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H-ARQ Techniques for Wireless Systems with Punctured Non-Binary LDPC as FEC Code

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***Abstract*—This paper presents the H-ARQ techniques comparison for OFDM systems with a new family of non-binary LDPC codes which has been developed within the EU FP7 DAVINCI project. The punctured NB-LDPC codes have been used in a simulated model of the transmission system. The link level performance has been evaluated in terms of spectral efficiency, codeword error rate and average number of retransmissions. The NB-LDPC codes can be easily and effective implemented with different methods of the retransmission needed if correct decoding of a codeword failed. Here the Optimal Symbol Selection method is proposed as a Chase Combining technique.

***Keywords*—H-ARQ, LDPC, Non-Binary, Punctured Codes.

I. Introduction

IT is well known that binary Low Density Parity Check (LDPC) codes achieve spectral efficiency close to the channel capacity for very long codewords [1]. That is why more and more LDPC solutions have been already proposed in standards of such system likes Digital Video Broadcasting (DVB) or Worldwide Interoperability for Microwave Access (Wi-MAX).

New advancements to LDPC codes include improvements in terms of non-binary versions of LDPC codes and LDPC codes with a variable number of non zero values in the parity check matrix [2]. In non-binary LDPC (NB-LDPC) codes information messages are encoded using symbols from a finite field (Galois Field) with more than two elements. The NB-LDPC codes used here are based on a Galois Field (GF) with $q = 64$ elements.

This paper presents performance of different H-ARQ methods when punctured DAVINCI codes are used. Performance results are shown in the form of spectral efficiency, Code Word Error Rate (CWER) and average number of retransmissions. On that basis the H-ARQ methods are compared.

The remainder of the paper is structured as it follows. The H-ARQ technique is presented in Section II. Punctured NB-LDPC codes and ACM are described in Section III. In the next Section IV different strategies of the combining for H-ARQ are discussed. In Section V the transmittance-based combining method for H-ARQ Chase Combining is described. The H-ARQ Incremental Redundancy method is explained in more details in Section VI. The simulation parameters are listed in Section VII, while the performance comparisons of H-ARQ methods are shown as numerical results in Section VIII. The final conclusions are drawn in Section IX.

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II. H-ARQ with Punctured Codes

Punctured codes are widely used in wireless communication systems. Main aim of this is to lower the complexity of coding and decoding algorithms. When such codes are implemented as FEC code, only one generator matrix is needed in generation of codewords as well as in decoding process.

Several H-ARQ strategies can be identified [3]. In H-ARQ Chase Combining (H-ARQ CC) method the coded transmission can be based on punctured codes or mother codes. The H-ARQ CC technique works in the same manner for both types of codes, i.e., each retransmission includes a part of the codeword previously decoded incorrectly. In the case of H-ARQ Incremental Redundancy (H-ARQ IR) method the retransmission process sends a new part of the same codeword (it is previously not sent part of the redundant partition of the codeword). The amount of retransmitted symbols in both cases depends on the length of currently used punctured codeword.

III. DAVINCI Punctured Codes

Punctured codes used for purposes of this research are a part of the ACM scheme designed during FP7 DAVINCI project. They are described in details in [4]. Briefly, this ACM scheme is based on a mother code of the length $N = 480$ Galois symbols with a rate equal to $R = 1/2$. The symbol-wise puncturing of a code of length N_{MOTHER} is performed. This kind of puncturing is made by simply deleting the last symbols, resulting in a higher code rate. The decoder to keep the same code generation matrix as the mother code fills LLR (Log-Likelihood Ratio) values of the missing symbols with zeros. Puncturing scheme assumes step size equal to 12 redundant symbols.

The ACM model defines 62 modulation and coding schemes (MCS) beginning from rate 1/2 and ending even with rate 5/6 for each modulation scheme. Symbol length of a codeword for this ACM model is expressed by:

$$N_{PUNCTURED} = 480 - 12p \; ; \; p \in < 0;20 > \tag{1}$$

Each codeword created from the mother code has the same amount of information in the form of $K = 240$ Galois symbols.

