaining the missed initial conditions. The velocity gradient, temperature, and concentration profiles are computed and discussed in details for various values of the different parameters.

Keywords—heat and mass transfer, stretching surface, chemical reaction, porous medium.

I. INTRODUCTION

THE problem of heat and mass transfer combined with chemical reaction is very important due to its industrial applications. It has been the subject of many works in recent years. Heat and mass transfer occur simultaneously in processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower, and the flow in a desert cooler. Other examples of industrial applications are curing of plastic, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc. Two types of chemical reaction can take place; homogeneous reaction which occurs uniformly throughout a given phase, while a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. Heat and mass transfer with chemical reaction has attracted the attention of many engineers and scientists because of the importance of the applications.

Das et al. [1] have studied the effect of chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux. Anderson et al. [2] have studied the diffusion of chemical reactive species from a linear stretching sheet. Muthucumaraswamy et al. [3] have studied the effect of homogeneous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Muthucumaraswamy et al. [4] have investigated the effect of a chemical reaction on the unsteady flow past an impulsively started semi-infinite vertical plate which is subjected to uniform heat flux.

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Muthucumaraswamy et al. [5] have analyzed the effect of chemical reaction on the unsteady flow past an impulsively started vertical plate which is subjected to uniform mass flux and in the presence of heat transfer. Muthucumaraswamy [6] studied the effects of suction on heat and mass transfer along a moving vertical surface in the presence of chemical reaction. Muthucumaraswamy et al. [7] have studied the effect of first order chemical reaction on unsteady flow past an uniformly accelerated isothermal infinite vertical plate in the presence of heat and mass transfer. EL-Arabawy [8] studied the effects of suction/injection and chemical reaction on mass transfer over a stretching surface. Loganathan, Kulandaivel et al. [9] have studied the effect of first order chemical reaction on flow past an impulsively started semi-infinite vertical plate in the presence of thermal radiation. Rajeswari et al. [10] have studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through vertical porous surface with heat source in the presence of suction. Kandasamy et al. [11] have studied the effect of variable viscosity, heat and mass transfer on nonlinear mixed convection flow over a porous wedge with chemical reaction in the presence of heat radiation. Muthucumaraswamy [12] has studied the heat and mass transfer of a continuously moving isothermal moving isothermal vertical surface with uniform suction and chemical reaction. It was concluded that the velocity increases during the generative reaction and decreases in the destructive reaction. It was also found that the concentration increases in the presence of the generative reaction. Mahmoud [13] studied the effect of slip velocity at the wall on flow and mass transfer of an electrically conduction visco-elastic fluid past a stretching sheet embedded in a porous medium in the presence of chemical reaction and concentration dependent viscosity.

The purpose of this work is to study the effects of variable chemical reaction on heat and mass transfer over unsteady stretching surface embedded in a porous medium.

II. FORMULATION OF THE PROBLEM

Consider an unsteady, laminar, incompressible, and viscous fluid on a continuous stretching surface with heat and mass transfer, and variable chemical reaction through a porous medium of permeability k' . The fluid properties are assumed to be constant in the limited temperature range. The concentration of diffusing species is very small in comparison to the other chemical species, the temperature

and the concentration of species far from the surface, T_∞ , C_∞ respectively are infinitesimally very small [14]. The chemical reactions are taking place in the flow and all physical properties are assumed to be constant. The x -axis runs along the continuous surface in the direction of the motion and the y -axis is perpendicular to it. The conservation equations for the unsteady flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k'} \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \quad (4)$$

with the associated boundary conditions:

$$y = 0 : u = U_w, v = 0, C = C_w, \quad (5)$$

$$y \rightarrow \infty : u = 0, T = T_\infty, C = C_\infty. \quad (6)$$

where u and v are velocity components in the x and y directions, respectively, t is the time, ν is the viscosity, T is the temperature of the fluid, C is the concentration of the fluid, α is the thermal diffusion coefficient, D is the effective diffusion coefficient.

