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Fuzzy Sliding Mode Control of an MR Mount for Vibration Attenuation

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***Abstract*—**In this paper, an magnetorheological (MR) mount with fuzzy sliding mode controller (FSMC) is studied for vibration suppression when the system is subject to base excitations. In recent years, magnetorheological fluids are becoming a popular material in the field of the semi-active control. However, the dynamic equation of an MR mount is highly nonlinear and it is difficult to identify. FSMC provides a simple method to achieve vibration attenuation of the nonlinear system with uncertain disturbances. This method is capable of handling the chattering problem of sliding mode control effectively and the fuzzy control rules are obtained by using the Lyapunov stability theory. The numerical simulations using one-dimension and two-dimension FSMC show effectiveness of the proposed controller for vibration suppression. Further, the well-known skyhook control scheme and an adaptive sliding mode controller are also included in the simulation for comparison with the proposed FSMC.

***Keywords*—**adaptive sliding mode controller, fuzzy sliding mode controller, magnetorheological mount, skyhook control.

I. Introduction

MAGNETORHEOLOGICAL fluids were proposed by Bitter and Elmore in 1930. MR fluids are materials that typically consist of non-colloidal suspensions of polarizable iron particles dispersed in a carrier medium such as mineral or silicon oil. One nice property of MR fluids is the maximum yield stress that monotonically increases with applied magnetic field. By this property, MR fluids are shown on the market but their application fields are restricted to devices such as valves, brakes, clutches, dampers, mounts, etc. MR mount is often used as a kind of semi-active suspension control. A semi-active control technique can provide real-time dissipation of the system energy, which has proved to provide better performance than the passive control. Karnopp et al. [1] used semi-active force generators to vibration control. Tseng and Hedrick [2] compared the suspension of vehicle system in semi-active control. Lu [3] studied active and semi-active air-spring suspension systems and compared the performances. MR fluids had been studied in many applications of semi-active control. Dyke and Spence [4] proposed the MR dampers for seismic protection. Kim and Roschke [5] employed a linearization scheme for MR damper behavior using a neural network. Yokoyama et al. [6] presented a model following sliding mode controller for semi-active suspension systems with MR dampers. In the same year, Shen et al. [7] improved the semi-active car suspension with MR damper. Choi [8, 9] studied the effects of H_∞ and skyhook control for full vehicle suspensions featuring MR by using the method of HILS (hardware-in-the-loop simulation). Although the dynamic equations of MR mount were derived in many research literatures, however it is difficult to solve the inherent problems of time-varying and nonlinear characteristics. Recently, many studies proposed all kinds of control methods to achieve better vibration suppression. Many of these robust control techniques can diminish disturbances, but requiring some uncertainties to be defined in several compact sets.

The first successful laboratory experiment of the fuzzy logic control was steam engine control (Mamdani, 1974) [10]. The fuzzy logic control (FLC) is suitable for controlling systems with complex, ill-defined, time-varying, and nonlinear dynamics. In recent years, FLC had been applied to control many devices/machines, like camcorders, air conditioners, servo motors, etc. In order to further improve the control performance, the FLC can be combined with other control algorithms, e.g. fuzzy neural network controller and adaptive fuzzy logic controller [11-14].

The traditional fuzzy logic controller depends on a human expert or an experienced operator to build the fuzzy knowledge base. Furthermore, it is not easy to prove the stability of a FLC system. In recent years, several researchers suggested combining the concept of a sliding mode control in the fuzzy logic control. Chen and Chang [15] employed an optimal design method to FSMC. Huang and Lin [16] used adaptive fuzzy control with sliding surface to vehicle suspension control. Yu et al. [17] applied FSMC to control an uncertain time-delayed system with nonlinear input.

In this paper, a FSMC is proposed to deal with modeling uncertainties and unknown disturbances. The stability of the proposed controller can be ensured by using Lyapunov stability theorem. Furthermore, the developed one-dimension and two-dimension FSMC will compare performances with the often used skyhook control and adaptive sliding mode control (ASMC, [18]). This paper is organized as follows. In Section II, a brief formulation of the MR mount model is given. In Section III, we derive the one-dimension and two-dimension FSMC in detail. Moreover, we use Lyapunov liked design to obtain the stability. In Section IV, results of one-dimension and two-dimension FSMC for semi-active control of an MR mount

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are presented. In addition, the well-known skyhook control scheme and adaptive sliding mode control are compared with the proposed FSMC. Finally, we make a briefly conclusion in Section V.

