Free Convection Boundary Layer Flow of a Viscoelastic Fluid in the Presence of Heat Generation

Abdul Rahman Mohd Kasim, Mohd Ariff Admon, and Sharidan Shafie

Abstract—The present paper considers the steady free convection boundary layer flow of a viscoelastics fluid with constant temperature in the presence of heat generation. The boundary layer equations are an order higher than those for the Newtonian (viscous) fluid and the adherence boundary conditions are insufficient to determine the solution of these equations completely. The governing boundary layer equations are first transformed into non-dimensional form by using special dimensionless group. Computations are performed numerically by using Keller-box method by augmenting an extra boundary condition at infinity and the results are displayed graphically to illustrate the influence of viscoelastic K, heat generation γ, and Prandtl Number, Pr parameters on the velocity and temperature profiles. The results of the surface shear stress in terms of the local skin friction and the surface rate of heat transfer in terms of the local Nusselt number for a selection of the heat generation parameter γ (=0.0, 0.2, 0.5, 0.8, 1.0) are obtained and presented in both tabular and graphical formats. Without effect of the internal heat generation inside the fluid domain for which we take $\gamma = 0.0$, the present numerical results show an excellent agreement with previous publication.

Keywords—Free Convection, Boundary Layer, Circular Cylinder, Viscoelastic Fluid, Heat Generation

I. INTRODUCTION

NATURAL convection has been the subject of research for many years due to its importance in the understanding of phenomena appearing in nature and their extensive engineering applications. Studies on the natural convection boundary layer flow past a horizontal cylinder have been conducted by several researchers. For example, Saville and Churchill [1] investigated the laminar natural convection boundary layer flow near horizontal cylinders and vertical axisymmetric bodies. Merkin [2] studied the natural convection boundary layer flow over a cylinder of elliptic cross section. Besides that, a study related to this topic was also carried out by Lien et al. [3] by examining the free convection heat transfer of micropolar fluid near a horizontal permeable cylinder at a non-uniform thermal condition while Bhattacharyya and Pop [4] studied the free convection heat transfer from an elliptical cylinder in micropolar fluids. Mansour et al. [5] studied the coupled heat and mass transfer in the magnetohydrodynamic flow of micropolar fluid on circular cylinders with uniform heat and mass flux, while

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Cheng [6] studied the natural convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluid. In addition, Hossain et al. [7] examined the effect of thermal radiation on natural convection over cylinders of elliptic cross section.

In recent years, the natural convection in viscoelastic fluids was investigated due to the applications these materials have in industry and geophysics. In the linear stability problem of viscoelastic fluids, only overstable convection was investigated as their stationary motion is identical to that of Newtonian fluids. Jitchote and Robertson [8] used a perturbation method to analyze the viscoelastic second order fluid flow in curved pipes of circular cross section for the case where the second normal stress difference is non-zero, as a model of polymeric liquid. Ariel and Teipel [9] investigated the laminar two-dimensional viscoelastic flow near a stagnation point using the orthogonal collocation point method with Laguerre polynomials. Meanwhile, the natural convection of a viscoelastic fluid with deformable free surface was studied by L.A. DaÂvalos-Orozco and E. V. Luis [10]. It was found that for different values of the Galileo number and relaxation times that are large enough, the curves of the critical Rayleigh numbers are lower than those of stationary convection and those of overstability of the Newtonian fluid with deformable free surface. When the lower surface is rigid, maxima in the curves of criticality against the relaxation time are found. Rasmussen and Hassager [11] used the Lagrangian Integral Method to model the classical problem of unsteady viscoelastic flow from a sphere in a cylinder. Conversely, Wood [12] investigated the unsteady start-up helical flows for Oldroyd-B and upper-convected Maxwell fluids in straight pipes of circular and annular cross-section.In chemical engineering systems, viscoelastic flows arise in numerous processes in chemical engineering systems. Such flows possess both viscous and elastic properties and can exhibit normal stresses and relaxation effects. Recently, the numerical study of transient free convective mass transfer in a Walters-B viscoelastic flow with wall suction was investigated by T. B Chang et al. [13]. Velocity was found to increase with a rise in viscoelasticity parameter with both time and distances close to the plate surface. An increase in Schmidt number and a separation from the plate was also observed to significantly decrease both velocity and concentration in time. Increasing species of Grashof number boosted the flow velocity at all times and caused a significant rise primarily near the plate surface.A large number of physical phenomena also involve

