International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:10, No:10, 2016

Frequency Response of Complex Systems with Localized Nonlinearities

E. Menga, S. Hernandez

Abstract**—**Finite Element Models (FEMs) are widely used in order to study and predict the dynamic properties of structures and usually, the prediction can be obtained with much more accuracy in the case of a single component than in the case of assemblies. Especially for structural dynamics studies, in the low and middle frequency range, most complex FEMs can be seen as assemblies made by linear components joined together at interfaces. From a modelling and computational point of view, these types of joints can be seen as localized sources of stiffness and damping and can be modelled as lumped spring/damper elements, most of time, characterized by nonlinear constitutive laws. On the other side, most of FE programs are able to run nonlinear analysis in time-domain. They treat the whole structure as nonlinear, even if there is one nonlinear degree of freedom (DOF) out of thousands of linear ones, making the analysis unnecessarily expensive from a computational point of view. In this work, a methodology in order to obtain the nonlinear frequency response of structures, whose nonlinearities can be considered as localized sources, is presented. The work extends the well-known Structural Dynamic Modification Method (SDMM) to a nonlinear set of modifications, and allows getting the Nonlinear Frequency Response Functions (NLFRFs), through an 'updating' process of the Linear Frequency Response Functions (LFRFs). A brief summary of the analytical concepts is given, starting from the linear formulation and understanding what the implications of the nonlinear one, are. The response of the system is formulated in both: time and frequency domain. First the Modal Database is extracted and the linear response is calculated. Secondly the nonlinear response is obtained thru the NL SDMM, by updating the underlying linear behavior of the system. The methodology, implemented in MATLAB, has been successfully applied to estimate the nonlinear frequency response of two systems. The first one is a two DOFs spring-mass-damper system, and the second example takes into account a full aircraft FE Model. In spite of the different levels of complexity, both examples show the reliability and effectiveness of the method. The results highlight a feasible and robust procedure, which allows a quick estimation of the effect of localized nonlinearities on the dynamic behavior. The method is particularly powerful when most of the FE Model can be considered as acting linearly and the nonlinear behavior is restricted to few degrees of freedom. The procedure is very attractive from a computational point of view because the FEM needs to be run just once, which allows faster nonlinear sensitivity analysis and easier implementation of optimization procedures for the calibration of nonlinear models.

Keywords**—**Frequency response, nonlinear dynamics, structural dynamic modification, softening effect, rubber.

Edoardo Menga is with the Component Loads & Component Dynamics – EGLCG, Airbus Operations SL, Getafe, Spain (e-mail: edoardo.menga@airbus.com).

Santiago Hernandez is with the School of Engineering, Mechanics of Structures, University of Coruña, Spain (e-mail: santiago.hernandez@udc.es).

I. Introduction

ANALYTICAL simulations, and in particular FEMs, are of very commonly used in many industrial fields to optimize products in terms of quality, time to delivery and costs. One of the key aspects is the tendency to replace, when possible, the experimental tests, which usually are expensive, with virtual-tests based on high-fidelity simulations. On the other side, as the number of components in the assembly increases, the calculation quality declines because the connection mechanisms between components are not represented sufficiently. That is particularly true in the case of structural dynamic simulations, where each component can be often considered acting in a linear field, but when the components are jointed together, nonlinear behavior appears because of interfaces. Commercial FE programs (i.e. Nastran, ANSYS, Abaqus, etc.) are able to run time-domain nonlinear simulations, and in spite of the percentage of nonlinear DOFs, maybe just few over thousands of linear ones, the whole structure is treated as nonlinear. Consequently, the computational costs become much higher than the equivalent linear case. The situation is even more complicated if the nonlinear results are required in the frequency-domain because frequency response solutions are based on a linear modal decomposition approach. If NLFRFs are required, one possible approach is running, for each desired frequency response point, a time domain analysis having the harmonic excitation force at that frequency, obtain the time response, and store the steady-state peak. Hence the collection of the so obtained, steady-state peaks, allows to build the NLFRFs. From the above considerations, the need to make these types of calculations more efficient is really deemed. The strategy for challenging the problem, takes advantages from the following concepts:

- ✓ *Localized nonlinearity*: the nonlinear behavior is restricted to few degrees of freedom out of thousands of linear ones. It can be simulated by the use of nonlinear lumped elements, which connect pairs of DOFs. Hence the nonlinear properties can be expressed in terms of stiffness and damping nonlinear properties.
- ✓ *Steady-state conditions*: the frequency response analysis, for definition, considers the response of the structure in steady-state conditions. In literature it is also called 'harmonic response', because the response is expected to be harmonic and at the same frequency of the harmonic excitation. It means that the response in the transient period can be avoided during the calculation and only the steady-state phase is of interest.
- ✓ *Modal Decomposition Approach*: as consequence of the

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:10, No:10, 2016

first two concepts, it can be said that the NLFRFs can be obtained updating the LFRFs, because the 'nonlinear modifications' do not affect significantly the linear modal base.

The methodology presented in this work extends the SDMM, usually applied to a linear set of lumped modifications [1], [2], to the nonlinear field.

II. SDMM

The objective of this chapter is to describe the SDMM, which is defined as the procedure which permits one to evaluate the impact of a set of changes on the structural dynamic behavior, without the need to continuously re-run the FEM Model. The modified dynamic behavior can be expressed as function of the baseline FEM Dynamic Database and the set of modifications.

