Fuel Cell/DC-DC Convertor Control by Sliding Mode Method

Farzad Abdous

Abstract— Fuel cell's system requires regulating circuit for voltage and current in order to control power in case of connecting to other generative devices or load. In this paper Fuel cell system and convertor, which is a multi-variable system, are controlled using sliding mode method. Use of weighting matrix in design procedure made it possible to regulate speed of control. Simulation results show the robustness and accuracy of proposed controller for controlling desired of outputs.

Keywords— DC-DC converter, Fuel cell, PEM, Slides mode control.

I. INTRODUCTION

HUMAN'S life nowadays depends on electrical energy more than ever, thus the importance of generating energy increases significantly. Because of economical and ecological problems of fossil fuel power plants, developing renewable energies for generating electrical power attracts many attentions. Fuel cells are alternative power sources that have recently attracted a great deal of attention. Fuel Cell generates electricity from hydrogen by a chemical process and their emissions are water. Fuel Cell can serve as an emergency source of energy in the event of a long-term power outage. Fuel cell could be used as portable power systems. The fuel cells are finding use in every aspect because of their clean and efficient way of supplying electric power. The fuel cells are used in the standalone purposes at homes, hospitals, industries and now are finding their use in numerous vehicles. The fuel cells are replacing the batteries and in the current trend are becoming the most widely used resources.

Between different kinds of Fuel Cell, Proton Exchange Membrane (PEM) is the most famous one. In PEM electrochemical power is generated by passing gas of Hydrogen from anode and Oxygen from cathode. There is an electrolyte between Anode and Cathode which fasten the electric charge (Fig.1) [1].

Voltage of each cell in normal condition is 1.2 volt. For supporting high power and voltage couple of cells must be connected in series. Use of this system as a support of energy needs power control. Because of this a DC/DC convertor is usually connected between fuel cell and load. DC/DC

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convertors are used in applications which an average output voltage is needed; this voltage can be more or less than input voltage.
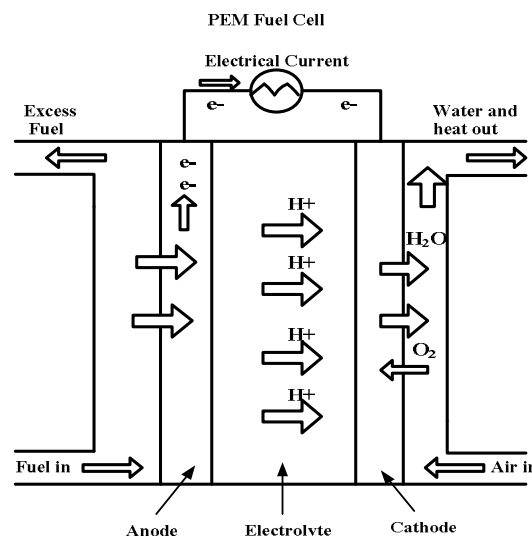

Fig. 1 Diagram of PEM Fuel Cell

Several switching strategies are introduced to gain such voltage [2-6].

Proposed models for fuel cell usually neglect the effect of DC/DC convertor. [7] Introduces state space model of DMFC, while [8] discusses such model for SOFC. In [1] state space model and transfer function are discussed for the combination of PEM fuel cell and related DC/DC convertor. Introduced model is a multivariable one with two input and output. In this model there exist two different control signals, one is continues and other is binary related to switching.

In nonlinear models it is possible to use different control methods such as: Feedback Linearization ،Fuzzy Control ، Robust Control and Sliding mode control [9-12]. In this paper sliding mode is chosen as control method.

This paper is organized as follows. Section II introduces proposed system and its model. Sliding mode controller is designed in section III. And finally, simulation results and conclusions are in sections IV and V respectively.

II. SYSTEM MODEL

A valid model for Fuel Cell and DC/DC convertor is introduced in [1]. Circuit of this model is illustrated in Fig.2.

Appendix 1 explains elements and equations between parameters of Fig.2.