IV. Comparison of Different Method of Combining

When H-ARQ CC is analyzed every usually have in mind the Maximal Ratio Combining (MRC) method. In this paper another method is considered. It is called here Optimal Symbol Selection method. It is based on the same strategy as the Antenna Selection method in MIMO (Multiple Input Multiple Output) [5] transmission. This technique chooses QAM symbols from firstly received and retransmitted OFDM blocks selecting symbols less affected by the channel imperfections.

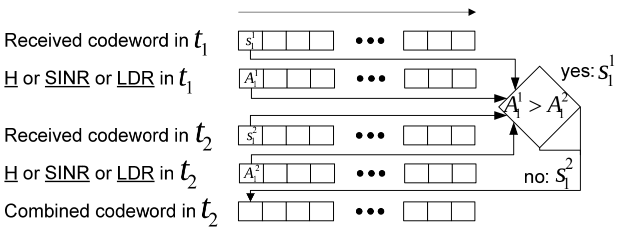

Fig. 1 Optimal Symbol Selection combining process

The decision could be made in dependence on e.g. estimated channel transfer function, SINR (Signal to Interference plus Noise Ratio) values or LDR (Log Density Ratio) values. These approaches are described below and general idea is presented in Fig. 1.

A. Transmittance-based combining (TBC)

This method is based on combing codewords in dependence on the channel transfer function, when they are still in form of QAM symbols, i.e. before Log-Likelihood Ratio (LLR soft demapping) is computed (Fig. 4). The TBC is applied as a Chase Combining H-ARQ technique analyzed in this paper (Sec. V).

B. SINR-based combining

This combining method uses information about current SINR per subcarrier (i.e. in relation to QAM symbols of an OFDM block) to select less degraded QAM symbols. Combining process is similar to the TBC method except that the main determinant of selection is not the channel transfer function but the SINR value. In simulations performed here the perfect SINR knowledge is assumed.

C. LDR (or LLR) level

In this case the combining process is performed after the soft decoding and before the decoder, and consists in selecting symbols with higher LLR values. This method characterizes a larger computational complexity than remaining two methods described above.

V. H-ARQ Type I (CC) – Part Codeword Combining

In this H-ARQ type each erroneous codeword is stored in the receiver. After a retransmission request the transmitter sends consecutive symbols taken from initially sent punctured codeword. When a retransmission is done the retransmitted symbols QAM are compared to equivalent symbols in the stored codeword and combined to maximize a probability of successive decoding. If after retransmission and combining a modified codeword is still not decodable a next retransmission request is sent to the transmitter, and the codeword stored in the receiver is then this one assembled after the combining process.

A combining algorithm selects such QAM symbols for which the influence of the channel was the least, i.e., where fadings and distortions affected on subcarriers were the weakest during transmission over the channel. To evaluate this influence of the channel, the transfer function at each subcarrier for the initially erroneous codeword and the current retransmitted part of the codeword is compared. Between two QAM symbols an algorithm selects a symbol for which the modulus of the transfer function is greater, i.e., this one for which an error probability due to channel imperfections is less. In Fig. 1 the above described solution is presented in the form of a block diagram. The combining process is the same as in case of the full codeword combining for the Chase Combining which diagram is presented in Fig. 2 (where maximum 4 retransmissions are assumed).