Many authors have formulated and completed the linear theoretical problem, highlighting that the method becomes particularly efficient if into the modification lumped elements are involved. Lumped modifications consist of whatever relationship between two DOFs, both of the structures or one DOF belonging to the structure and another one belonging to an external fixed point. Usually the relationship is expressed as a combination of lumped masses, spring and damper elements. A very good theoretical approach can be found in the work of [1]. Significant effort and work can be found in the publications of [2], [3]. In the following two paragraphs, first the Linear SDMM is completed and secondly the NL SDMM is formulated.

A. Linear SDMM

In this paragraph, the theoretical base of the linear SDM is given, because the understanding of the nonlinear algorithm requires the deep understanding of the linear one. The objective of the SDMM is to develop a mathematical formulation, which allows one to express the modified dynamic behavior as function of the baseline FEM Dynamic Database and a set of modifications. This set of modification is defined as the Modification Matrix.

The baseline FEM Dynamic Database can be expressed in terms of the Modal Matrix [3] or Frequency Response Functions [1], [2]. In this study the Modal Matrix Database is considered, in which case the Eigenvalues, Eigenvectors of the baseline model need to be available. The Modal Matrix needs to include the results of all the DOFs of interest, including those one where the Modification Matrix is applied.

The dynamic equilibrium of a structure in the frequency domain can be expressed as:

$$[-M\omega^2 + j(\omega C + G) + K]u(\omega) = F(\omega) \tag{1}$$

M is the mass matrix, C and G, respectively, the viscous and the structural damping matrices, K the stiffness matrix, u the field of displacements and F the force applied. If the field of displacements is expressed as a linear combination of the eigenvectors associated with the system (1), the latter is transformed from the physical coordinates to the modal coordinates by means of the following transformation:

$$u(\omega) = FRF(\omega) = \Phi q(\omega) \tag{2}$$

The field of displacements is defined Frequency Response Functions (FRFs).

Considering now, the modal matrix Φ normalized with respect to the mass, and multiplying all the terms for the transpose Φ^T the system (1) becomes finally:

$$[\widetilde{R} + j(\omega\widetilde{C} + \widetilde{G})]q(\omega) = \widetilde{F}(\omega) \tag{3}$$

where

$$\widetilde{R} = [-\Phi^T M \Phi \omega^2 + \Phi^T K \Phi] = \left[-I\omega^2 + \Lambda\right] \tag{4}$$

The second relationship of equation (XX) is valid only in the case that the Modal Matrix is Mass Normalized. The mass normalization gives the advantage that from the baseline FEM only the spectral data (eigenvalues, eigenvectors) concerning the DOFs we are interested in, are required. Therefore, [I] is the unitary matrix, [$\widetilde{C}$] and [$\widetilde{G}$] are the normalized damping matrices, [Λ] is the normalized stiffness matrix, whose terms are the square of the eigenvalues, and $\widetilde{F}(\omega)$ the vector of modal forces.

Finally, the FRFs are obtained as:

$$FRF(\omega) = \Phi q(\omega) = \Phi[\widetilde{R} + j(\omega\widetilde{C} + \widetilde{G})]^{-1}\widetilde{F}(\omega) \tag{5}$$

Let us consider now a set of dynamic modifications, involving mass and stiffness variations of the baseline terms, expressed by the Modification Matrix:

$$\Delta R(\omega) = \left[-\omega^2 \Delta M(\omega) + \Delta K(\omega)\right] \tag{6}$$

If the modification matrix is included in the system (1), following the previous steps it is obtained:

$$FRF_{\mathrm{mod}}(\omega) = \Phi[\widetilde{R} + \Delta\widetilde{R} + j(\omega\widetilde{C} + \widetilde{G})]^{-1}\widetilde{F}(\omega) \tag{7}$$

where

$$\Delta\widetilde{R} = \Phi^T \Delta R(\omega)\Phi \tag{8}$$

The Modified FRFs, in terms of displacements, are obtained by means of (7). If the FRFs are escalated to the magnitude of the input force, or, which is equivalent, in the case of unitary force input, the FRFs take the name of Transfer Functions (TFs). In the linear case, in spite of the magnitude of the input, the TFs are constant values because they express the relationship between the outputs and the input.

It is clear that, the knowledge of the FE Baseline Modal Matrix and the definition of the Modification Matrix, allows the direct calculation of the modified field of displacements without any need to re-run the FEM.

Looking at (7), it is worth pointing out that the linear SDM formulation allows dealing with frequency-dependent lumped elements. The use of the frequency dependent Modification Matrix in the frequency domain is still a linear problem. Also, it should be noted that the size of the FEMs, in terms of DOFs, is not an issue, in fact, the baseline Modal Matrix can be restricted to few DOFs. This set must, at least, include the DOFs where the change is required and those ones where the dynamic response needs to be evaluated. More details on this last point are given in references [1]-[3].

B. Non-Linear SDMM

The frequency-dependency of the modification matrix does not affect the linearity of the problem; therefore, the nonlinear lumped elements are considered those elements whose constitutive equations express dependency on the relative amplitude. In the linear case, the Transfer Function (TF), defined for each DOF as the ratio between the output acceleration, or displacement, and the input force, is an invariant of the system. But, if the structure acts nonlinearly, the TF depends on the magnitude of the input force. Hence in the nonlinear case, even if, the same symbols are still used, it is more correct to speak about the NLFRFs Matrix instead of the TF Matrix. The NL TF Matrix can be obtained later, dividing the NLFRFs Matrix by the input force value. Evidently, the NL TF Matrix is not unique and is dependent on the magnitude of the input force. Equations (1) and (2) are still valid but if we define 'u' as the relative displacement experimented by the lumped elements, the Modification Matrix, in the nonlinear case looks like:

$$\Delta R(\omega,u)=\left[-\omega^2\Delta M(\omega,u)+\Delta K(\omega,u)\right] \tag{9}$$