In illustrated system i_l, v_c, and v_{CFC} are state variables. E_{Nernst} of Fuel Cell (controllable by pressure of H_2 and O_2) and switching signal d of DC/DC convertor are assumed to be control signals. Output current of Fuel Cell (i_L) and output voltage of DC/DC convertor (v_C) are considered as outputs of system. Defined variables are as followed:

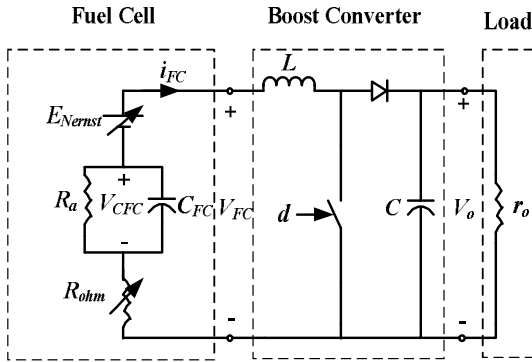$$\dot{x} = [x_1\, x_2\, x_3]^T = [i_L\, v_C\, v_{CFC}]^T \tag{1}$$

Fig. 2 circuit model of Fuel Cell and DC/DC convertor

$$u = [u_1\, u_2]^T = [E_{Nernst}\, d]^T \tag{2}$$

$$y = [y_1\, y_2]^T = [i_L\, v_C]^T \tag{3}$$

For reaching state space equations of proposed system, switching signal d considered to be 0 or 1.

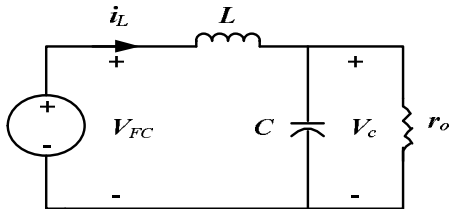In case of d equal zero, shown in Fig.3, state space equations are:

Fig.3 circuit of system for d=0

$$\begin{cases} \dot{i}_L = \frac{1}{L}(v_{CF} - v_C) \\ \dot{v}_C = \frac{1}{C}\left(i_l - \frac{v_C}{r_o}\right) \\ \dot{v}_{CFC} = \frac{1}{C}\left(i_l - \frac{v_C}{r_o}\right) \end{cases} \tag{4}$$

For d equal one, illustrated in Fig.4, it can be written:

$$\begin{cases} \dot{i}_L = \frac{1}{L}v_{CF} \\ \dot{v}_C = \frac{1}{C}\frac{v_C}{r_o} \\ \dot{v}_{CFC} = \frac{1}{C}\left(i_l - \frac{v_C}{r_o}\right) \end{cases} \tag{5}$$
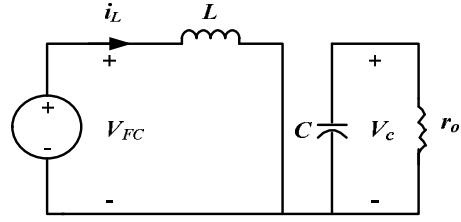

Fig. 4 circuit of system for d=1

With control signal u_2, (4) and (5) can be written as:

$$\begin{cases} \dot{i}_L = \frac{1}{L}(v_{CF} - v_C + v_C u_2) \\ \dot{v}_C = \frac{1}{C}\left(-\frac{v_C}{r_o} + \frac{2v_C}{r_o}u_2\right) \\ \dot{v}_{CFC} = \frac{1}{C}\left(i_l - \frac{v_C}{r_o}\right) \end{cases} \tag{6}$$

With considering KCL equation in Fuel Cell as it mentioned in (7) and substituting it in (6), we deduced:

$$v_{CF} = Nv_{FC} = u_1 + v_{CFC} - NR_{ohm}i_L \tag{7}$$

$$\begin{cases} \dot{i}_L = \frac{1}{L}(u_1 + v_{CFC} - NR_{ohm}i_L - v_C + v_C u_2) \\ \dot{v}_C = \frac{1}{C}\left(-\frac{v_C}{r_o} + \frac{2v_C}{r_o}u_2\right) \\ \dot{v}_{CFC} = \frac{1}{C}\left(i_L - \frac{v_C}{r_o}\right) \end{cases} \tag{8}$$