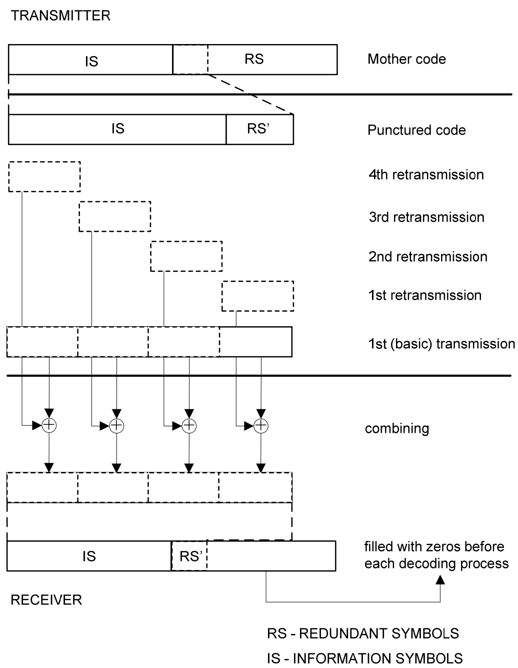

Fig. 2 H-ARQ type I process scheme where combining is made on the reliability of QAM symbols expressed by the channel transfer function recorded for each of them

VI. H-ARQ Type II (IR)

In H-ARQ IR each erroneous packet is also stored in the receiver as in the CC technique. After a retransmission request the transmitter sends in steps consecutive redundant symbols taken from the previously sent and erroneously decoded codeword. When a retransmission is done the retransmitted symbols are assembled with the previously symbols belonging to erroneously decoded codeword. It results in a new codeword of the less code rate, containing the same information symbols and incremental amount of the redundant symbols. If after a retransmission of a consecutive IR part and assembling a new codeword it is still not correctly decodable, a next retransmission request is sent to the transmitter, and the current codeword stored in the receiver is then this one constructed after last assembling process. This approach is presented in Fig. 3. After each retransmission the receiver expands previously incorrectly decoded codeword (in the form of QAM symbols) with incoming redundant symbols during retransmissions until successful decoding or a maximum number of retransmissions is achieved (here 4). The IR mechanism increases the redundant part of the codeword by assembling information symbols and consecutive parts of redundant partition in a given codeword.

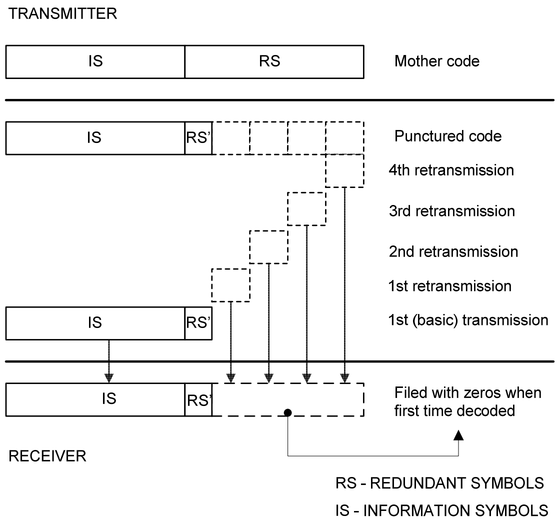

Fig. 3 H-ARQ type II process scheme

VII. System simulations

All methods of the Chase Combining strategy (Sec. IV) are evaluated for part and full codeword combining, i.e., each retransmission can concern a certain part of the codeword or at once the full codeword. However, for comparison purposes, the simulation results (Sec. VIII) are only presented for the full codeword combining. From whole range of available MCS schemes created from the mother code which length is $N_{mother} = 480$ Galois symbols and code rate is $R = 1/2$ they are chosen three MCS's (Table I). This approach allows to notice in easy way a relation between codeword length (and resulting from it, a number of redundant symbols in the punctured code vs. the mother code) and efficiency of different H-ARQ methods.Other parameters related to the channel and OFDM transmission model for executed simulations are collected in Table II.

