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From Experiments to Numerical Modeling: A Tool for Teaching Heat Transfer in Mechanical Engineering

D. Zabala, Y. Cárdenas, and G. Núñez

***Abstract*—**In this work the numerical simulation of transient heat transfer in a cylindrical probe is done. An experiment was conducted introducing a steel cylinder in a heating chamber and registering its surface temperature along the time during one hour. In parallel, a mathematical model was solved for one dimension transient heat transfer in cylindrical coordinates, considering the boundary conditions of the test. The model was solved using finite difference method, because the thermal conductivity in the cylindrical steel bar and the convection heat transfer coefficient used in the model are considered temperature dependant functions, and both conditions prevent the use of the analytical solution. The comparison between theoretical and experimental results showed the average deviation is below 2%. It was concluded that numerical methods are useful in order to solve engineering complex problems. For constant k and h, the experimental methodology used here can be used as a tool for teaching heat transfer in mechanical engineering, using mathematical simplified models with analytical solutions.

***Keywords*—**Heat transfer experiment, thermal conductivity, finite difference, engineering education.

I. Introduction

TRANSIENT heat transfer is a complex matter which requires mathematical tools in order to get reliable results. For example, one of the applications of interest is the heating in furnaces of solid materials with different geometries in material science [1]-[7] or food processing and the heat exchanger modeling [8], [9]. For teaching numerical modeling of transient heat transfer problems, it is necessary to get a mechanism that facilitates the student approach to the physical phenomenon and its governing equations, together with the solutions of these. Trying to improve education in engineering, Molina [10] revised the main factors having influence in mechanical engineering education in Latin America, concluding that professors with no enough pedagogic formation make a mechanical transmission of the knowledge and the fragmented study programs without a clear relationship between disciplines and courses, are responsible for having as final product a dependant graduate student with low creativity. Other authors [11], [12] reviewed learning theories in order to improve the educational software development. The use of videos, educational software and internet was evaluated by Pineda *et al.* [12], determining higher motivational level and student performance in the teaching-learning of a physics course. Considering that "learning to do" is one of the four pillars of the education [13], an easy experiment can be used in order to illustrate the problem of transient heat transfer in a metallic cylindrical probe and its boundary conditions. This experiment combines different disciplines like heat transfer, material science, thermal treatment and numerical methods. The temperatures obtained from the experimental measurements are compared with the temperatures predicted by the mathematical model. In this way, student experience and relationship between disciplines can be established for understanding the physical phenomenon and the mathematical tool for solving the problems.

D. Zabala is with Universidad de Carabobo, CIMEC, Escuela de Ingeniería, Mecánica, Av. Universidad, Naguanagua, Edo. Carabobo, código postal 2005, Venezuela (e-mail: dzabala@ uc.edu.ve).

Y. Cárdenas was with Universidad de Carabobo. He is now with Metalúrgica Carabobo, Zona Industrial Sur, Valencia, Venezuela (e-mail: yonathan16_7@ hotmail.com).

G. Núñez was with Universidad de Carabobo. He is now free agent (e-mail: g_a_n_r_@ hotmail.com).

II. Mathematical Model

Heat transfer conduction problems are described by the heat equation. For cylindrical coordinates, the general heat equation is (1).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(T)r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k(T)\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right)+\dot{q}=\rho C_p\frac{\partial T}{\partial t} \quad (1)$$

The infinite cylinder is an idealization which allows assuming one-dimensional conduction in radial direction [10]. It is an adequate approximation if the aspect ratio for the cylinder is $L/r_0 \geq 10$. With this assumption and considering no internal generation, equation (1) transforms into equation (2):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k(T)r\frac{\partial T}{\partial r}\right)=\rho C_p\frac{\partial T}{\partial t} \quad (2)$$

To solve equation (2), two boundary conditions and an initial condition are needed. For an infinite cylinder (Fig.1) with an initial uniform temperature T_i, the boundary conditions are shown in equations (3) and (4).

$$\left.\frac{\partial T}{\partial r}\right|_{r=0}=0 \quad (3)$$

$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_0} = h\left(T(r_0,t)-T_\infty\right) \tag{4}$$

The equation (2) solution for uniform initial temperature T_i and the boundary conditions (3) and (4) can be solved numerically. There is an analytical solution [15] if the equation (2) is transformed to a dimensionless expression (5) in the case of k and h are constants. Boundary conditions are transformed too.