For converting (8) in standard form of (9), nonlinear functions $f(x)$ and $g(x)$ are defined as followed:

$$\dot{x} = f(x) + g(x)u \tag{9}$$

$$f(x) = \begin{bmatrix} -\frac{NR_{ohm}}{L} & -\frac{1}{L} & \frac{1}{L} \\ 0 & -\frac{1}{Cr_o} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{Cr_o} \end{bmatrix} \tag{10}$$

$$g(x) = \begin{bmatrix} \frac{1}{L} & \frac{x_2}{L} \\ 0 & \frac{2x_2}{r_o C} \\ 0 & 0 \end{bmatrix} \tag{11}$$

III. SLIDING MODE CONTROL

Objective control for proposed system is to deliver system outputs to desire values. Sliding mode method of control is used for this goal. In sliding mode method, control surfaces can be expressed as:

$$S_1 = x_1 - x_{1_r} \tag{12}$$

$$S_2 = x_2 - x_{2_r} \tag{13}$$

$$S = [s_1\, s_2]^T \tag{14}$$

x_{1_r} and x_{2_r} are desired values of desired values of state variables x_1 and x_2 respectively.

Lyapunov-like function is defined as follows:

$$V = \frac{1}{2}S^T W S \tag{15}$$

Based on (15) it can be written:

$$\dot{V} = S^T W \dot{S} \tag{16}$$

From (14), S' is:

$$\dot{S} = [\dot{x}_1 \ \dot{x}_2]^T = f_1(x) + g_1(x)u \tag{17}$$

$f_1(x)$ and $g_1(x)$ are sub matrices related to state variables x_1, x_2 from matrices $f(x)$ and $g(x)$.

With substituting (17) in (16) we deduced:

$$\dot{V} = S^T W f_1(x) + S^T W g_1(x)u \tag{18}$$

It is assumed that signal control u_e exists that can make control surface equal zero.

$$S\big|_{u_e} = 0 \tag{19}$$

From (18) and (19) it can be written:

$$0 = S^T W f_1(x) + S^T W g_1(x)u_e \tag{20}$$

Reducing (18) from (20) it can be written:

$$\dot{V} = S^T W g_1(x)(u - u_e) \tag{21}$$

Matrix $g_1(x)$ is considered as:

$$g_1(x) = [g_{11}(x) \ g_{12}(x)] \tag{22}$$

From (21) and (22) it can be written:

$$\dot{V} = S^T W g_{11}(x)(u_1 - u_{e_1}) + S^T W g_{12}(x)(u_2 - u_{e_2}) \tag{23}$$

u_{e1} and u_{e2} are signal control of u_1 and u_2 respectively.

Signal controls u_1 and u_2 must be in the way that:

$$\dot{V} < 0 \tag{24}$$

$$S^T W g_{11}(x)(u_1 - u_{e_1}) < 0 \tag{25}$$

$$S^T W g_{12}(x)(u_2 - u_{e_2}) < 0 \tag{26}$$

Considering conditions (25) and (26), u_1, u_2 can be expressed as:

$$\begin{cases} u_1 < u_{e_1} & S^T W g_{11}(x) > 0 \\ u_1 > u_{e_1} & S^T W g_{11}(x) < 0 \end{cases} \tag{27}$$

$$\begin{cases} u_2 < u_{e_2} & S^T W g_{12}(x) > 0 \\ u_2 > u_{e_2} & S^T W g_{12}(x) < 0 \end{cases} \tag{28}$$