The number of Galois symbols in each retransmission can be different and is given by the expression (2) for H-ARQ CC and by the expression (3) for H-ARQ IR.

$$R_{unit} = \frac{N_{PUNCTURED}}{Max\_RT} \tag{2}$$

$$R_{unit} = \frac{N_{MOTHER} - N_{PUNCTURED}}{Max\_RT} \tag{3}$$

So it is assumed limited number of retransmissions (the results presented in the next Section were generated assuming $Max\ RT = 4$) per erroneous codeword and that each retransmission unit is different. One can see that the shorter codeword the less symbols can be retransmitted when using H-ARQ CC. For H-ARQ IR the number of retransmitted symbols taken from the redundant partition of a codeword is inversely proportional to the codeword length if the above puncturing scheme is implemented. For all code rates the amount of symbols that are retransmitted is equal or higher for H-ARQ CC method than for H-ARQ IR (merely for the highest code rate $R = 1$ this amount is equal for both cases). Simplified block scheme of the simulation model is presented in Fig. 4.

Table I
Parameters of codes used in simulation scenarios

$N_{PUNCTURED}$	R	$N_{MOTHER} - N_{PUNCTURED}$	p
300	0.8	180	15
360	0.66	120	10
420	0.57	60	5

Table II
Common simulation parameters

FFT size	1024
Data carriers	1024
CP size	128
Channel	ITU pedestrian B
Mobile user speed	3 km/h
Bandwidth	10 MHz
Carrier frequency	2.5 GHz
Frame duration	5 ms
Others	Perfect CSI, flexible RTU, Freq. Interleaver

VIII. Simulation results

A. Different methods of combining

All results were obtained for:

- punctured codeword length $N_{PUNCTURED} = 360$
- rate: $R = 2/3$
- modulation scheme: 64-QAM.

Results are presented in figures of codeword error rate (Fig. 5), spectral efficiency (Fig. 6) and average number of retransmissions (Fig. 7) and average delay per codeword (Fig. 8).

