Fuzzy Metric Approach for Fuzzy Time Series Forecasting based on Frequency Density Based Partitioning

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Abstract—In the last 15 years, a number of methods have been proposed for forecasting based on fuzzy time series. Most of the fuzzy time series methods are presented for forecasting of enrollments at the University of Alabama. However, the forecasting accuracy rates of the existing methods are not good enough. In this paper, we compared our proposed new method of fuzzy time series forecasting with existing methods. Our method is based on frequency density based partitioning of the historical enrollment data. The proposed method belongs to the kth order and time-variant methods. The proposed method can get the best forecasting accuracy rate for forecasting enrollments than the existing methods.

Keywords—Fuzzy logical groups, fuzzified enrollments, fuzzy sets, fuzzy time series.

I. INTRODUCTION

 \mathbf{I}_{role}^{T} is obvious that forecasting activities play an important role in our daily life. During last few decades, various approaches have been developed for time series forecasting. Among them ARMA models and Box-Jenkins model building approaches are highly famous. But the classical time series methods can not deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets [11], [23]. Therefore, Song and Chissom [18] presented the theory of fuzzy time series to overcome this drawback of the classical time series methods. Based on the theory of fuzzy time series, Song et al. presented some forecasting methods [16], [18], [19], [20] to forecast the enrollments of the University of Alabama. In [1] Chen and Hsu and in [2], Chen presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series. It has the advantage of reducing the calculation, time and simplifying the calculation process. In [8], Hwang, Chen and Lee used the differences of the enrollments to present a method to forecast the enrollments of the University of Alabama based on fuzzy

time series. In [5] and [6], Huang used simplified calculations with the addition of heuristic rules to forecast the enrollments using [2]. In [4], Chen presented a forecasting method based on high-order fuzzy time series for forecasting the enrollments of the University of Alabama. In [3], Chen and Hwang presented a method based on fuzzy time series to forecast the daily temperature. In [15], Melike and Konstsntin presented a new first order time series model for forecasting enrollments of the University of Alabama. In [14], Li and Kozma presented a dynamic neural network method for time series prediction using the KIII model. In [21], Su and Li presented a method for fusing global and local information in predicting time series based on neural networks. In [22], Sullivan and Woodall reviewed the first-order time-variant fuzzy time series model and the first-order time-invariant fuzzy time series model presented by Song and Chissom [18], where their models are compared with each other and with a time-variant Markov model using linguistic labels with probability distributions. In [13], Lee, Wang and Chen presented two factor high order fuzzy time series for forecasting daily temperature in Taipei and TAIFEX. In [9], Jilani and Burney and in [10], Jilani, Burney and Ardil presented new fuzzy metrics for high order multivariate fuzzy time series forecasting for car road accident casualties in Belgium.

In this paper, we present a comparison of our proposed method and existing fuzzy time series forecasting methods to forecast the enrollments of the University of Alabama. Our proposed method belongs to the class of k-step first-order univariate time-variant method. The proposed method gives the best forecasting accuracy rate for forecasting enrollments when compared with existing methods. The rest of this paper is organized as follows. In Section 2, we briefly review some basic concepts of fuzzy time series. In Section 3, we present our method of fuzzy forecasting based on frequency density based partitioning of the enrollment data. In Section 4, we compared the forecasting results of the proposed method with the existing methods. The conclusions are discussed in Section 5.

II. SOME BASIC CONCEPTS OF FUZZY TIME SERIES

There are number of definitions for fuzzy time series.

Manuscript received June 15, 2007. This work was supported in part by the Higher Education Commission of Pakistan.