Control signal u_1 depends on Fuel Cell's voltage, thus, is continues signal. Based on (27) it can be written:

$$u_1 = u_{e_1} - \mathbf{K}\, \mathbf{sgn}(S^T W g_{11}(x)) \tag{29}$$

Where K is a positive number and usually chosen to be small. For avoiding chattering other functions like *sat* can be used instead of *sgn*, so it can be written:

$$u_1 = u_{e_1} - \mathbf{K}\, \mathbf{sat}\left(\frac{S^T W g_{11}(x)}{\varphi}\right) \tag{30}$$

j defines margins of linear function *sat*.

u_{e_1} **can be concluded from (19):**

$$S\big|_{u_e} = 0 \rightarrow \dot{S}\big|_{u_e} = 0 \tag{31}$$

From (17) and (31) we deduced:

$$f_1(x) + g_1(x)u_e = 0 \tag{32}$$

Thus u_e is:

$$u_e = -\mathbf{inv}[g_{11}^T(x)g_{11}(x)]g_{11}^T(x)f_1(x) \tag{33}$$

u_2 is a control signal for the switch, as a result has a binary nature. Signal u_{e_2} is between 0 and 1, thus from (28) it can be written:

$$\begin{cases} u_2 = 0 & S^T W g_{12}(x) > 0 \\ u_2 = 1 & S^T W g_{12}(x) < 0 \end{cases} \tag{34}$$

Based on (31) u_2 is:

$$u_2 = \frac{1}{2}\left[1 - \mathbf{sgn}(S^T W g_{12}(x))\right] \tag{35}$$

For simplification matrix W is usually chose to be diagonal as follows:

$$w = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{36}$$

IV. SIMULATION

This section discusses operation of designed controller. For introduced model in section2, parameters are as indicated in appendix 2. As a result f(x) and g(x) can be expressed as:

$$f(x) = \begin{bmatrix} -499.14 & -13333 & 13333 \\ 0 & -1385.8 & 0 \\ 2.564.1 & 0 & -1385.8 \end{bmatrix} \tag{37}$$

$$g(x) = \begin{bmatrix} 13333 & 13333x_2 \\ 0 & 2771.5x_2 \\ 0 & 0 \end{bmatrix} \qquad (38)$$

From state equation (37) sub matrices $f_1(x)$ and $g_1(x)$ can be written:

$$f_1(x) = \begin{bmatrix} -499.14 & -13333 & 13333 \\ 0 & -1385.8 & 0 \end{bmatrix} \qquad (39)$$

$$g_1(x) = \begin{bmatrix} 13333 & 13333x_2 \\ 0 & 2771.5x_2 \end{bmatrix} \qquad (40)$$

From (30),(35), (36) , (40) we deduced:

$$u_1 = u_{e_1} - \mathbf{K} \, \mathbf{sat}\left(\frac{13333 \, a(x_1 - x_{1_r})}{\varphi}\right) \qquad (41)$$

$$u_2 = \frac{1}{2}\left[1 - \mathbf{sgn}\left(13333 \, a \, x_2[x_1 - x_{1_r}] + 2771.5 \, b \, x_2[x_2 - x_{2_r}]\right)\right] \qquad (42)$$

Parameters K, b, a and φ are based on Table I:

TABLE I:
CONTROL PARAMETERS VALUES

PARAMETER	VALUE
a	1
b	2
K	0.5
φ	1/800

With assuming outputs i_l and v_c to be 45.9 and 75.1 and must go to 46.99 and 75.35. Time response of system with designed controller is shown in Fig.5.

Effect of weighing parameters a, b on rate of response is noticeable. With choosing a , b as it indicated in Table II, time response in same conditions of previous case is as shown in Fig.6.