natural convection that is driven by heat generation. Vajravelu and Hadjinicolaou [14] investigated the heat transfer characteristics in a laminar boundary layer flow of a viscous fluid over a linearly stretching continuous surface with viscous dissipation/frictional heating and internal heat generation, in which they considered the volumetric rate of heat generation, $q'''[w/m^3]$ as;

$$q''' = \begin{cases} Q_0(T-T_\infty) & for \quad T \geq T_\infty \\ 0 & for \quad T < T_\infty \end{cases}$$

where Q_0 is the heat generation constant. The above relation is valid for the state of some exothermic processes having T_∞ as the onset temperature. Furthermore, Chamkha and Issa [15] studied the effect of the heat generation or absorption and thermophoresis on a hydromagnetic flow with heat and mass transfer over a flat plate, while Mendez and Trevino [16] studied the effects of the conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform heat generation. Motivated by the work above, this paper investigates the problem of free convection boundary layer flow of viscoelastics fluid past a horizontal circular cylinder with the constant temperature in the presence of heat generation. The results obtained herein are compared with the solutions by Merkin [17] and Molla [18] with the publications titled free convection boundary layer on an isothermal horizontal circular cylinders and natural convection flow from an isothermal horizontal circular cylinder in presence of heat generation respectively, in order to verify the accuracy of the present results.

II. PROBLEM FORMULATION

The problem studied is the steady free convection boundary layer flow for an isothermal horizontal circular cylinder placed in a viscoelastic fluid. Figure 1 illustrates the geometry of the problem and the corresponding coordinate system. It was assumed that the constant temperature of the surface of the cylinder is T_w, and that of the ambient fluid is T_∞, where $T_w > T_\infty$ corresponds to a heated cylinder (assisting flow) and $T_w < T_\infty$ corresponds to a cooled cylinder (opposing flow), respectively. The physical configuration considered is as shown in Fig. 1.
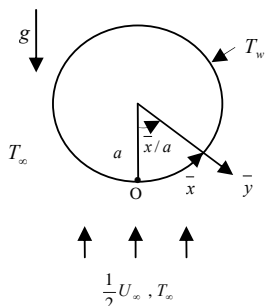

Fig. 1 Physical model and coordinate system

Under the usual Bousinesq and boundary layer approximations, the equations for mass continuity, momentum and energy took the following form:

Continuity equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

Momentum equation:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_e \frac{\partial \bar{u}_e}{\partial \bar{x}} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$
$$-k_0 \left\{ \frac{\partial}{\partial \bar{x}}\left(\bar{u}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} \right\} \tag{2}$$
$$+ g\beta(T-T_\infty)\sin(\bar{x}/a)$$

Energy equation:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} + \frac{Q_0}{\rho C_p}(T-T_\infty) \tag{3}$$

Subject to the boundary conditions;

$$\bar{u} = \bar{v} = 0, \qquad T = T_w \quad on \quad \bar{y} = 0,$$
$$\bar{u} = \bar{u}_e(x), \quad \partial \bar{u}/\partial \bar{y} = 0, \quad T = T_\infty \quad as \quad y \to \infty \tag{4}$$

Where ($\bar{u}$, $\bar{v}$) are the velocity components along the axes and the velocity outside the boundary layer, while T, ρ, g, β, μ, k_0, and α are the fluid temperature, density, gravitational acceleration, coefficient of thermal expansion, dynamic viscosity, vortex viscosity and thermal diffusivity of the fluid respectively.

Below are the following non-dimensional variables:

$$x = \bar{x}/a, \quad y = Gr^{1/4}(\bar{y}/a), \quad u = \frac{a}{\nu}Gr^{-1/2}\bar{u}, \quad v = \frac{a}{\nu}Gr^{-1/4}\bar{v}$$
$$u_e = \frac{a}{\nu}Gr^{-1/2}\bar{u}_e, \quad \theta = (T-T_\infty)/(T_W-T_\infty) \tag{5}$$

Substitution (5) into (1) to (3) led to the following non-dimensional equations:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\frac{\partial^2 u}{\partial y^2}$$

$$-K(\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y})-\theta\sin x \qquad (7)$$