II. PROBLEM FORMULATION

The schematic configuration and its corresponding hydraulic model of a one-dimensional MR mount [9] is shown in Fig. 1. The dynamic equation of this MR suspension system can be derived as

$$m\ddot{x}(t)+\left\{b-A\frac{\eta}{h}\right\}(\dot{x}(t)-\dot{y}(t))+\left\{k+\frac{A_p^{\,2}}{C_1+C_2}\right\}(x(t)-y(t))$$
$$+\frac{C_2A_p}{C_1+C_2}\left\{\frac{2n+1}{n}\frac{2}{\mathrm{W}h^2}\left(\frac{A}{2}-\frac{C_2A_p}{C_1+C_2}\right)\left|\dot{x}(t)-\dot{y}(t)\right|\right\}^n\frac{2\eta L}{h} \quad (1)$$
$$=-F_{MR}$$

where

$$F_{MR}=\left(\frac{C_2A_p}{C_1+C_2}D\frac{2L\tau_{yf}(H)}{h}+A\tau_{ys}(H)\right)\mathrm{sgn}(\dot{x}(t)-\dot{y}(t)) \quad (2)$$

$$D=\frac{2n+1}{n+1}-\frac{3n(1+2n)(1-n)}{16(n+1)^2}-\frac{3n(n^2+1)}{40(n+1)} \quad (3)$$

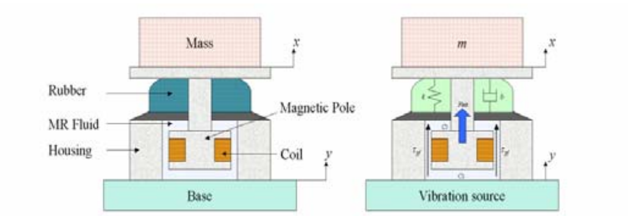

Fig. 1 Sketch of an MR mount (left) and its hydraulic model (right)

It is noticed that A is the flow area, A_p is the piston area of the upper chamber, b is the damping constant of the rubber, C_1 and C_2 are the compliance of the upper and lower chamber, h is the gap of the magnetic pole, k is the stiffness of the rubber, L is the length of the magnetic pole, m is the mass, n is the flow behavior index of Herschel-Bulkey model, W is the width of the magnetic pole, $x(t)$ and $y(t)$ represent the respective displacements at the mass and base, η is the viscosity of the MR fluid. We may represent Eqs. (1)-(3) in state space form:

$$\begin{cases}\dot{z}_1=z_2\\ \dot{z}_2=f(\mathrm{z},t)+Bu\end{cases} \quad (4)$$

where $z_1=x$ and $z_2=\dot{x}$ are state variables, B and u represent the unknown input gain and control input.

$$f(\mathrm{z},t)=\frac{1}{m}\left\{b-A\frac{\eta}{h}\right\}(\dot{y}-z_2)+\frac{1}{m}\left\{k+\frac{A_p^{\,2}}{C_1+C_2}\right\}(y-z_1)$$
$$-\frac{1}{m}\frac{C_2A_p}{C_1+C_2}\left\{\frac{2n+1}{n}\frac{2}{Wh^2}\left(\frac{A}{2}-\frac{C_2A_p}{C_1+C_2}\right)\left|\dot{y}-z_2\right|\right\}^n\frac{2\eta L}{h} \quad (5)$$

$$B=\frac{1}{m}\left(\frac{C_2A_p}{C_1+C_2}D\frac{2L}{h}\right)>0 \quad (6)$$

$$u=\tau_{yf}(H)\,\mathrm{sgn}(\dot{y}-z_2) \quad (7)$$

Assumption:

$f(\mathrm{z},t)$ is an unknown function with unknown variation bound, but it remains continuous and bounded for all admissible z and for all $t\in[t_0,\infty)$.

Remark:

Eq. (4) is obtained by assuming that yield stress of the MR fluid in driven flow mode $\tau_{yf}(H)$ is much greater than that in direct shear mode $\tau_{ys}(H)$ which is thus neglected.

III. FUZZY SLIDING MODE CONTROLLER DESIGN

In this section, traditional design procedures of sliding mode controller for the MR mount of Fig. 1 are first briefly given. In the beginning, we define the sliding surface $s=\dot{e}+\lambda e$, where $e=z_{1d}-z_1$, $\dot{e}=z_{2d}-z_2$, z_{id} represents the desired value of state, $i=1,2$, $\lambda>0$ is a parameter to be properly selected. The time derivative of s can be derived as

$$\dot{s}=-f(z,t)-Bu+\dot{z}_{2d}+\lambda\dot{e} \quad (8)$$

Next by selecting u as

$$u=\frac{1}{B}[-f(z,t)+\dot{z}_{2d}+\lambda\dot{e}+k\,\mathrm{sgn}(s)] \quad (9)$$