We assume that the stretching velocity U_w , the fluid temperature T_w , the fluid Concentration C_w , and the chemical reaction coefficient k_1 are of the form :

$$U_w(x, t) = \frac{ax}{1-\gamma t}, \quad T_w(x, t) = \frac{bx}{1-\gamma t}, \quad C_w(x, t) = \frac{cx}{1-\gamma t},$$

and $K_1 = \frac{k}{x}$. Where a, b, c , and γ are constants with $a > 0, b \geq 0, c \geq 0$ (with $\gamma t < 1$), and k is the chemical reaction rate.

We now introduce the following dimensionless functions f, θ and ϕ , and the similarity variables η

$$\eta = \sqrt{\frac{U_w}{\nu x}} y, \quad \psi = \sqrt{\nu x U_w} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

Where $\psi(x, y, t)$ is a stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, which identically satisfies the mass conservation equation (1) substituting into eqs.(2),(3), and (4) we obtain

$$f''' + f f'' - A \left(\frac{1}{2} \eta f'' + f' \right) - f'^2 - \lambda f' = 0 \quad (8)$$

$$\theta'' + Pr(f \theta' - f' \theta) - APr \left(\frac{1}{2} \eta \theta' + \theta \right) = 0 \quad (9)$$

$$\phi'' + Sc(f \phi' - f' \phi) - AS c \left(\frac{1}{2} \eta \phi' + \phi \right) - L \phi = 0 \quad (10)$$

where $A = \frac{\gamma}{a}$ is the unsteadiness parameter, $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $\lambda = \frac{\nu x}{k' U_w}$

is the permeability parameter, and $L = \frac{k S c}{U_w}$ is the chemical reaction parameter.

The boundary conditions (5) now becomes

$$\eta = 0 : f = 0, f' = 1, \theta = 1, \phi = 1$$

$$\eta \rightarrow \infty : f' = 0, \theta = 0, \phi = 0 \quad (11)$$

III. NUMERICAL SOLUTION OF THE PROBLEM

We first convert the three Equations (8 – 10) into a system of first order equations, by using

$$g_1 = f, g_2 = f', g_3 = f'', g_4 = \theta, g_5 = \theta', g_6 = \phi, g_7 = \phi'$$

$$g'_1 = g_2, \quad (12)$$

$$g'_2 = g_3, \quad (13)$$

$$g'_3 = g_2^2 - g_1 g_3 + A(g_2 + \frac{1}{2} \eta g_3) + \lambda g_2 \quad (14)$$

$$g'_4 = g_5, \quad (15)$$

$$g'_5 = APr(g_4 + \frac{1}{2} \eta g_5) - Pr(g_1 g_5 - g_2 g_4) \quad (16)$$

$$g'_6 = g_7, \quad (17)$$

$$g'_7 = AS c(g_6 + \frac{1}{2} \eta g_7) - Sc(g_1 g_7 - g_2 g_6) + L g_6 \quad (18)$$

The initial conditions are

$$g_1(0) = 0, \quad g_2(0) = 0, \quad g_3(0) = m, \quad g_4(0) = 1,$$

$$g_5(0) = n, \quad g_6(0) = 1, \quad g_7(0) = s. \quad (19)$$

Where m, n and s are priori unknown.

Using Mathematica, we define a function $F[m, n, s] := \text{NDSolve}[\text{System}(12 - 19)]$. The values of m, n , and s are determined upon solving the equations $g_2(\eta_{max}) = 0$, $g_4(\eta_{max}) = 0$, and $g_6(\eta_{max}) = 0$. Once m, n , and s are determined, the system is closed. It can be solved numerically using the subroutine NDSolve. Consequently, only one integration path is enough to solve the problem instead of an iteration technique like shooting method. The accuracy of the numerical method was checked by performing various comparisons at different conditions with exact solutions (see table 1).