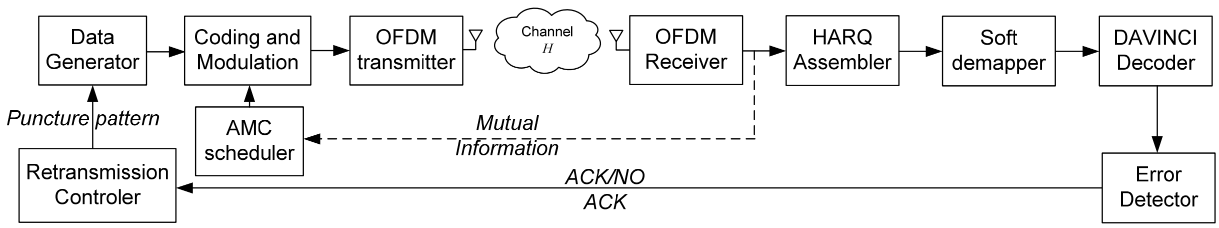

Fig. 4 Simulation model

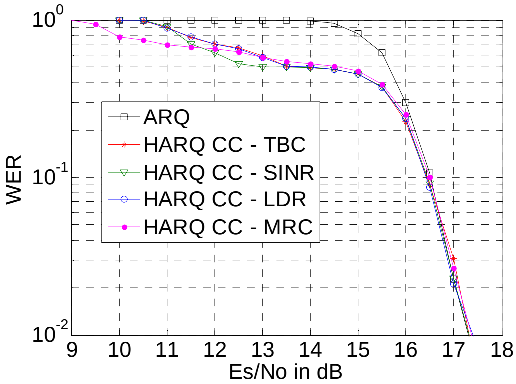

Fig. 5 Word Error Rate for different methods of retransmission

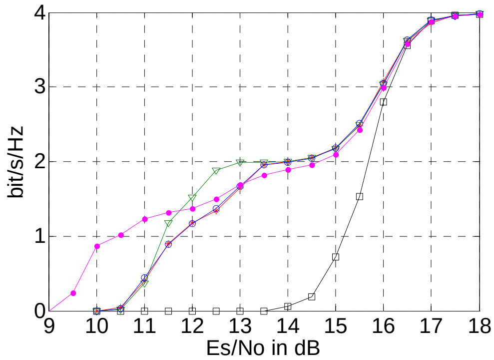

Fig. 6 Spectral efficiency for different methods of retransmission

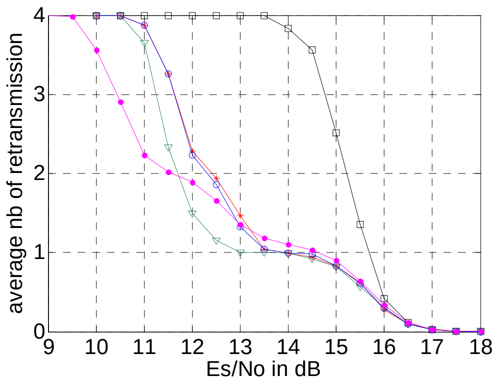

Fig. 7 Average number of retransmissions for different methods of combining

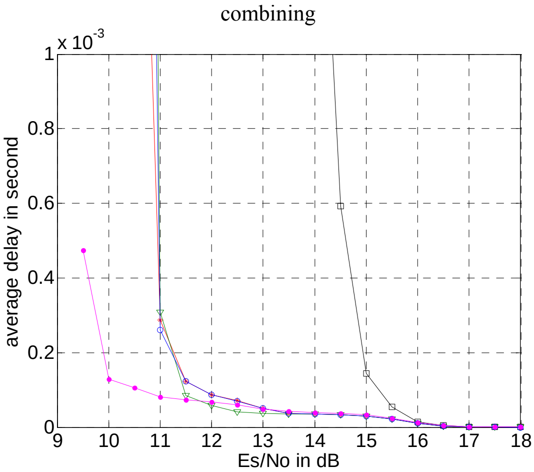

Fig. 8 Average delay of retransmissions for different methods of combining

Average delay denotes the average time that system has to wait to have correctly decoded codeword. It is expressed by:

$$T_{delay} = \frac{N_{ret} \cdot T_{interval}}{N_{CorrectCodeword}} \tag{4}$$

where:
N_{ret} – number of total retransmissions,
$T_{interval}$ – time gap between erroneous codeword and its retransmission (here assumed 102.857 µs),
$N_{CorrectCodeword}$ – number of correctly decoded codeword.

Average delay is computed per each SNR value. The value of $T_{interval}$ in not so important, it could by any positive value greater than length of OFDM symbol duration because the characteristic of function T_{delay} falls dramatically beginning from a certain SNR for which transmission starts to become acceptable for a user service. This threshold SNR is a exactly wanted value. One can see that all methods for codeword combining are far better than for simple ARQ technique (Fig. 5). In a lower SNR region (9 dB-11.5 dB) MRC method overcomes other methods. In detail, when analyzing spectral efficiency comparison (Fig. 6) one can observe close to 1 bit/Hz/s gain in the range 10 dB-11 dB. In the case of average number of retransmissions (Fig. 7) the MRC method needed up to 1.5 retransmission fewer than SINR based combining method. Also average delay per codeword (Fig. 8) when MRC mechanism is used falls faster and become acceptable at least 1 dB before other methods. From about 11.5 dB for SINR based method and from 13 dB for other methods, the MRC method is no longer the most efficient technique.

When analyzing Optimal Symbol Selection methods (using different comparison measures) all results show a little gain of SINR based combining over TBC and LDR based combining which both demonstrate similar efficiency. That state of things can be explained by the fact that the CSI in SINR based combining is more comprehensive than in two other methods. In all figures at SNR of about 13.5 dB all curves converge. One can also notice performance improvements for greater SNR that can be explained by full codeword combining and steep waterfall region of WER curves for DAVINCI codes. Considering H-ARQ technique with the ACM mechanism which tends to keep WER bound equal to 10^{-2}, there will be no practical performance differences among three described H-ARQ type I methods (Sec. IV). Taking into account an additional computationally complexity needed by SINR-based method and LDR-based one, in all following simulations presented further the TBC and MRC approach is used to compare with H-ARQ IR method.