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- **Definition 1:** Imprecise data at equally spaced discrete time points are modeled as fuzzy variables. The set of this discrete fuzzy data forms a fuzzy time series.
- **Definition 2:** Chronological sequences of imprecise data are considered as time series with fuzzy data. A time series with fuzzy data is referred to as fuzzy time series.
- **Definition 3:** Let Y(t), (t = ..., 0, 1, 2, ...) be the universe of and $Y(t) \subseteq R$. discourse Assume that $f_i(t)$, i = 1, 2, ... is defined in the universe of discourse Y(t) and F(t) is a collection of $f(t_i), (i = ..., 0, 1, 2, ...)$, then F(t) is called a fuzzy time series of Y(t), i = 1, 2, ... Using fuzzy relation, we define $F(t) = F(t-1) \circ R(t,t-1)$ where R(t,t-1) is a fuzzy relation and "o" is the max-min composition operator, then F(t) is caused by F(t-1) where F(t) and F(t-1) are fuzzy sets.

Let F(t) be a fuzzy time series and let R(t, t-1) be a firstorder model of F(t). If R(t, t-1) = R(t-1, t-2) for any time t, then F(t) is called a time-invariant fuzzy time series. If R(t, t-1) is dependent on time t, that is, R(t, t-1) may be different from R(t-1, t-2) for any t, then F(t) is called a time-variant fuzzy time series. In [19], Song et al. proposed the time-variant fuzzy time-series model and forecasted the enrollments of the University of Alabama based on the model.

III. A NEW METHOD FOR FORECASTING ENROLLMENTS USING FUZZY TIME SERIES

In this section, we present our method to forecast the enrollments of the University of Alabama based on fuzzy time series based on [9] and [10]. The historical enrollments of the University of Alabama are shown in Table I, [19].

Firstly, based on [1], we defined the partition the universe of discourse into equal length intervals. Then based on frequency density portioning, we redefine the intervals. After this, define some membership function for each interval of the historical enrollment data to obtain fuzzy enrollments to form a fuzzy time series. Then, it establishes fuzzy logical relationships (FLRs) based on the fuzzified enrollments in Table IV. Finally, it uses our proposed method. The proposed method bases on Hsu and Chen approach, [7] of partitioning universe of discourse are as follows:

Step 1: Define the universe of discourse U and partition it into several even and equal length intervals $u_1, u_2, ..., u_n$. For example, assume that the universe of discourse U= [13000, 20000] is partitioned into seven even and equal length intervals.

TABLE I THE HISTORICAL ENROLLMENTS OF THE UNIVERSITY OF ALABAMA

		[19]		
YEAR	ENROLLMENTS		YEAR	ENROLLMENTS
1972	13055	-	1982	15433
1972	13563		1983	15497
1973	13847		1984	15145
1974	14696		1985	15163
1975	15460		1986	15984
1976	15311		1987	16859
1977	15603		1988	18150
1978	15861		1989	18970
1979	16807		1990	19328
1980	16919		1991	19337
1981	16388		1992	18876

Step 2: Get a weighted aggregation [24] of the fuzzy distribution of the historical enrollments in each interval. Sort the intervals based on the number of historical enrollment data in each interval from the highest to the lowest as given in [1]. Find the interval having the largest number of historical enrollment data and divide it into four sub-intervals of equal length. Find the interval having the second largest number of historical enrollment data and divide it into three sub-intervals of equal length. Find the interval having the third largest number of historical enrollment data and divide it into two sub-intervals of equal length. Find the interval with the fourth largest number of historical enrollment data and let the length of this interval remain unchanged. If there are no data distributed in an interval then discard this interval. For example, the distributions of the historical enrollment data in different intervals are summarized as shown in Table II, [7].

After executing this step, the universe of discourse [13000, 20000] is re-divided into the following intervals [7], see Table III.

Step 3: Define each fuzzy set A_i based on the re-divided intervals and fuzzify the historical enrollments shown in Table I, where fuzzy set A_i denotes a linguistic value of the enrollments represented by a fuzzy set. We have used triangular membership function to define the fuzzy sets A_i [10]. The reason for fuzzifying the historical enrollments into fuzzified enrollments is to translate crisp values into fuzzy sets to get a fuzzy time series.