It means that the system described by (7) becomes non-linear

$$FRF_{NL,\mathrm{mod}}(\omega,u)= \Phi[\widetilde{R}+\Delta\widetilde{R}(\omega,u)+j(\omega\widetilde{C}+\widetilde{G})]^{-1}\widetilde{F}(\omega) \tag{10}$$

The nonlinear system (10), where all the nonlinear terms are included in the Modification Matrix, requires an iterative algorithm in order to be solved. The convergence of the solution is assured when the relative displacements of the lumped modified elements reach stabilized values. For each values of interest in the frequency domain, the relative displacements of all lumped elements involved in the Modification Matrix, need to be evaluated. At the end of each frequency step, the peaks of the steady-state time-domain responses are stored in order to build-up the NL FRFs response.

The NL SDM allows the calculation of the nonlinear response without any need to re-run the FEM. The nonlinear response is calculated thru updating the underlying linear FRFs. It evidently means a drastic reduction of the computational costs, allowing faster sensitivities analysis and easier implementation of inverse methods for optimization and calibration of the FEMs. To test the method, in the next chapter, a two DOFs system is analyzed.

III. NL SDMM: Two- DOFs System Example

The reliability and robustness of NL SDM algorithm are tested by the following example.

Let us consider the following 2-DOFS system (Fig. 1), whose linear behavior is update thru the bilinear stiffness element (Fig. 2):

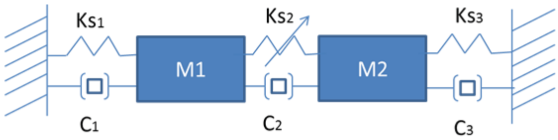

Fig. 1 Two- DOFs system

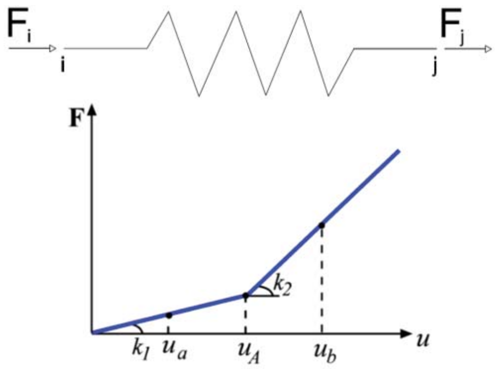

Fig. 2 Bilinear Stiffness Element

The Modification Matrix associated with the spring Ks2 is:

$$\Delta K_{elem}(\omega,u)=\begin{bmatrix}\Delta K_{elem}(\omega,u) & -\Delta K_{elem}(\omega,u)\\ -\Delta K_{elem}(\omega,u) & \Delta K_{elem}(\omega,u)\end{bmatrix} \tag{11}$$

Being the constitutive equation of the element:

$$\begin{cases}\Delta K_{elem}(\omega,u)=k_1-k_0 & \text{if} \quad u_i-u_j=u_a<=u_A\\ \Delta K_{elem}(\omega,u)=k_{equ}-k_0 & \text{if} \quad u_i-u_j=u_b>u_A\\ \text{where:} & \\ k_{equ}=\dfrac{k_1\cdot u_A+k_2\cdot(u-u_A)}{u} & \end{cases} \tag{12}$$

Even if not strictly required, it is expected that the underline linear element has the stiffness value k_0 , which is not so far from the nonlinear values. For instance, k_0 could be the mean value between k_1 and k_2 . The values of the problem are given in Table I.

The NL FRFs are calculated updating the stiffness Ks2 to a nonlinear behaviour. It is considered bilinear with stiffening effect. The underline linear value is 20 [N/mm] and the second branch has a value of 40 [N/mm]. The relative displacement

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:10, No:10, 2016

Ua, which defines the boundary between the two branches, has a value of 4 mm.

TABLE I
TWO DOFS SYSTEM: VALUES OF PARAMETERS

PARAMETER	VALUE
M1 [Ton]	3.00E-2
M2 [Ton]	5.00E-2
Ks1 [N/mm]	20
Ks2 [N/mm]	20
Ks3 [N/mm]	80

The problem is formulated in terms of both: time-domain and frequency-domain equations.

The time-domain dynamic problem is solved by mean of ODE45 already included in MATLAB [6], which uses an explicit Runge-Kutta integration procedure of the State-Space equations.

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t) \quad (13)$$

The State-Space formulation allows the solution of N DOFs system with N second order differential equations to 2*N first order differential equations. The first order form of the equations of motion is known as *State Space Form*. Details on the numerical implementation of state-space equations can be found in [5], also a very good explanation, with an intuitive and pragmatic approach to this formulation, is carried out in [6]. With no further details, because of the scope of this paper, the State-Space problem can be formulated as:

$$\dot{p}(t) = Ap(t) + BF(t) \quad (14)$$

being

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}$$

The validation of the NL SDM is completed in three main steps:

1. First the comparison between the structural response obtained in the frequency domain and the FRFs obtained collecting the peaks of the steady-state part of the time-domain response (Fig. 3) is done. The results, which are in very good agreement, are shown in Fig. 4. It is worth to mention that the damping formulation in the frequency domain and the damping matrix used in the time-domain equations need to be equivalent in order to compare the results. In order to define such equivalency, the damping matrix in the time domain is considered proportional to the mass matrix thru the coefficient α and to the stiffness matrix thru the coefficient β. The equivalent modal damping is:

$$\xi(\omega_i) = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \quad (15)$$

In this exercise, the value used for α is 1.5 and β is set to zero.