A. Chase Combinning vs. Incremental Redundancy

Results of three different MCS schemes with punctured codes are presented in the form of codeword error rate (Fig. 9), spectral efficiency (Fig. 10), average number of retransmissions (Fig. 11) and average delay per codeword (Fig. 12). All MCS schemes used here are equidistance each other taking the length of applied mother code (expressed in a number of symbols, here 60 Galois symbols) into consideration.

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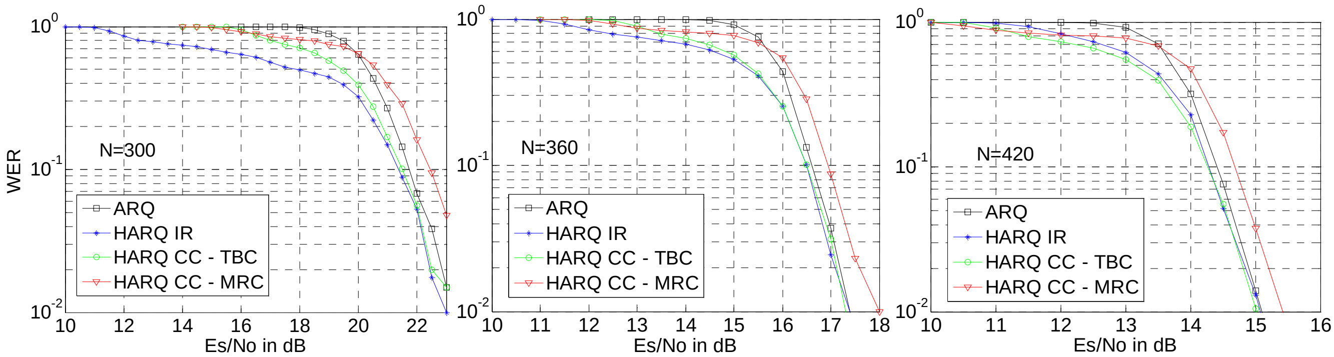