Step 4: Establish fuzzy logical relationships based on the fuzzified enrollments where the fuzzy logical relationship " $A_p, A_q, A_r \rightarrow A_s$ " denotes that "if the fuzzified enrollments of year p, q and r are A_p, A_q and A_r respectively, then the fuzzified enrollments of year (r) is A_r ".

TABLE II THE FREQUENCY DENSITY BASED DISTRIBUTION OF THE HISTORICAL ENROLLMENT DATA [7]

ENROLLMENT DATA [7]								
Intervals	Number of historical enrollment data							
[13000, 14000]	3							
[14000, 15000]	1							
[15000, 16000]	9							
[16000, 17000]	4							
[17000, 18000]	0							
[18000, 19000]	3							
[19000, 20000]	2							

TABLE III FUZZY INTERVALS USING FREQUENCY DENSITY BASED PARTITIONING

Linguistic	Intervals						
ul	[13000, 13500]	_					
u2	[13500, 14000]						
u3	[14000, 15000]	[14000, 15000]					
u4	[15000, 15250]						
u5	[15250,15500]						
u6	[15500,15750]						
u7	[15750,16000]						
u8	[16000,16333]						
u9	[16333,16667]						
u10	[16667,17000]						
u11	[18000,18500]						
u12	[18500,19000]	[18500,19000]					
u13	[19000,20000]	[19000,20000]					
	Intervals						
$\boxed{\frac{1}{\frac{1}{a_1}}}$	$+ \frac{0.5}{a_2}$, <i>if</i> $j = 1$,						
$t_j = \begin{cases} \frac{1}{0} \\ \frac{1}{a_j} \end{cases}$	$\frac{0.5+1+0.5}{5}, \frac{1}{a_j} + \frac{1}{a_j} + \frac{0.5}{a_{j+1}}, \text{, if } 2 \le j \le n-2,$						
$\left \frac{0}{\frac{0}{a_n}}\right $	$\frac{5+1}{5} + \frac{1}{a_n}$, if $j = n$.						

where a_{j-1}, a_j, a_{j+1} are the mid points of the fuzzy intervals A_{j-1}, A_j, A_{j+1} respectively. Based on the fuzzify historical enrollments obtained in Step 3, we can get the fuzzy logical relationship group (FLGR) as shown in Table IV.

Divide each interval derived in Step 2 into subintervals of equal length with respect to the corresponding frequency density of the interval. We have assumed thirteen partitions of the universe of discourse of the main factor fuzzy time series. Assuming that $0 \neq A_i$, $\forall A_i$, i = 1, 2, ..., 13. The proposed method satisfies the following axioms:

TABLE IV Third-Order Fuzzy Logical Relationship Groups [9]								
Group 1:	$X_2, X_2, X_3 \rightarrow X_5$	Group 10:	$X_9, X_5, X_5 \rightarrow X_4$					
Group 2:	$X_2, X_3, X_5 \rightarrow X_5$	Group 11:	$X_5, X_5, X_4 \rightarrow X_4$					
Group 3:	$X_3, X_5, X_5 \rightarrow X_6$	Group 12:	$X_5, X_4, X_4 \rightarrow X_7$					
Group 4:	$X_5, X_5, X_6 \rightarrow X_7$	Group 13:	$X_4, X_4, X_7 \mathop{\rightarrow} X_{10}$					
Group 5:	$X_5, X_6, X_7 \rightarrow X_{10}$	Group 14:	$X_4, X_7, X_{10} \rightarrow X_{11}$					
Group 6:	$X_6, X_7, X_{10} \rightarrow X_{10}$	Group 15:	$X_7, X_{10}, X_{11} \rightarrow X_{12}$					
C	V V V V	C	$X_{10}, X_{11}, X_{12} \rightarrow X$					
Group 7:	$\Lambda_7, \Lambda_{10}, \Lambda_{10} \rightarrow \Lambda_9$	Group 16:	X_{13} $X_{11}, X_{12}, X_{13} \rightarrow$					
Group 8:	$X_{10}, X_{10}, X_9 \mathop{\rightarrow} X_5$	Group 17:	X ₁₃					
		_	$X_{12}, X_{13}, X_{13} \rightarrow$					
Group 9:	$X_{10}, X_9, X_5 \rightarrow X_5$	Group 18:	X ₁₂					