where $k>0$ is a constant, Eq. (8) then can be reduced to

$$\dot{s}=-k\ \mathrm{sgn}(s) \quad (10)$$

Now we may select the Lyapunov function candidate as

$$V=\frac{1}{2}s^2 \quad (11)$$

Taking time derivative of Eq. (11), we have

$$\dot{V} = s\dot{s} = -k|s| \le 0 \tag{12}$$

Hence the closed loop stability of the system can be guaranteed. From Eq. (9), the traditional sliding mode control needs to know the exact values of B and $f(x,t)$. The authors [18] had applied a function approximation technique to approximate the constant B and the unknown function $f(x,t)$, in which a Fourier series of 41 terms were used. The update laws of the unknown coefficients in these estimates were derived using the Lyapunov stability theory. Although the developed adaptive sliding mode controller (ASMC) was effective in vibration attenuation, two problems needs to be stated. One is computational efficiency associated with the number of terms used in the Fourier series: the lower the terms used, the higher the computational efficiency at the expense of approximation accuracy; the higher the terms used, the better the approximation but with slower computational efficiency resulting in reducing the sampling rate. The other problem is singularity encountered in approximating the constant B. When the estimated B is too close to zero, the control force becomes large and may go unbound. In order to tackle these problems, we propose a FSMC which uses sliding surface as input variable and requires neither function approximation nor singularity avoidance.

3.1 One-dimension FSMC

Fuzzy logic controller is employed to control the MR mount, in which the sliding surface *s* is used as the input variable. Designing procedures of the one-dimension FSMC are described in the following steps.

Step 1:
Select the sliding surface $s = \dot{e} + \lambda e$ to be the input variable of the one-dimension FSMC.

Step 2:
Define the linguistic variables and fuzzy partition for the input signal s. Here, we use five linguistic states F_i $(i = 1,2\cdots,5)$: Negative Big (NB), Negative Medium (NM), Zero (ZE), Positive Medium (PM), and Positive Big (PB). The five linguistic states with corresponding Gaussian membership functions are shown in Fig. 2. The expression of Gaussian function is as follows

$$\mu_i(s) = \exp\left(\frac{-(s-\overline{m}_i)^2}{2\sigma_i^{\,2}}\right) \tag{13}$$

where $i = 1,2,\cdots 5$; $\overline{m}$ is the mean value and σ is the standard deviation. These parameters are given in Table I.

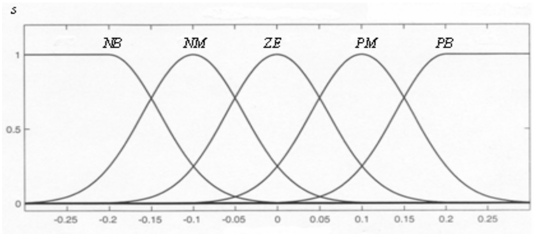

Fig. 2 The membership functions of the input variable s

TABLE. I
THE PARAMETERS OF GAUSSIAN FUNCTIONS OF S

s	NB	NM	ZE	PM	PB
$\overline{m}$	-0.2	-0.1	0	0.1	0.2
σ	0.06	0.06	0.06	0.06	0.06

Step 3:
Construct the fuzzy control rules according to the Lyapunov stability criterion:

$$\text{Rule } i: \text{If } s \text{ is } F_i, \text{ Then } u \text{ is } \theta_i \tag{14}$$

θ_i is the output linguistic state, $i = 1,2,\cdots 5$, which is chosen the same as those used for input variable *s* but with different membership functions. Here, fuzzy singleton is adopted as the membership functions of θ_i for quick fuzzy inference, whose nonzero membership value is listed in Table II. Therefore the fuzzy control system mentioned in Eq. (14) is in a simple form of the TSK fuzzy system [19]. As stated, one requires $\dot{V} = s\dot{s} \le 0$ for closed loop stability. That is to say, s and $\dot{s}$ should be in opposite signs for stability. From Eq. (8), it is clearly observed that sign of $\dot{s}$ can be controlled by the term -*Bu*. Henceforth, the fuzzy control rules can be derived accordingly as shown in Table 2, where only 5 rules are needed.