Energy equation:

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{a^2 Q_0}{v\rho C_p Gr^{1/2}}\theta \qquad (8)$$

Subject to the boundary conditions

$$u=v=0,\qquad \theta=1 \quad \text{on}\quad y=0,$$
$$u=u_e(x),\quad \partial u/\partial y=0,\quad \theta=0 \quad \text{as}\quad y\to\infty \qquad (9)$$

Where $Gr = g\beta(T_W - T_\infty)a^3/v^2$ is the Grashof number and was denoted with the viscoelastic parameter, K as

$$K=\frac{k_0 Gr^{5/2}}{a^2},$$

III. SOLUTION PROCEDURES

In order to solve (6) to (8) according to the boundary condition (9), the following variables were assumed:

$$\psi=xf(x,y),\qquad \theta=\theta(x,y) \qquad (10)$$

Where ψ is the stream function defined as:

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x} \qquad (11)$$

By substituting (10) and (11) into (6) to (8), the following equations were obtained:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (12)$$

$$\frac{\partial^3 f}{\partial y^3}+f\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial f}{\partial y}\right)^2-x\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x\partial y}+x\frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}$$
$$+\frac{\sin x\cos x}{x}+K\left\{2\frac{\partial f}{\partial y}\frac{\partial^3 f}{\partial y^3}-f\frac{\partial^4 f}{\partial y^4}-\frac{\partial^2 f}{\partial y^2}\frac{\partial^2 f}{\partial y^2}\right\}$$
$$+x\frac{\partial^2 f}{\partial x\partial y}\frac{\partial^3 f}{\partial y^3}+x\frac{\partial f}{\partial y}\frac{\partial^4 f}{\partial x\partial y^3}-x\frac{\partial f}{\partial x}\frac{\partial^4 f}{\partial y^4}$$
$$-x\frac{\partial^2 f}{\partial y^2}\frac{\partial^3 f}{\partial x\partial y^2}+\frac{\theta\sin(x)}{x}=0 \qquad (13)$$

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+f\frac{\partial\theta}{\partial y}+\frac{a^2 Q_0}{v\rho C_p Gr^{1/2}}\theta=x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \qquad (14)$$

With respect to the following boundary conditions

$$f=\frac{\partial f}{\partial y}=0,\qquad \theta=1 \quad \text{on}\quad y=0,$$
$$\frac{\partial f}{\partial y}=\frac{\sin x}{x},\quad \frac{\partial^2 f}{\partial y^2}=0,\quad \theta=0 \quad \text{as}\quad y\to\infty \qquad (15)$$

At the lower stagnation point of the cylinder, $x\approx 0$, (12) to (14) was reduced to the following ordinary differential equation:

$$ff''+f'''-\left(f'\right)^2+1+\theta+K\left\{ff''''-2f'f'''+(f'')^2\right\}=0 \qquad (16)$$

$$\frac{1}{\Pr}\theta''+f\theta'+\gamma\theta=0 \qquad (17)$$

with the boundary conditions

$$f(0)=f'(0)=0,\quad \theta(0)=1$$
$$f'(\infty)=0,\quad f''(\infty)=0,\quad \theta(\infty)=0 \qquad (18)$$

where primes denote the differentiation with respect to y.

The physical quantities of principal interest are shearing stress and the rate of heat transfer in terms of skin friction coefficient C_f and the Nusselt number Nu respectively, which can be written as:

$$C_f=\frac{\tau_w}{\rho U_\infty^2},\quad Nu=\frac{aq_w}{k(T_w-T_\infty)}$$

where

$$\tau_w=\mu(\frac{\partial\bar{u}}{\partial\bar{y}})_{\bar{y}=0}+k_0\left(\bar{u}\frac{\partial^2\bar{u}}{\partial\bar{x}\partial\bar{y}}+\bar{v}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+2\frac{\partial\bar{u}}{\partial\bar{x}}\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},$$

and

$$q_w=-k(\frac{\partial T}{\partial\bar{y}})_{\bar{y}=0}$$

Using the variables (5) and the boundary condition into (18) resulted in the following:

$$C_f Gr^{1/4}=xf''(x,0) \qquad (19)$$
$$Nu Gr^{-1/4}=-\theta(x,0) \qquad (20)$$

The results of the velocity and temperature distributions were then calculated respectively from the following relations