The modal matrix, normalized to the mass, and the normal modes are:

$$\Phi = \begin{bmatrix} -5.07 & 2.76 \\ -2.14 & -3.93 \end{bmatrix}; \; \lambda = \begin{bmatrix} 5.16 & 0 \\ 0 & 7.60 \end{bmatrix}$$

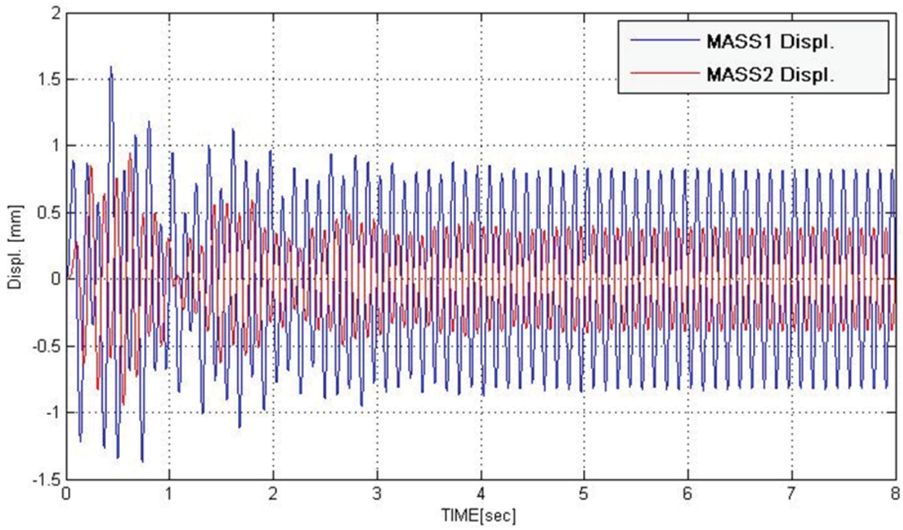

Fig. 3 Time Domain results for a given frequency value

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:10, No:10, 2016

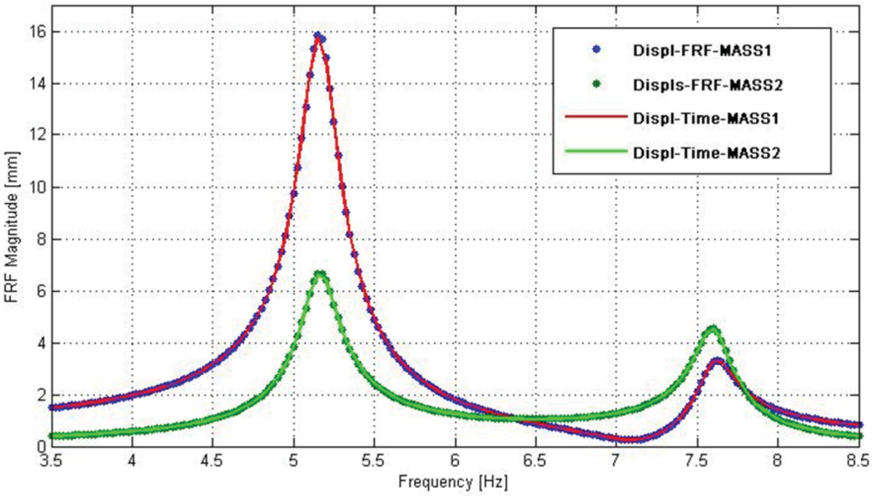

Fig. 4 Frequency Domain vs. Steady-State Time-Domain

2. Secondly, the NLFRFs are obtained thru the resolution of the State-Space Time Domain equations taking into account the nonlinear stiffness formulation and, as in the previous point, collecting the steady-state peaks. The results are shown in Fig. 5. The effect of having a bilinear stiffness, which increases in the second branch, has two effects: the magnitude of the peaks at resonance decreases and the frequency of resonance shifts to higher values. This is the typical stiffening effect.

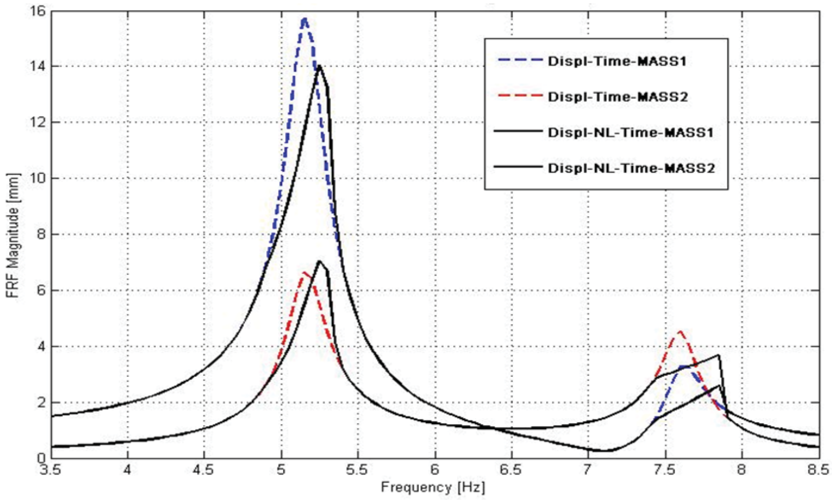

Fig. 5 Linear vs. Nonlinear Frequency Response

3. Finally, the NLFRFs are obtained by mean of the NL SDM formulation. The State-Space equations in this case are formulated in modal coordinates. The Modification Matrix updates the unitary mass matrix and the eigenvalues stiffness matrix. Hence, there is no need to re-run the FEM because the nonlinear behavior is obtained as an 'update' of the underlying linear behavior. In this case, the state variables 'p' (16) are obtained in modal coordinates, and therefore a back transformation to the physical coordinates is required by mean of the spectral matrix (17).