Step 4:
The firing strength ω_i of rule *i* is the grade of membership of *s* belonging to variable F_i:

$$\omega_i = \mu_i(s) \tag{15}$$

The resulting discrete output signal composing effects of the 5 control rules can be obtained by using the weighted sum as

$$U = \frac{\sum_{i=1}^{5} \omega_i \cdot \theta_i}{\sum_{i=1}^{5} \omega_i} \tag{16}$$

Remark :

From Eqs. (4) and (7), it is noted that the MR mount control force *u* is applied only when the following actuating condition is met:

$$u = \begin{cases} U & for \quad U(\dot{y} - z_2) > 0 \\ 0 & for \quad U\ (\dot{y} - z_2) \le 0 \end{cases} \tag{17}$$

TABLE II
THE DERIVED FUZZY CONTROL RULES

$s \in F_i$	NB	NM	ZE	PM	PB
$u \in \theta_i$	NB	NM	ZE	PM	PB
$value$	-10^5	$-0.5x10^5$	0	$0.5x10^5$	10^5

3.2 Two-dimension FSMC

In this subsection, two-dimension FSMC is to be developed with an aim of giving more flexibility to the controller design. The design steps are given as follows:

Step 1:
Select the sliding surface and its derivative as the two inputs to the proposed two-dimension FSMC, namely s and $\dot{s}$.

Step 2:
Define linguistic variables and fuzzy partition for $\dot{s}$. The five linguistic states $G_i, i = 1,2\cdots 5$ are shown in Fig. 3, whereas parameters of the corresponding Gaussian membership functions are given in Table III.

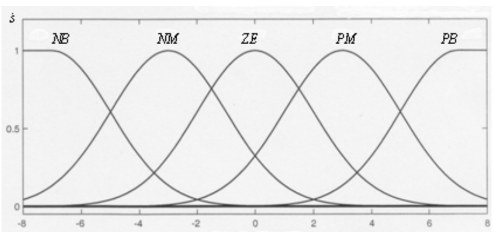

Fig. 3 The membership functions of $\dot{s}$

TABLE III
THE PARAMETERS OF GAUSSIAN FUNCTIONS OF $\dot{s}$

$\dot{s}$	NB	NM	ZE	PM	PB
$\overline{m}$	-7	-3	0	3	7
σ	2	2	2	2	2

Step 3:
Construct the two-dimension fuzzy IF-THEN rules in following form:

Rule: IF s is F_i and $\dot{s}$ is G_i, THEN u is θ_i (18)

We can derive the 25 fuzzy control rules using again the Lyapunov stability criterion:

- If s is negative and $\dot{s}$ is negative, then u is negative. From Eq. (8), if we select negative control output u to increase $\dot{s}$ from negative to positive. Then the Lyapunov condition may be satisfied ($\dot{V} = s\dot{s} < 0$).
- If s is positive and $\dot{s}$ is positive, then u is positive. The arguments for this case are just the opposite to the above case. If we select positive control output u to decrease $\dot{s}$ from positive to negative. Then the Lyapunov condition can be satisfied ($\dot{V} = s\dot{s} < 0$).
- If s or $\dot{s}$ is zero, then u is zero. That is to say, $\dot{V} = s\dot{s} = 0$ at the moment, no control force is thus applied for the sake of conserving energy.
- If signs of s and $\dot{s}$ are opposite, then the Lyapunov condition is satisfied. No control is needed at all.

The resulting fuzzy control rules are given in Table IV.

TABLE IV
FUZZY CONTROL RULES OF THE TWO-DIMENSION FSMC

$s / \dot{s}$	NB	NM	ZE	PM	PB
NB	NB	NM	ZE	ZE	ZE
NM	NB	NM	ZE	ZE	ZE
ZE	ZE	ZE	ZE	ZE	ZE
PM	ZE	ZE	ZE	PM	PB
PB	ZE	ZE	ZE	PM	PB

Step 4:
The firing strength ω_i of rule i is computed by using Mamdani's product rule as

$$\omega_i = \mu^F(s) \cdot \mu^G(\dot{s}) \tag{19}$$

The resulting discrete output signal can be obtained by using the weighted sum as

$$U = \frac{\sum_{i=1}^{25} \omega_i \cdot \theta_i}{\sum_{i=1}^{25} \omega_i} \tag{20}$$

where this control force is applied only when the actuation condition of Eq. (17) is satisfied (namely, U and $(\dot{y} - z_2)$ has the same sign).

IV. SIMULATION RESULTS

The system and control parameters used in the simulation are shown in Table V. The sinusoidal disturbance $y(t)$ with single frequency between 1Hz~15Hz and amplitude at 1 mm is employed as the base excitation. Numerical simulation is carried out by using Runge-Kutta order 4 solver with sampling rate at 1000Hz. Simulation results for vibration control are shown in Fig. 4. Good vibration attenuations are readily seen by the proposed controllers and are even better than the often used skyhook controller. Vibration amplitude reductions by the one- and two-dimension FSMC near resonant frequency are illustrated in Fig. 5. The system had also been studied using an adaptive sliding mode control (ASMC) by the Authors, where 41 terms of Fourier orthogonal basis to approximate the uncertainty function $f(z,t)$ were adopted. Fig. 6 compares the vibration suppression capabilities of the proposed controllers with that of ASMC. Comparable performances for vibration attenuation among theses controllers are easily observed.