The influence of the mass transfer, unsteadiness parameter, chemical reaction parameter, and Schmidt number on the velocity gradient, dimensionless temperature and dimensionless concentration are shown in figures

IV. SKIN-FRICTION COEFFICIENT, NUSSELT NUMBER, AND SHERWOOD NUMBER.

The parameters of physical and engineering interest for the present problem are the local Skin-Friction Coefficient, local Nusselt number, and the local Sherwood number which indicate physically surface shear stress, rate of heat transfer and rate of mass transfer respectively.

1-The surface skin-friction is defined as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = \mu U_w \sqrt{\frac{U_w}{\nu x}} f''(0) \quad (20)$$

Hence the skin-friction coefficient is given by

$$C_f = \frac{2\tau_w}{\rho U_w^2} \quad \text{or} \quad f''(0) = \frac{1}{2} C_f \sqrt{Re} \quad (21)$$

2-The surface heat flux is defined as

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} = -k(T_w - T_\infty) \sqrt{\frac{U_w}{\nu x}} \theta'(0) \quad (22)$$

Hence the Nusselt number is given by

$$Nu = \frac{x q_w}{k(T_w - T_\infty)} \quad \text{or} \quad -\theta'(0) = Nu \sqrt{Re} \quad (23)$$

3-The surface mass flux is defined as

$$M_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} = -D(C_w - C_\infty) \sqrt{\frac{U_w}{\nu x}} \phi'(0) \quad (24)$$

Hence the Sherwood number is given by

$$Sh = \frac{x M_w}{D(C_w - C_\infty)} \quad \text{or} \quad -\phi'(0) = \frac{Sh}{\sqrt{Re}} \quad (25)$$

The above coefficients are obtained numerically and are sorted in tables (3) and (4).

TABLE I: VALUES OF $-\phi'(0)$ FOR VARIOUS VALUES OF Sc AT $A = 0$, $L = 0$ AND $\lambda = 0$

Sc	Exact Solution [8]	Present Results
0.024	0.466150	0.466150
0.7	0.939550	0.939547
7	3.105680	3.105682

TABLE II: VALUES OF $-f''(0)$, $-\phi'(0)$, $-\theta'(0)$ FOR VARIOUS VALUES OF (L) AT $Pr = 7$, $A = 0.1$, $Sc = 0.7$ AND $\lambda = 0.5$

L	$-f''(0)$	$-\phi'(0)$	$-\theta'(0)$
0	1.25373	3.09052	0.77539
0.3	1.25373	3.09052	0.99342
0.5	1.25373	3.09052	1.10340
1	1.25373	3.09052	1.32905

TABLE III: VALUES OF $-f''(0)$, $-\phi'(0)$, $-\theta'(0)$ FOR VARIOUS VALUES OF (A) AT $Pr = 1$, $L = 2$, $Sc = 0.7$ AND $\lambda = 0.5$

A	$-f''(0)$	$-\phi'(0)$	$-\theta'(0)$
0.1	1.25373	0.981073	1.67932
0.7	1.42024	1.20376	1.76586
1	1.49839	1.30013	1.80832
3	1.94788	1.81007	2.07668

TABLE IV: VALUES OF $-f''(0)$, $-\phi'(0)$, $-\theta'(0)$ FOR VARIOUS VALUES OF (Sc) AT $Pr = 7$, $L = 0.3$, $A = 0.1$ AND $\lambda = 0.5$

Sc	$-f''(0)$	$-\phi'(0)$	$-\theta'(0)$
0.1	1.25373	3.09052	0.61764
0.3	1.25373	3.09052	0.75086
0.5	1.25373	3.09052	0.87587
0.7	1.25373	3.09052	0.99342

TABLE V: VALUES OF $-f''(0)$, $-\phi'(0)$, $-\theta'(0)$ FOR VARIOUS VALUES OF λ AT $Pr = 7$, $L = 0.3$, $A = 0.1$ AND $Sc = 0.7$