Fig. 9 Word Error Rate for different lengths of punctured codewords

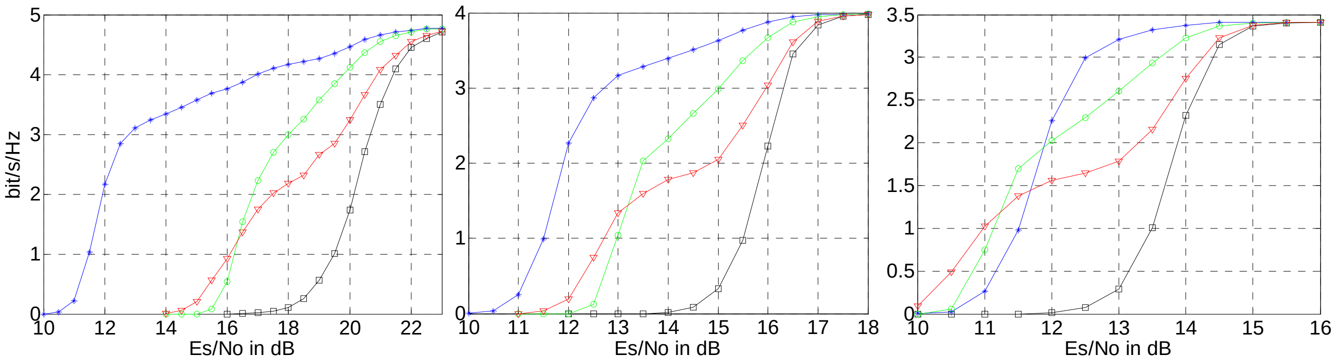

Fig. 10 Spectral efficiency for different lengths of punctured codewords

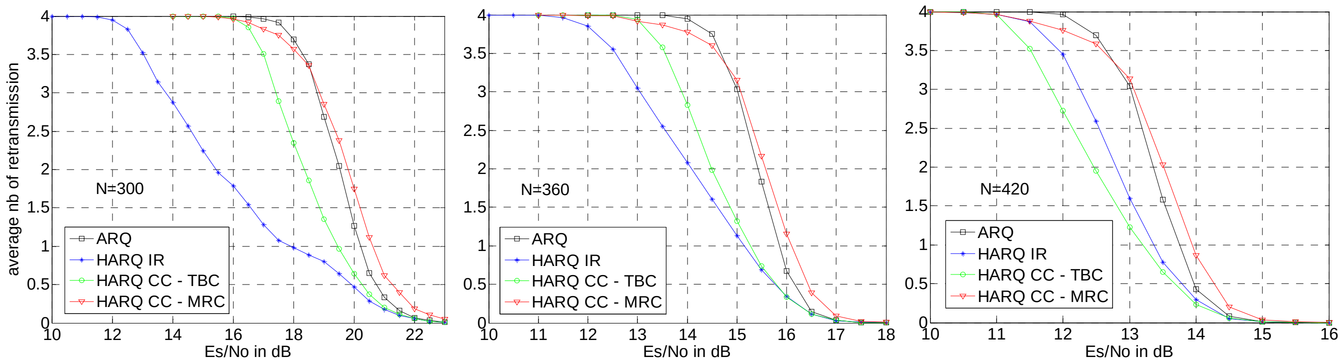

Fig. 11 Average number of retransmissions for different of lengths punctured codewords

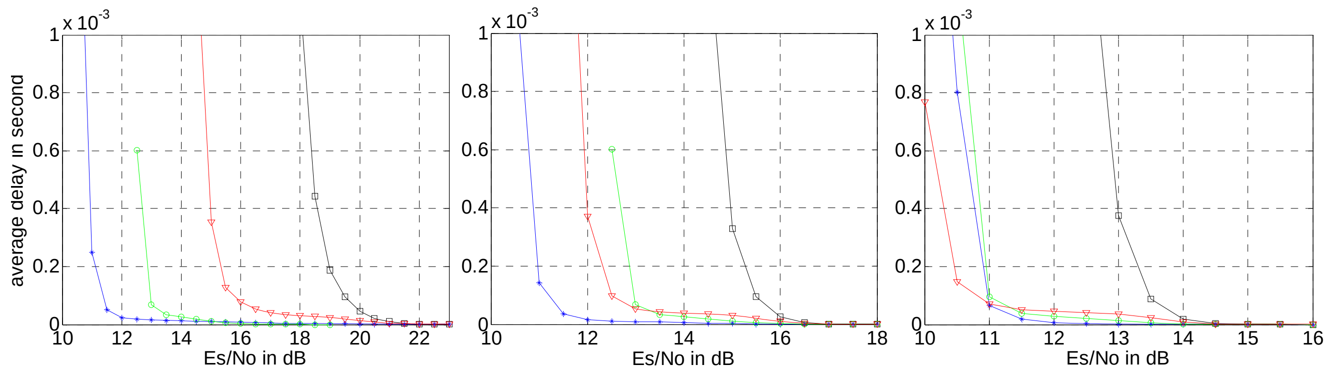

Fig. 12 Average delay per codeword for different lengths of punctured codewords

The left figure is for $N = 300$, the middle one for $N = 360$, and the right one for $N = 420$.