TABLE V
SYSTEM PARAMETERS

Parameter	*Specification*	*Value*	*Unit*
m	load mass	60	kg
b	rubber damping	610	Ns/m
A	flow area	0.0095	m^2
η	MR fluid viscosity	0.8	Ns/m^2
h	gap of magnetic pole	0.01	m
k	rubber stiffness	133240	N/m
A_p	piston area of upper chamber	0.009	m^2
C_1 , C_2	compliance of upper and lower chamber	$\approx 3\times10^{-8}$	m^3/N
L	length of the magnetic pole	0.03	m
W	magnetic pole width	0.09	m
n	flow behavior index of Herschel-Bulkey model	0.8	
λ		10	

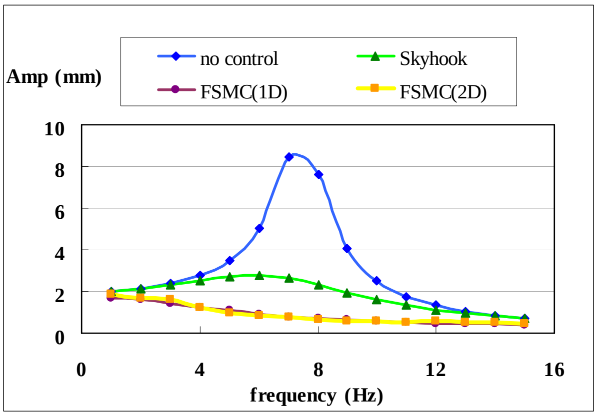

Fig. 4 Frequency responses for 1Hz~15Hz disturbance

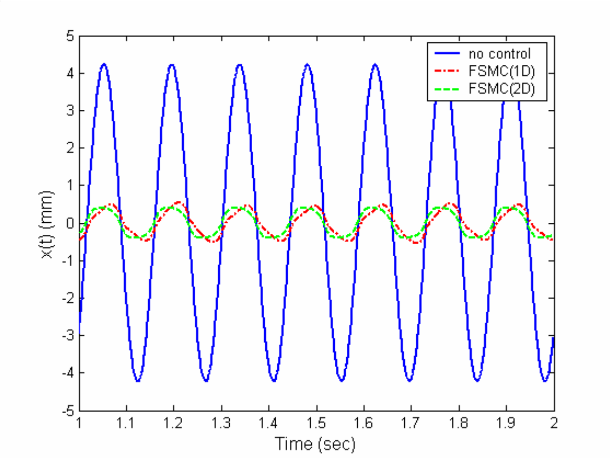

Fig. 5 Time responses of the proposed controllers (7 Hz).

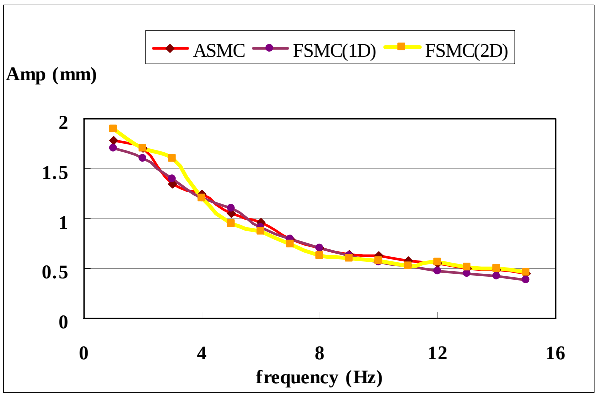

Fig. 6 Comparison of frequency responses by different controllers

V. CONCLUSION

This paper proposed a fuzzy sliding mode control for an MR mount for vibration suppression. In section III, we have given a detailed derivation of the controller by combining the fuzzy logic control and sliding mode control. Fuzzy control rules are obtained by applying the Lyapunov stability theory. Information on system model concerning parameters and time varying functions, as well as the disturbances needs not be known for the developed controller. Good vibration controls are obtained from the results of numerical simulations in section IV. In addition, the proposed controller is easy to derive and implement, without using any function approximation technique and without bumping into singularity as required by the adaptive sliding mode controller. In the near future, the experiments for vibration suppression of an MR mount will be carried out to validate the effectiveness of the proposed FSMC.

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