Axiom 1: t(0) = 0 and t(1) = 1

(Boundary

Axiom 2: $t^{\alpha_i}(a_1, a_2, ..., a_n) \le t^{\alpha_i}(b_1, b_2, ..., b_n)$ provided $a_i \le b_i, i-1, 2, \dots, n$

$$u_i \leq b_i, i = 1, 2, ..., n$$

(Monotonicity)

Axiom 3: $t^{\alpha}(a_1, a_2, ..., a_n)$ is continuous

Axiom 4:
$$(A_k)_{\min} \le t^{\alpha}(a) \le (A_k)_{\max}$$
; $k = 1, 2, ..., 8$

(Symmetry)

Condition)

Axiom 5: $t^{\alpha}(a_1, a_2, ..., a_n) = a, \forall a \in [0, 1]$

(Idempotency)

In the next section, we have given comparison of different fuzzy time series forecasting methods.

IV. A COMPARISON OF DIFFERENT FORECASTING METHODS

In the following, Table V summarizes the forecasting results of the proposed method from 1972 to 1992, where the universe of discourse is divided into thirteen intervals based on frequency density based partitioning. In the following, we use the average forecasting error rate (AFER) and mean square error (MSE) to compare the forecasting results of different forecasting methods, where A_i denotes the actual enrollment and F_i denotes the forecasting enrollment of year i, respectively.

$$AFER = \frac{|A_i - F_i|/A_i}{n} \times 100\%$$
$$MSE = \frac{\sum_{i=1}^{n} (A_i - F_i)^2}{n}$$

In Table VI, we compare the forecasting results of the proposed method with that of the existing methods. From Table III, we can see that when the number of intervals in the universe of discourse is thirteen and the intervals are subpartitioned based on frequency density, the proposed method shows smallest values of the MSE and AFER of the forecasting results as compared to other methods of fuzzy time series forecasting. That is, the proposed method can get a

International Journal of Information, Control and Computer Sciences ISSN: 2517-9942 Vol:4, No:7, 2010

higher forecasting accuracy rate for forecasting enrollments than the existing methods.

Year	Enrollments (A_i)	Fuzzy Rule	FLRG	Forecast (F_i)	$A_i - F_i$	$(A_i - F_i)^2$	$\frac{\left A_{i}-F_{i}\right }{A_{i}}$
1971	13055	A ₁	A ₁ , A ₂	13579	-524	274778	0.040153
1972	13563	A_2	A ₁ , A ₂ , A ₃	13798	235	55344	0.017345
1973	13847	A_2	A1, A2, A3	13798	-49	2376	0.003520
1974	14696	A_3	A2, A3, A4	14452	-244	59427	0.016588
1975	15460	A_5	A4, A5, A6	15373	-87	7575	0.005630
1976	15311	A_5	A4, A5, A6	15373	62	3840	0.004047
1977	15603	A_6	A5, A6, A7	15623	20	400	0.001282
1978	15861	A_7	A ₆ , A ₇ , A ₈	15883	22	487	0.001391
1979	16807	A_{10}	A9, A10, A11	17079	272	73765	0.016160
1980	16919	A_{10}	A9, A10, A11	17079	160	25471	0.009433
1981	16388	A_9	A ₈ , A ₉ , A ₁₀	16497	109	11800	0.006629
1982	15433	A_5	A4, A5, A6	15373	-60	3604	0.003890
1983	15497	A_5	A4, A5, A6	15373	-124	15384	0.008004
1984	15145	A_4	A3, A4, A5	15024	-121	14599	0.007978
1985	15163	A_4	A3, A4, A5	15024	-139	19272	0.009155
1986	15984	A_7	A ₆ , A ₇ , A ₈	15883	-101	10188	0.006315
1987	16859	A_{10}	A9, A10, A11	17079	220	48223	0.013025
1988	18150	A_{11}	A_{10}, A_{11}, A_{12}	17991	-159	25136	0.008735
1989	18970	A_{12}	A ₁₁ , A ₁₂ , A ₁₃	18802	-168	28221	0.008856
1990	19328	A ₁₃	A ₁₂ , A ₁₃	18994	-334	111886	0.017306
1991	19337	A ₁₃	A ₁₂ , A ₁₃	18994	-343	117988	0.017764
1992	18876	A ₁₂	A11, A12, A13	18916	40	1600	0.002119