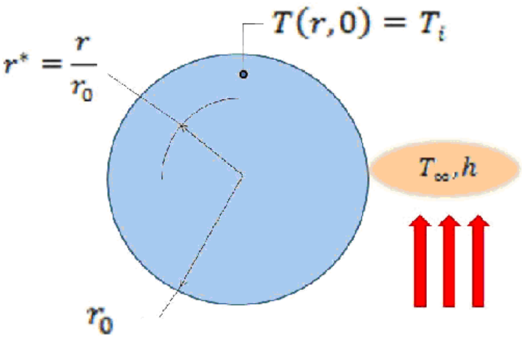

Fig. 1 Infinite cylinder with uniform initial temperature, in sudden convection conditions. [14]

$$\frac{\partial^2\theta}{\partial R^2} = \frac{\partial \theta}{\partial Fo} \tag{5}$$

The variables in equation (5) are:

$\theta = \frac{T - T_\infty}{T_i - T_\infty}$, dimensionless temperature

$R = \frac{r}{r_0}$, dimensionless radius

$Fo = \frac{\alpha t}{r_0{}^2} = \frac{kt}{\rho C_p r_0{}^2}$, dimensionless time

The solution for the dimensionless temperature inside the cylinder is shown in equation (6).

$$\theta = \sum_{n=1}^{\infty} A_n \exp(-\lambda_n^2 Fo) J_0(\lambda_n R) \tag{6}$$

In equation (6) λ_n are the roots of the transcendental equation (7), where $Bi=hr_0/k$ and J_0 y J_1 are the Bessel functions of the first kind. The variable A_n is expressed by equation (8). For AISI 1045 steel used in this experiment, the average values for *k* and *h*, are k= 40.5 W/mK and h=7.19 $W/(m^2K)$, obtained from the general values shown in equation (9) and Table 1. Other AISI 1045 steel properties are Cp=434 J/(kgK) and density, $\rho=7870$ kg/m^3.

$$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = Bi \tag{7}$$

$$A_n = \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} \tag{8}$$

For the numerical solution of equation (2) by the finite difference method [16], the thermal conductivity is expressed by equation (9), where *k* [W/mK] and *T* [K].The *h* (T) values are obtained from Table 1. The values in Table 1, were calculated for an average air temperature T_∞=100°C, using the Churchill and Chu correlation [17].

$$k(T) = 0{,}005T + 39 \tag{9}$$

TABLE I
FREE CONVECTION COEFFICIENT AS A FUNCTION OF SURFACE TEMPERATURE (T_S) IN A HORIZONTAL CYLINDER

Ts (K)	**h (W/m^2K)**
298	9.196
303	9.041
308	8.88
313	8.71
318	8.531
323	8.342
328	8.133
333	7.887
338	7.623
343	7.338
348	7.023
353	6.666
358	6.244
363	5.709
368	4.929

III. EXPERIMENTAL SETUP

A probe, with cylindrical geometry (Fig.2) with initial temperature T_i equals to environment temperature, is inserted in a preheated oven to a fixed temperature, T_∞.

Fig. 2 Thermocuple J type, attached to the cylindrical probe

The probe surface temperature is measured by a thermocouple and recorded into the computer during the test time by a data acquisition card (Fig.3). The dimensions of the probe are length (L= ½" (1.27 cm) and diameter (D= 6.5" (16.5 cm)). With these dimensions, the aspect ratio is L/r_0 =26, then the infinite cylinder assumption is valid.