$$u=\frac{\partial f}{\partial y},\quad \theta=\theta(x,y)$$

IV. RESULT AND DISCUSSION

The systems of Equations (16) and (17) were solved numerically for some values of the heat generation $\gamma=(0.0,0.2,0.5,0.8,1.0)$ and viscoelastic parameter K, using the implicit finite-difference method known as Keller-box method that was very well described in the book by Cebeci and Bradshaw [19]. In this paper, the case in question is when

the Prandtl number Pr is 0.7 and 1. The present results for the skin friction coefficient $C_f Gr^{1/4}$ and the local Nusselt number, $NuGr^{1/4}$ were also compared with those of Merkin [17] and Molla [18] in order to validate the numerical results obtained. The comparison showed that the numerical solutions (see Table I and II) obtained by the present authors concurs very well with those of previous authors.

TABLE I

COMPARISON OF THE PRESENT NUMERICAL RESULTS FOR LOCAL SKIN FRICTION WITH THOSE MERKIN [17] AND MOLLA [18] AT $\gamma = 0$ FOR PR =1.0

X	$C_f Gr^{1/4}$			
	Merkin [17]	Molla [18]	Series solutions by Molla [18]	Present result
0.0	0.0000	0.0000	0.0000	0.000000
$\pi/6$	0.4151	0.4145	0.4144	0.412366
$\pi/3$	0.7558	0.7539	0.7544	0.751733
$\pi/2$	0.9579	0.9541	0.9550	0.955196
$2\pi/3$	0.9756	0.9696	0.9701	0.977381
$5\pi/6$	0.7822	0.7739	0.7824	0.786216

TABLE II

COMPARISON OF THE PRESENT NUMERICAL RESULTS FOR LOCAL NUSSELT NUMBER WITH THOSE MERKIN [17] AND MOLLA [18] AT $\gamma = 0$ FOR PR =1.0

X	$NuGr^{-1/4}$			
	Merkin [17]	Molla [18]	Series solutions by Molla [18]	Present result
0.0	0.4214	0.4241	0.4216	0.421411
$\pi/6$	0.4161	0.4161	0.4164	0.416371
$\pi/3$	0.4007	0.4005	0.4009	0.401111
$\pi/2$	0.3745	0.3740	0.3751	0.375214
$2\pi/3$	0.3364	0.3355	0.3389	0.337486
$5\pi/6$	0.2825	0.2812	0.2923	0.283199

Besides that, the comparison of the present results with those of previous works on skin friction and heat transfer variation was illustrated as shown in Fig. 2, while the comparison on velocity and temperature profile can be seen in Figure 3; both illustrations show a strong assent between the comparisons. The present authors are therefore confident that the present results are very accurate.

The variation of local skin friction and local Nusselt number with various values of heat generation $\gamma = (0.0, 0.2, 0.5, 0.8, 1.0)$ with the fixed values of K = 1 and Pr = 1 are illustrated in Fig. 4(a) and 4(b). From these figures, it can be seen that an increase in heat generation parameter, γ, leads to an increase in the local skin-friction coefficient and decrease on the local Nusselt number. This is expected as the heat generation mechanism creates a layer of hot fluid near the surface, which subsequently causes the rate of heat transfer from the surface to decrease. Owing to enhanced temperature, both the viscosity of the fluid and the corresponding local skin-friction coefficient increased. A similar trend was also observed by Molla [22] on the study of natural convection flow from a horizontal circular cylinder with uniform heat flux in the presence of heat generation. Fig. 6 illustrates the variation of local skin friction and heat transfer with various values of viscoelastic parameter, which indicated that both local skin friction and heat transfer decreased with an increase of K.

The effect of Pr on the local skin friction and heat transfer is illustrated by Fig. 8(a) and 8(b), which shows that an increase in the value of Prandtl number leads to an increase in both the value of the rate of heat transfer and the local skin-friction. The absolute maxima of the local skin-friction may also be observed to shift toward the middle of the surface. Fig. 9(a) and 9(b) shows the distribution of velocity and temperature profile respectively, whereby an increase of Prandtl number is shown to decrease the velocity distribution, while the opposite trend was observed for the temperature profile.