TABLE II:
CONTROL PARAMETERS VALUES

PARAMETER	VALUE
a	0.5
b	2

It can be concluded that with changing weighing parameters, it is possible to achieve different control characteristics especially in time response speed. Based on this some other methods like genetic algorithm can be used too, however this option is not considered in this paper.
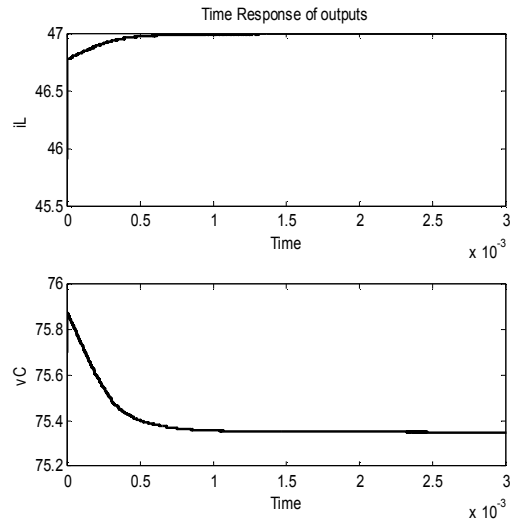

Fig. 5: Time response of outputs.
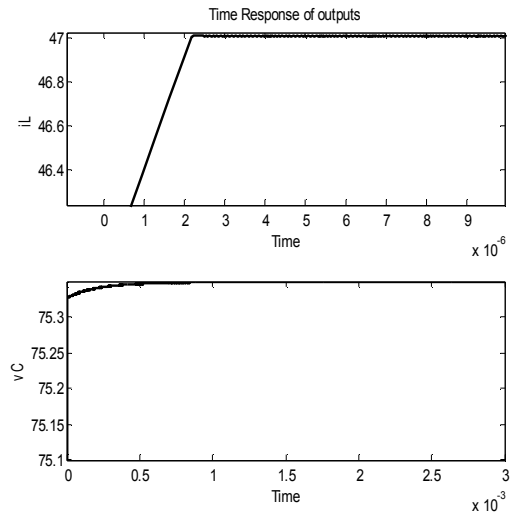Top: current i_l , Down: voltage v_c

Fig. 6: Time response of outputs.
Top: current i_l , Down: voltage v_c

V. CONCLUSION

Controlling related systems is an important aspect of control. This paper is considering a system composed of a Fuel Cell and its DC/DC convertor as a unique system and discusses about its control methods. Designed controller is based on sliding mode method, which has the capability to control output voltage and current of Fuel Cell at any level. Considering weighing matrix made the procedure more flexible. Simulation results show the effectiveness and robustness of proposed method.

APPENDIX

APPENDIX 1

PARAMETER	DEFINITION	EQUATION
E_{Nerst}	THE CELL THERMODYNAMIC POTENTIAL DROP	$\kappa_1 + \kappa_2 T + k_3 Tln(P_{H_2}) + k_4 Tln(P_{O_2})$
R_a	EQUIVALENT RESISTANCE OF THE FUEL CELL	$\dfrac{v_{act} + v_{CONC}}{I_{FC}}$
C_{FC}	EQUIVALENT CAPACITANCE OF THE FUEL CELL	---------------
R_{ohm}	OHMIC RESISTANCE OF TRANSFER IN FUEL CELL	$\dfrac{k_5 + k_6 I_{FC} + k_7 T^2 I_{FC}^{2.5}}{k_8 + k_9 I_{FC} + EXP\left(k_{10} + \dfrac{k_{11}}{T}\right) + k_{12}}$
L	INDUCTOR OF THE BOOST CONVERTER	--------------
C	CAPACITANCE OF THE BOOST CONVERTER	---------------
r_o	LOAD RESISTANCE	---------------

APPENDIX 2

PARAMETER	VALUE	PARAMETER	VALUE
N	33	k_4	2.1515E-5
T	338	k_5	0.0796
C_{FC}	15F	k_6	5.8831E-5
L	75μH	k_7	5.1192E-12
C	390μF	k_8	18.866
R_0	1.85	k_9	0.0739
k_1	1.4823	k_{10}	4.18
k_2	0.85E-3	k_{11}	1.2665E-3
k_3	4.13E-5	k_{12}	1E-4

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