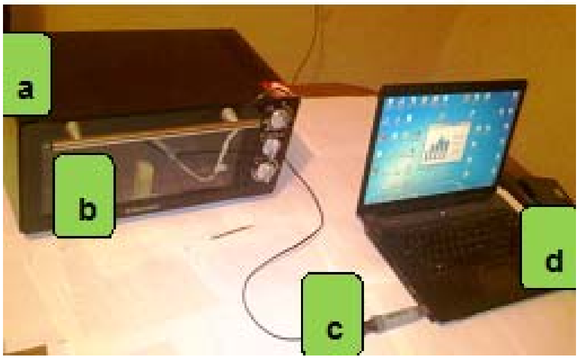

Fig. 3 Experiment. a.- Domestic oven, b.- Thermocouple and cylinder, c.-USB data card, d.- Computer

IV. Results

Results are showed in figures 4 to 6. The variations in the experimental curves of surface temperature are shown in figure 4. The reason for the variations is the initial temperature is different because T_i is the laboratory environmental temperature and it changes every day, because there is not temperature control system. On the other hand, the probe is introduced inside the oven when the oven temperature reaches the desired value, T_∞. The door opening produces a temperature descent and there is a fluctuation during the test in the oven temperature (Fig. 5).

In figure 6**,** the comparison between experimental temperature values and the ones obtained by both solution methods (numerical and analytical) is done. It can be observed that both theoretical methods represent well the behavior of the temperature in the cylinder surface.

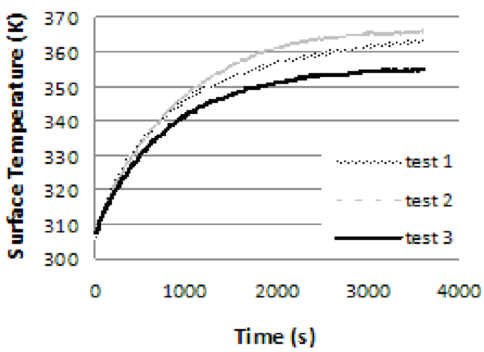

Fig. 4 Repeatability of the heating process in the cylinder. Test 1 (T_∞ = 363K, Ti= 307.1 K), test 2 (T_∞ = 370K, Ti= 306.2 K), test 3 (T_∞ = 360K, Ti= 307.3 K)

The numerical solution has the best adjustment (deviation 1.4% respect to experimental values) and it is because it takes into account the variation of *k* and *h* with temperature, which affects the process even with the small variation observed for this material in particular. For analytical solution deviation is 1.9%.

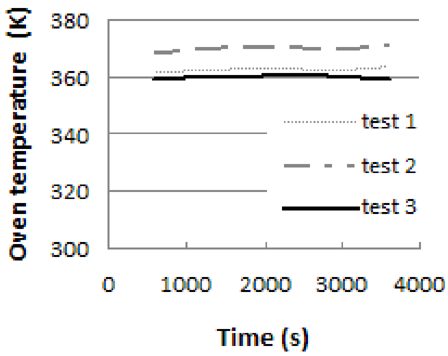

Fig. 5 Variations in oven temperature

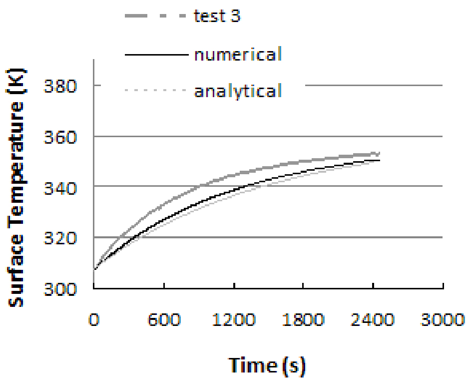

Fig. 6 Experimental and theoretical surface temperature

V. Conclusion

The results of both mathematical models agree with the experimental values of the cylinder surface temperature. This simple application shows to the student the relationship between the different disciplines in mechanical engineering like heat transfer, differential equations, thermal treatments and numerical methods.

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