Since there are numerous detailed profiles of the transient velocity and temperature fields, the velocity and temperature distributions at the lower stagnation point were given at Pr = 1 with various values of heat generation coefficient and viscoelastics parameter, K. These are illustrated in Fig. 4 and 9 respectively. Based on Fig. 5(a) and 5(b), it was noticed that the fluid velocity increases with γ as it contributed to the acceleration of the flow and enhanced the level of local skin-friction coefficient at the same time. On the other hand, the temperature profile increased gradually and at $\gamma \geq 0.5$ it exceeded the level of surface temperature ($\theta = 1$), along with some critical levels of temperature appearing close to the surface of the cylinder. For $\gamma = 1.0$ the fluid temperature nearly doubled the surface temperature.

Based on Fig. 7(a), it was noticed that, for $\gamma = 1$ the velocity distributions decreased when the value of viscoelastic parameter, K was increased. The values of these profiles are lower for a viscoelastic fluid than for a Newtonian fluid (K = 0). Therefore, the thickness of the velocity boundary layer for a viscoelastic fluid is higher than for a Newtonian fluid. Meanwhile, the opposite behaviour was observed for the temperature profiles as shown in Fig. 7(b). This figure shows that the decrease of the velocity and increase of the viscoelastics parameter have a similar trend with the vertical free flows (see Katagiri [23]). This behavior reflects the coupling of the energy equation to the momentum equation through the temperature dependent body forces.
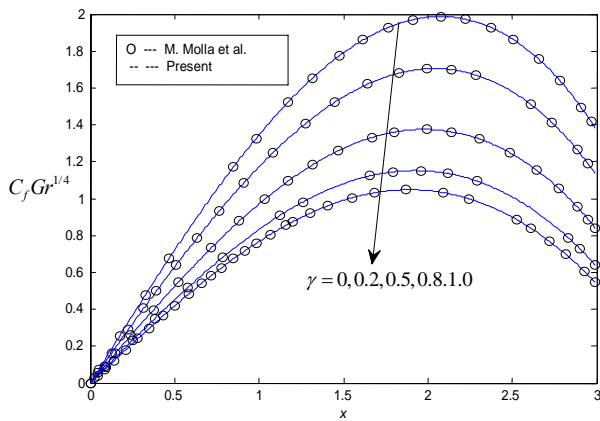
Fig. 2 (a) Comparison of skin-friction for different values
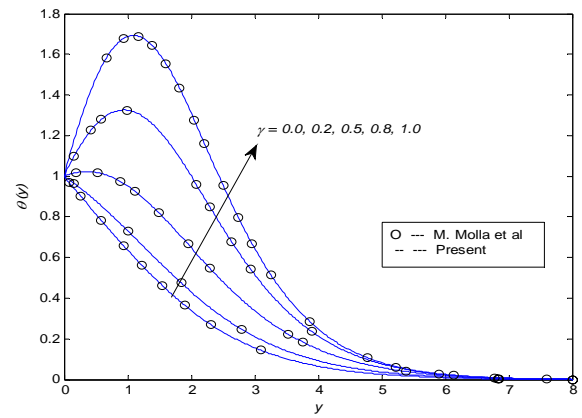of γ with Pr = 0.7

Fig. 3 (b) Comparison of temperature distribution for
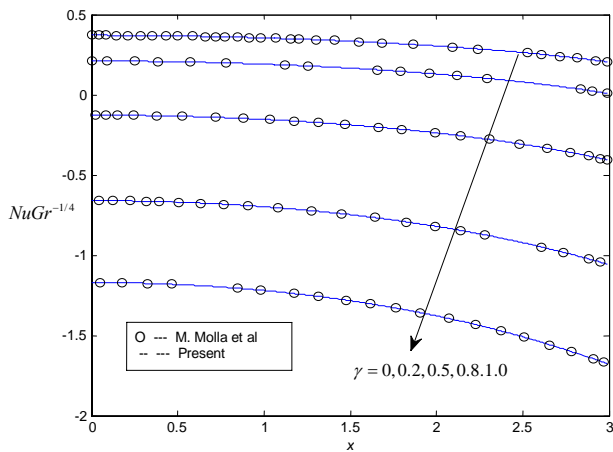different values of γ with Pr = 0.7