This back-transformation, modal to physical coordinate, needs to be included in the NL SDM iterative procedure, being the nonlinear properties of the elements based on their relative physical displacements.

Fig. 6 shows the good agreement of the results.

$$\dot{p}(t) = Ap(t) + B\widetilde{F}(t) \quad \textit{being} \quad A = \begin{bmatrix} 0 & I \\ -\Lambda & -\widetilde{C} \end{bmatrix} \textit{ and } B = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{16}$$

$$s(t) = \Phi p(t) \tag{17}$$

In the next chapter, the NL SDMM is tested on a real

International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:10, No:10, 2016

industrial application. A much more complex model is used and the underlying linear Modal Database is extracted from the FEM built in NASTRAN.

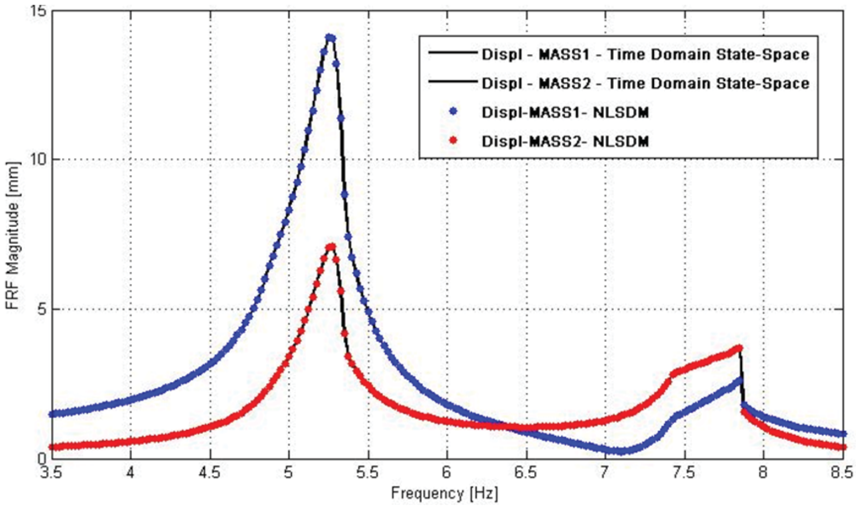

Fig. 6 Non Linear SDM Method results

IV. NL SDMM Applied to the APU Suspension System FE Model

For testing the strength and robustness of the method on a real industrial application, the FEM of the Auxiliary Power Suspension System is considered. The nonlinear behavior of this system is characterized by the rubber mounts, which present a dynamic stiffness with a softening effect. In the NASTRAN FEM, these elements are simulated by linear springs. First, the Eigenvalues and Eigenvectors are extracted by solving the normal modes SOL103 in NASTRAN, also the LFRFs are calculated, solving the system in (7) and considering a null Modification Matrix. This step is completely equivalent in solving the problem in NASTRAN SOL111. Secondly, the underlying linear model is updated through the NL SDM Method, introducing in the system nonlinear springs instead of linear ones. The NLFRFs are obtained and the nonlinear dynamic behavior of the structure is analyzed to check the coherency with the type of nonlinearity introduced in the system.

A. Description of the APU Suspension System Model

The focus of this study is on the interfaces between the Auxiliary Power Unit (APU) and its Suspension System. The APU is installed in the aircraft Tail-Cone by its Suspension System, which has a double purpose: to sustain the inertia loads at which the APU is submitted and to isolate the airframe from the APU's vibrations.

The Suspension System consists of three principals' subassembly called:

- ✓ Left-Hand: three rods, three APU lugs on the structure side, one Rubber Mount
- ✓ Right -Hand: two rods, two APU lugs on the structure side, one Rubber Mount
- ✓ Aft –Hand: two rods, two APU lugs on the structure side, one Rubber Mount

Each rubber mount is done by a steel isolator housing with an elastomeric inside.

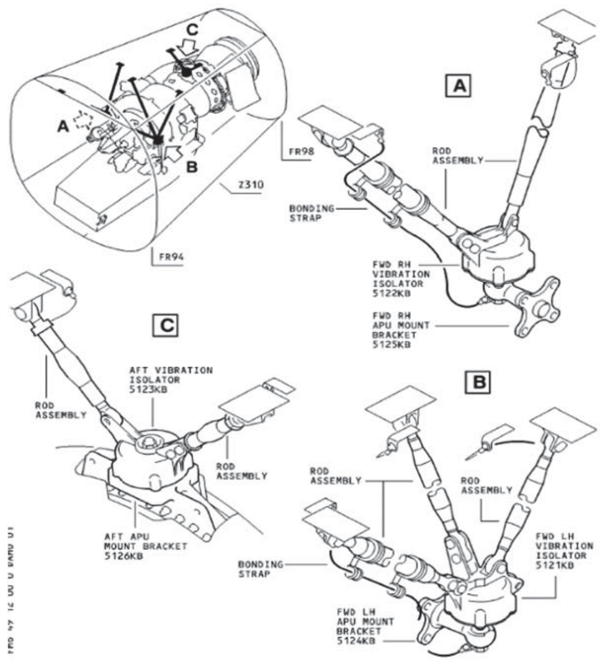

Fig. 7 APU Suspension System

Experimentally, it is seen that the dynamic stiffness of the rubber mounts depends both on the frequency and on the dynamic amplitude. Fig. 7 shows an example of the trend of the dynamic stiffness of the rubber mount when it is stretched by an axial force.