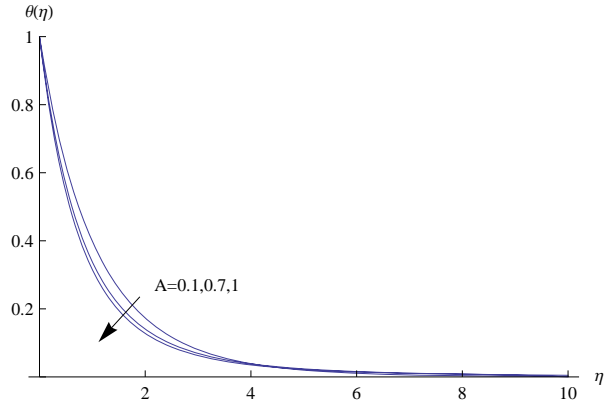


Fig. 1. Variation of temperature profiles with A , where $Sc = 0.7$, $L = 0.3$, $\lambda = 0.5$, and $Pr = 7$.

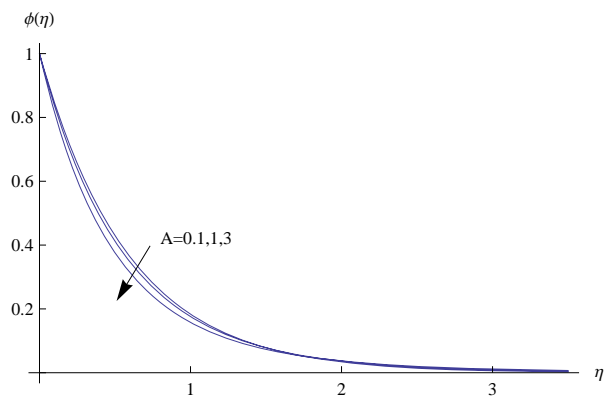


Fig. 2. Variation of concentration profiles with A , where $Sc = 0.7$, $L = 0.3$, $\lambda = 0.5$, and $Pr = 7$.

λ	$-f''(0)$	$-\phi'(0)$	$-\theta'(0)$
0.1	1.08174	3.13206	1.01718
0.3	1.17097	3.11053	1.00459
0.5	1.25373	3.09052	0.99342
0.9	1.40450	3.05392	0.97425

V. DISCUSSIONS

The influence of the mass transfer, chemical reaction parameter (L) and unsteadiness parameter (A) on the velocity gradient, dimensionless temperature and dimensionless concentration are shown in figures (1)-(6).

Figures(1)-(3) show the effect of the unsteadiness parameter A on the temperature profiles, concentration profiles, and the velocity profiles, respectively. One can note that the increase of A results in decreasing the temperature, the concentration, and the velocity of the fluid. So the increase of A leads to thinning of the temperature, the concentration, and the velocity boundary layers. The dependence of the skin friction, Nusselt number, and Sherwood number on the unsteadiness parameter A is shown in Table(3). It is clear that the three numbers increases with the increase of A . Sherwood number

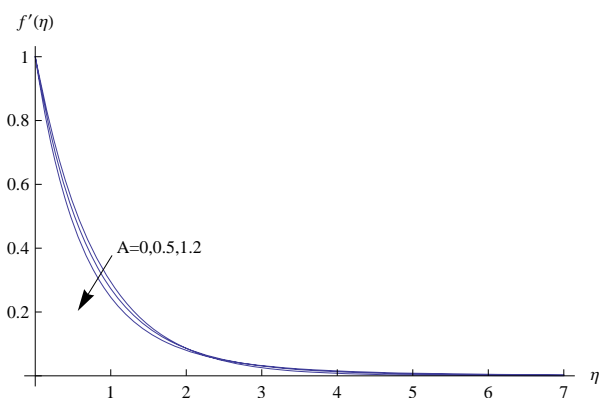


Fig. 3. Variation of velocity profiles with A , where $Sc = 0.7$, $L = 0.3$, $\lambda = 0.5$, and $Pr = 7$.