Fig. 2 (b) Comparison of heat transfer for different values
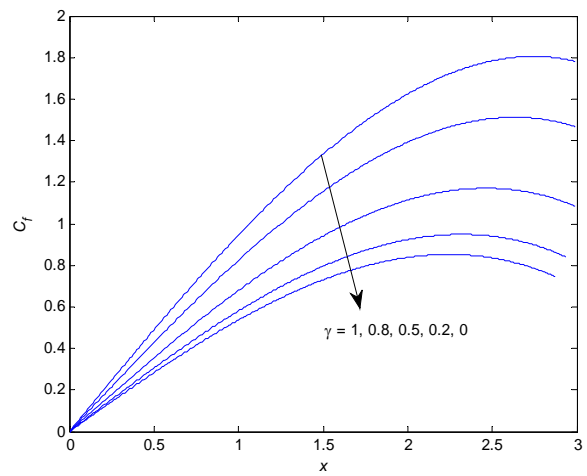of γ with Pr = 0.7

Fig. 4 (a) Variation of the local skin friction for different
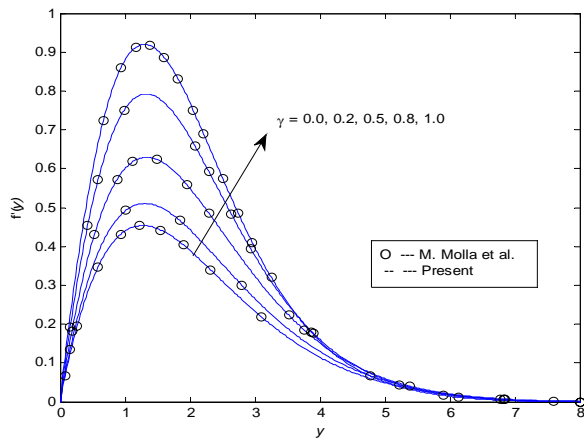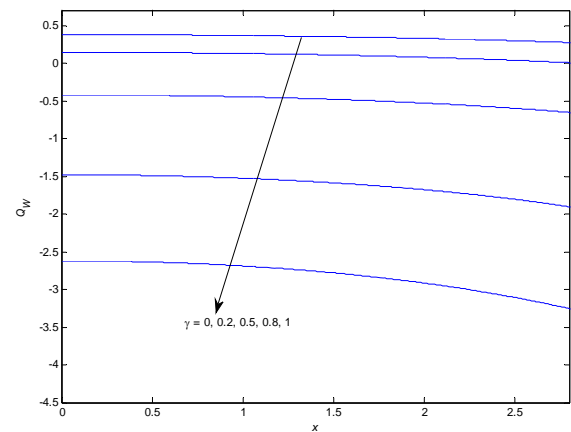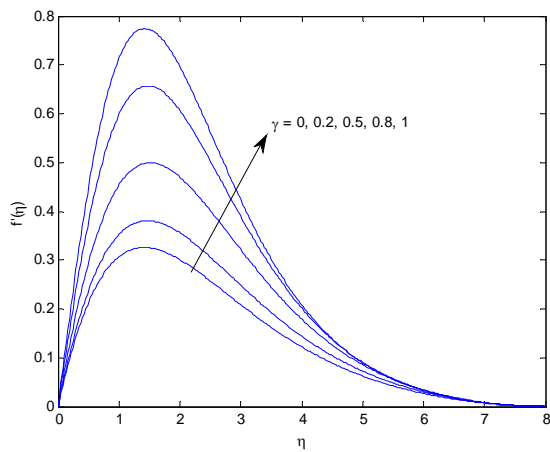values of γ at Pr = 1 and
K=1

Fig. 3 (a) Comparison of velocity distribution for different
values of γ with Pr = 0.7

Fig. 4. (b) Variation of the local heat transfer for different
values of γ at Pr = 1 and K=1