All results let easily notice how great advantage has the IR method over the CC method for shorter codewords and how it is decreasing according to increasing number of redundant symbols in the basic (first) transmission. In details, for each puncture code length:

- $N_{PUNCTURED} = 300$

H-ARQ IR outperforms the H-ARQ CC methods for almost whole SNR range. For the WER (Fig. 9), result curves for H-ARQ converge from about 22 dB except H-ARQ CC-MRC which achieves the same WER level for SNR greater by 1 dB. For the spectral efficiency (Fig. 10) a gain of IR method over CC methods is up to 5 dB and up to 8 dB over simple ARQ method.

Considering average delay (Fig. 12), the acceptable transmission begins from 11.5 dB for IR, 13 dB for CC-TBC, 16 dB for CC-MRC and 19.75 dB for ARQ. In the case of average retransmission number (Fig. 11) for an SNR range of up to 16 dB the IR method needs twice fewer retransmissions than the other methods.

- $N_{PUNCTURED} = 360$

Conclusions are the same as for $N_{PUNCTURED} = 300$ puncture pattern but the WER curve convergence for H-ARQ IR and H-ARQ CC-TBC is at 16 dB except H-ARQ CC-MRC which achieves the similar WER level at SNR greater by 0.5 dB (Fig. 9). The gain in spectral efficiency over simple ARQ is up to 4 dB while in the previous case it is up to 8 dB (Fig. 10, $N_{PUNCTURED} = 300$). A difference in spectral efficiency between CC methods and IR methods are also smaller but the IR method has still up to 2.25 dB gain over TBC method and up to 3.25 dB over MRC method (Fig. 10).

Considering average delay (Fig. 12), the acceptable transmission starts from 11.5 dB for IR, 12.5 dB for CC-MRC, 13 dB for CC-TBC and 15.5 dB for ARQ. In the case of average retransmission number (Fig. 11) for an SNR range of up to 16 dB, IR method needs up to 1 retransmission fewer than CC-TBC method and up to 2 retransmissions fewer than the other methods.

- $N_{PUNCTURED} = 420$

In this case, H-ARQ CC methods outperform H-ARQ IR method in certain ranges of SNR. The WER curve convergence for H-ARQ IR and H-ARQ CC-TBC is at 14.2 dB except H-ARQ CC-MRC which achieves the similar WER level for SNR greater by 0.4 dB (Fig. 9). For SNR smaller than 14.2 dB the CC-TBC has the best WER. In spectral efficiency for smaller SNR (up to about 11.6 dB) both CC methods have better performance than the IR method (Fig. 10). For greater SNR on the contrary the IR technique achieves a gain of up to 1 dB over CC-TBC and 1.75 dB over CC-MRC method.

Considering average delay (Fig. 12), the acceptable transmission begins from about 11 dB for IR and CC-TBC methods so for a smaller SNR of 2.5 dB than for ARQ method. A little extra gain of about 0.5 dB is observer for CC-MRC. In the case of average retransmission number (Fig. 11), IR method needs up to 0.75 retransmission more than CC-TBC method and up to 1.5 retransmissions fewer than other methods.

IX. Conclusions

The receiver structure can be based on H-ARQ type I (CC) based on proposed Optimal Symbol Selection method (using TBC measure) instead of Maximal Combining Ratio (MRC) and on H-ARQ type II (IR) strategies in order to improve the decoding performance. But one can see that H-ARQ type II model outperforms H-ARQ type I scheme in term of reliability of transmission, especially for greater code rates. The Chase Combining technique on the other hand, when the code rate is closing to one of the mother rate (i.e. 1/2), comes up or even outperforms the Incremental Redundancy method — one can observe 0.5 dB gain in a short range of SNR on the spectral efficiency (Fig. 10, $N = 420$). Moreover, Chase Combining is easier to implement and requires less memory span. Due to IR technique the average number of retransmissions can be reduced over twice as compared to the CC approach (Fig. 11, $N = 300$, $E_s/N_0 = 16$ dB). It seems that joining of both methods for certain MCS schemes would be also a good choice that can improve an overall system performance.

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