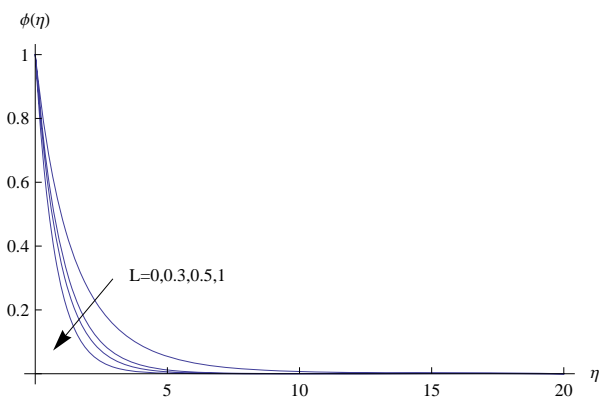


Fig. 4. Variation of concentration profiles with L , where $Sc = 0.7$, $\lambda = 0.5$, $A = 0.1$, and $Pr = 7$.

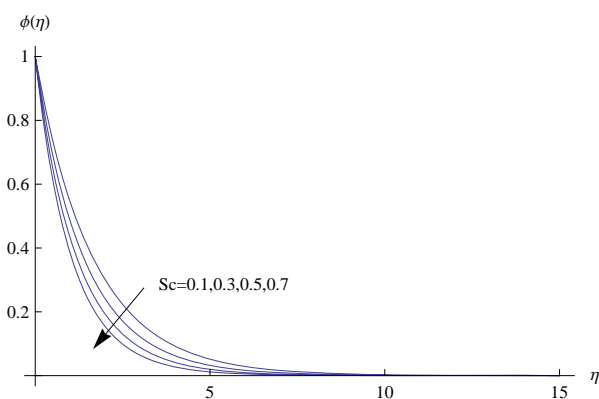


Fig. 5. Variation of concentration profiles with Sc , where $\lambda = 0.5$, $L = 0.3$, $A = 0.1$, and $Pr = 7$.

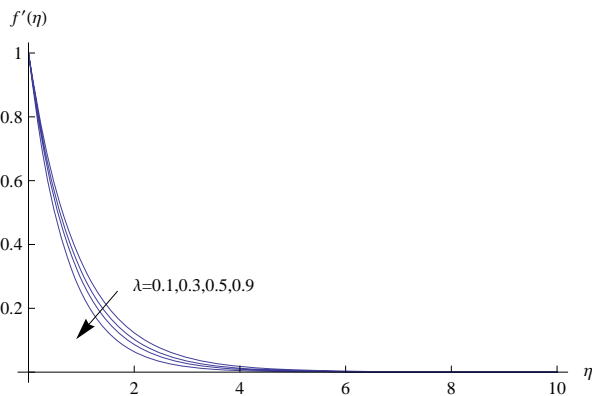


Fig. 6. Variation of velocity profiles with λ , where $Sc = 0.7$, $L = 0.3$, $A = 0.1$, and $Pr = 7$.

increases with the increase of L as shown in Table (2). A similar behavior of Sherwood number can be noted when Sc is increased as shown in table (4). From table (5) it is noted that the skin friction increases with the increase of λ which results in decreasing the velocity of the fluid as shown in figure (6). The effect of λ on Nusselt number is shown in table (5). There is a decrease in Nusselt number when λ is increased. The Sherwood number decreases also with the increase of λ . The variation of the concentration profiles with the chemical reaction parameter is shown in figure (4). One can find that the concentration decreases with the increase of the chemical reaction L . The concentration decreases also with the increase of the Schmidt number Sc as shown in figure (5). These results are due to the fact that the increase of the Schmidt number leads to thinning of the concentration boundary layer. Figure (6) shows that the velocity of the fluid decreases with the increase of the permeability parameter λ .

VI. CONCLUSIONS

Numerical solution has been obtained for the effects of chemical reaction on mass and heat transfer characteristics over an unsteady stretching surface. It has been found that the unsteadiness parameter has a considerable effect surface shear stress, rate of heat transfer, and rate of mass transfer. The temperature, concentration, and velocity are also affected by the unsteadiness parameter.

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