International Journal of Information, Control and Computer Sciences ISSN: 2517-9942 Vol:4, No:7, 2010

TABLE VI

A COMPARISON OF THE FORECASTING RESULTS OF DIFFERENT FORECASTING METHODS										
Year	Enrollments	Song Chissom Method [18]	Song Chissom Method [19]	Chen's [2]	Hwang, Chen & Lee's [8]	Huarng's [5]	Chen's [4]	Jilani and Burney [9]	Jilani, Burney and Ardil [10]	Proposed Method
1971	13055								14464	13579
1972	13563	14000		14000		14000			14464	13798
1973	13867	14000		14000		14000			14464	13798
1974	14696	14000		14000		14000	14500	14730	14710	14452
1975	15460	15500	14700	15500		15500	15500	15615	15606	15373
1976	15311	16000	14800	16000	16260	15500	15500	15614	15606	15373
1977	15603	16000	15400	16000	15511	16000	15500	15611	15606	15623
1978	15861	16000	15500	16000	16003	16000	15500	15611	15606	15883
1979	16807	16000	15500	16000	16261	16000	16500	16484	16470	17079
1980	16919	16813	16800	16833	17407	17500	16500	16476	16470	17079
1981	16388	16813	16200	16833	17119	16000	16500	16469	16470	16497
1982	15433	16789	16400	16833	16188	16000	15500	15609	15606	15373
1983	15497	16000	16800	16000	14833	16000	15500	15614	15606	15373
1984	15145	16000	16400	16000	15497	15500	15500	15612	15606	15024
1985	15163	16000	15500	16000	14745	16000	15500	15609	15606	15024
1986	15984	16000	15500	16000	15163	16000	15500	15606	15606	15883
1987	16859	16000	15500	16000	16384	16000	16500	16477	16470	17079
1988	18150	16813	16800	16833	17659	17500	18500	18482	18473	17991
1989	18970	19000	19300	19000	19150	19000	18500	18481	18473	18802
1990	19328	19000	17800	19000	19770	19000	19500	19158	19155	18994
1991	19337	19000	19300	19000	19928	19500	19500	19155	19155	18994
1992	18876		19600	19000	19537	19000	18500	18475	18473	18916
MSE	423027	775687	407507	321418	226611	86694	86694	82269	227194	41426
AFER	3.2238%	4.3800%	3.1100%	3.1169%	2.4452%	1.5294%	1.5294%	1.4064 %	2.3865%	1.0242%

V. CONCLUSION

In this paper, we have presented frequency density based partitioning of the historical enrollment data of the University of Alabama and applied improved fuzzy metric for forecasting. The proposed method belongs to the first order and time-variant methods. From Table VI, we can see that the AFER and MSE of the forecasting results of the proposed method are the smallest than that of the existing methods. In the future, we will extend the proposed method to deal with other forecasting problems based on fuzzy time series. We also will develop new methods for forecasting enrollments based on fuzzy parametric and semi-parametric approaches to get a higher forecasting accuracy.

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