In the FEM, the rubber mounts can be modelled by means of spring/damper elements connecting a pair of nodes. Each rubber mount is described by three lumped spring/damper elements, one for the axial direction and other two for the radial one. In this work the focus is on the stiffness properties, the damping is assumed to be fixed and its values are calculated from the area of the hysteretic loop. It is introduced in the FEM as localized structural damping. Interesting experimental research to characterize lumped nonlinear properties can be also found in [4].

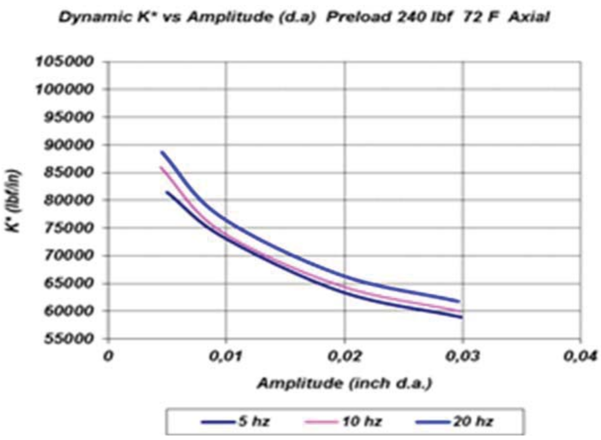

Fig. 8 Rubber Mounts dynamic stiffness

A full FE aircraft model is used in order to show that the methodology does not present any limitation concerning the size of the model. In Figs. 9 and 10 some details can be seen. The linear behavior of the rubber mounts is introduced in NASTRAN by CBUSH cards. The input load is applied to the APU Centre of Gravity in vertical (Z+) direction, see Fig. 10. The APU is modelled like a heavy mass, using the CONM2 card of NASTRAN.

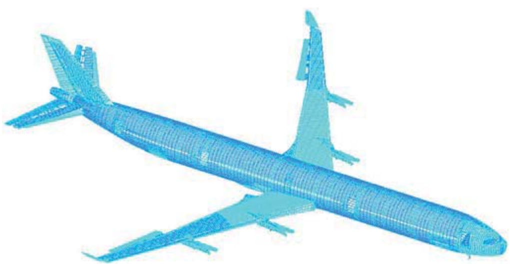

Fig. 9 Full FE A/C Model

B. Nonlinear 'Update' of the Rubber Mounts Stiffness

According to Fig. 8, the rubber mounts present a nonlinear behavior characterized by a softening effect: the higher the relative displacements, the lower the dynamic stiffness.

The NL SDMM is used in order to include this nonlinear behavior and see the effect on the FRFs of the model. The frequency range of interest is between 10 Hz and 20 Hz, where the APU Suspension System Modes are. Experimentally, it is also known that the rubber mount dynamic behavior affects the response of the system above 12-14 Hz. Having a softening effect, some peaks of the FRFs are expected moving to lower frequencies.

Before running the NL analysis, the linear FRFs obtained from both codes, using the same modal base, are checked. In the Fig. 11, the frequency response has been compared in all the DOFs of interest; as example Fig. 11 shows, the response of the pair of DOFs which defines the spring in the Z direction of the forward rubber mount. The very good agreement between both codes, ABAQUS and NL SDMM, implemented in MATLAB, can be seen.

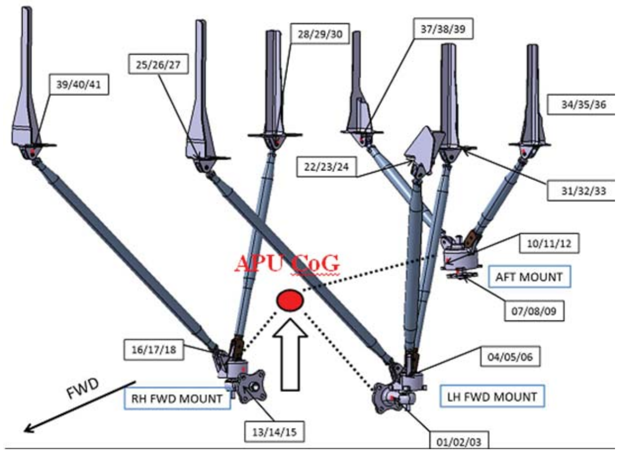

Fig. 10 Suspension System Model

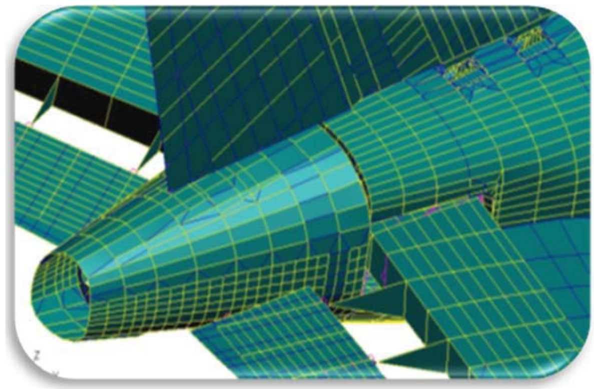

Fig. 11 Detail of FEM Tail-Cone

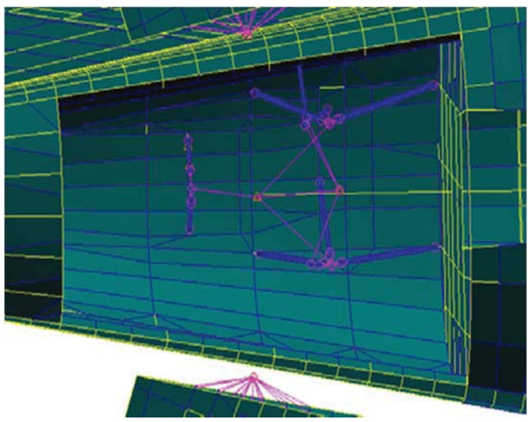

Fig. 12 Detail of APU Suspension System

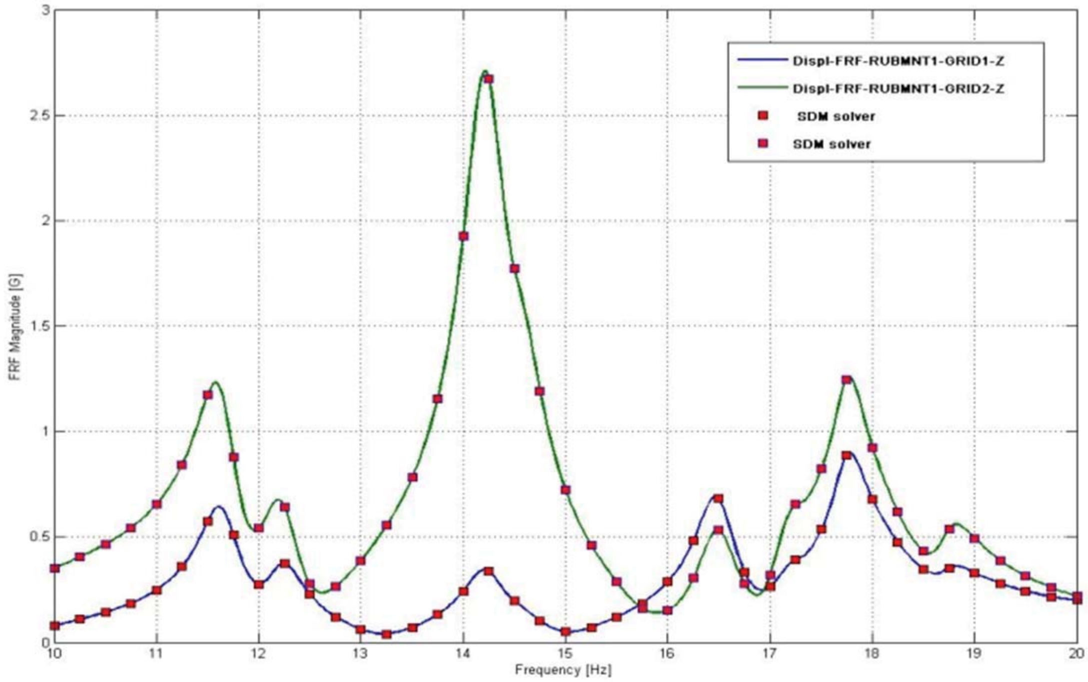

Fig. 13 NASTRAN Frequency Response vs. SDMM (linear case)

The stiffness behavior of the spring element representative of the forward rubber mount behavior in Z-direction is updated from its linear behavior to a bilinear stiffness, with a softening effect. The second branch is characterized by half of the underlying linear stiffness. The nonlinear results are compared with those obtained by means of the nonlinear ABAQUS code. The model has run in ABAQUS several times with different values of the frequency of the input force and consequently, storing the peaks of the steady-state response, the FRFs are built-up. On the other side, the NL SDMM obtains the results always with the same linear modal base and the NL FRFs are obtained with the same iterative procedure described in the easier case of the 2-DOFs system.

Fig. 12 shows that, as expected, the FRF peak of the frequency about 14.5 Hz, due to the softening effect, shifts to lower frequency, about 13.5 Hz. Evidently to introduce the nonlinearity in the system affects the frequency range where the local APU suspension system modes are involved. The agreement between the NL SDMM and the nonlinear results obtained in ABAQUS is very good.

Lastly in Fig. 13, the results coming from NL SDMM are presented, when instead of updating just one of the linear springs to nonlinear behavior, all the springs representative of the rubber mounts are considered as nonlinear. In this last ‘updating’, nine springs, hence 18 DOFs, are included into the Modification Matrix. Also in this case, the results respond to the expected behavior and the agreement with the results obtained in ABAQUS code are very good.

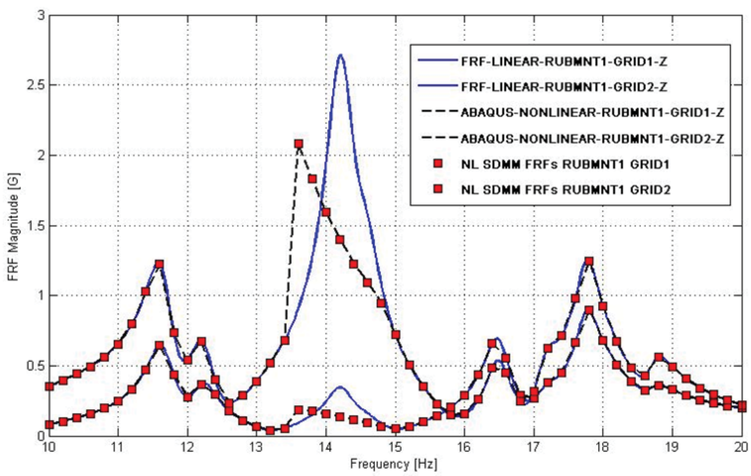

Fig. 14 ABAQUS Frequency Response vs. NL SDMM (nonlinear case)

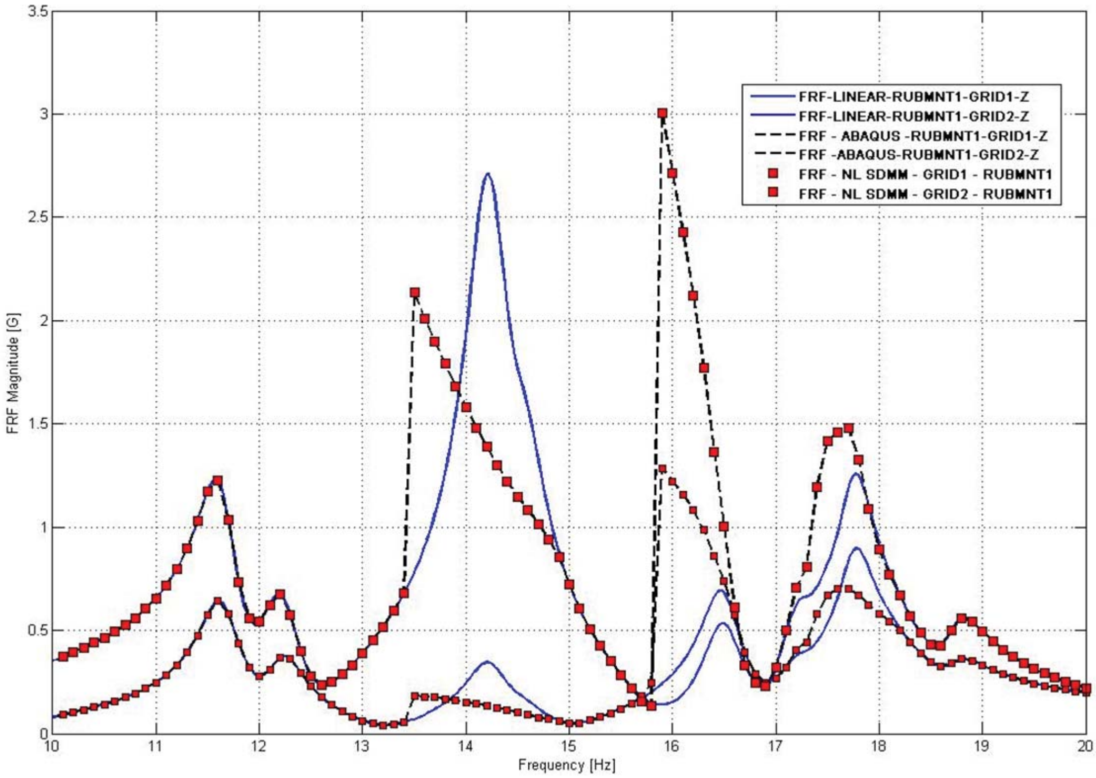

Fig. 15 ABAQUS Frequency Response vs. NL SDMM (nonlinear case)

V. Conclusion

A novel approach to update the linear FRFs with localized nonlinearities has been presented. It extends the well-known SDM Method, based on Modal Database updating, to a set of nonlinear modifications by means of an iterative procedure in the State-Space form. The linear method is reviewed and the nonlinear one is explained through a bilinear stiffness element.

In any case, the methodology has very general applicability, and in fact, there is no limitation to the size of the model in terms of the DOFs. The Modal Matrix is required only to contain the DOFs involved in the Modification Matrix, those where the loads are applied and those where the dynamic responses are required. After that, the estimation of the nonlinear behavior in the frequency domain is obtained without any need to re-run the FEM, being just an update of the underlying linear behavior.

The approach is very practical and theoretically, whatever the type of nonlinear lumped element that can be implemented. Both dependency on frequency and dynamic amplitude can be considered at the same time. The method has been successfully implemented in MATLAB and its reliability and robustness have been demonstrated thru two examples with different levels of complexity. The first nonlinear dynamic study regards a 2-DOFs system, and the second, a much more complex full A/C FEM. Both cases of the study show very good agreement between the nonlinear code and the NL SDMM method.

Finally, the approach seen for elements having nonlinear stiffness can be extended to those ones having both, nonlinear stiffness and nonlinear damping properties, i.e. spring-damper elements. That offers a rather easy and straight way of dealing with elements having a quite complex structural dynamic behavior which usually makes the simulations extremely time-expensive.

Acknowledgment

This piece of research has been funded by the Spanish Ministry of Economy and Competitiveness (MINECO) in the DREPANO project with reference RTC-2014-1593-4.

References

[1] Sestieri A., *Structural Dynamic Modification*, Sadhana, Vol.25, Part.3, pp. 247–259, 2000.

[2] Menga E., *Structural junction identification methodology* - Structures under Shock and Impact XII – 2012.

[3] S. Hernandez, E. Menga, A. Baldomir, C. Lopez, M. Cid and S. Moledo. *A methodology for identification of dynamic parameters in assembled aircraft structures.* 16th International Conference on Computational Methods and Experimental Measurements, A Coruña, Julio 2-4, 2013.

[4] Stanbridge, A.B., Sanliturk, K. Y., Ewins, D.J. and Ferreira, J. V., *Experimental Investigation of Dry Friction Damping and Cubic Stiffness Nonlinearity*, presented at ASME Design Technical Conferences, Pittsburgh, Pennsylvania, 2001.

[5] Kahaner, D. , C. Moler, and S. Nash, *Numerical Methods and Software*, Prentice-Hall, New Jersey, 1989.

[6] Michael Hatch - *Vibration Simulation Using MATLAB and ANSYS* - CHAPMAN & HALL/CRC - 2002