Fig. 5 (a) Velocity distribution for different values of γ at
Pr = 1 and K=1
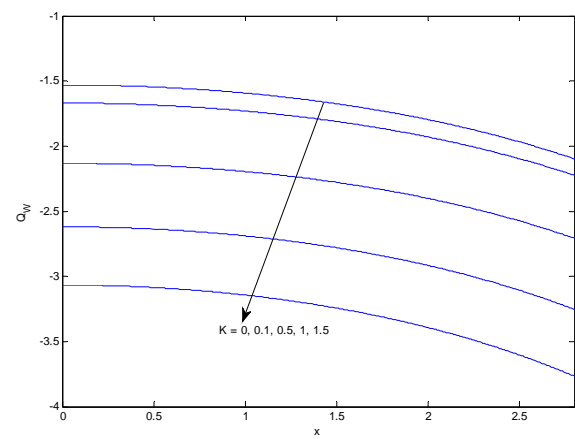
Fig. 6 (b) Variation of the local heat transfer coefficient
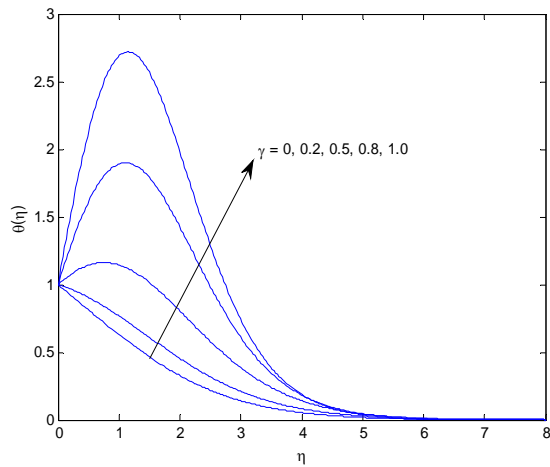Q_w for various value of K at $\gamma = 1$ and Pr
=1

Fig. 5 (b) Temperature distribution for different values of
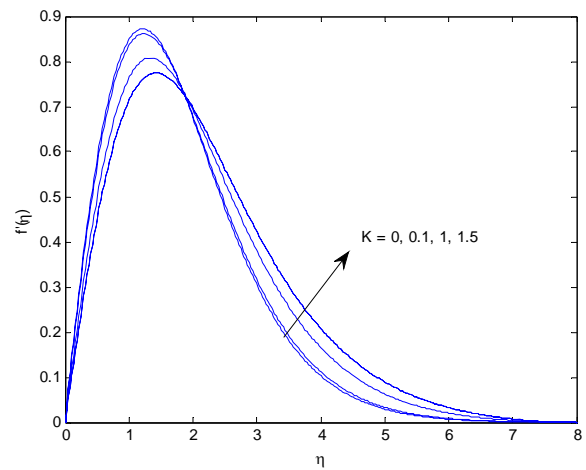γ at Pr = 1 and K=1

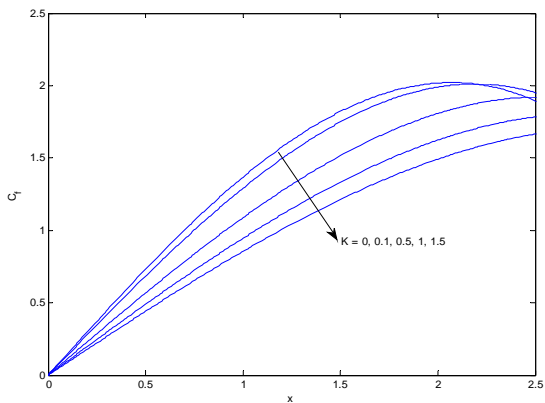Fig. 7 (a) Velocity distribution for different values of K at
$\gamma = 1$ and Pr = 1

Fig. 6 (a) Variation of the local skin friction C_f for
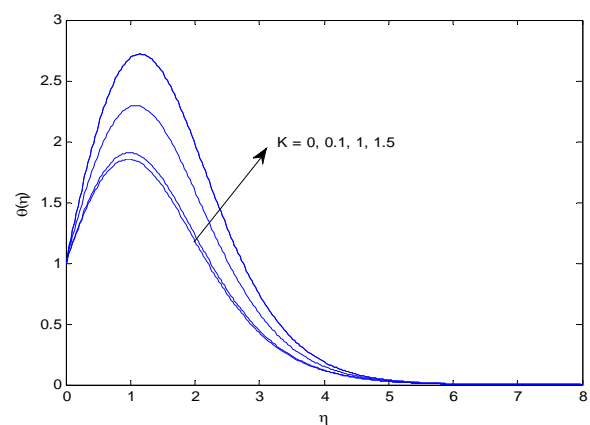various value of K at $\gamma = 1$ and Pr=1

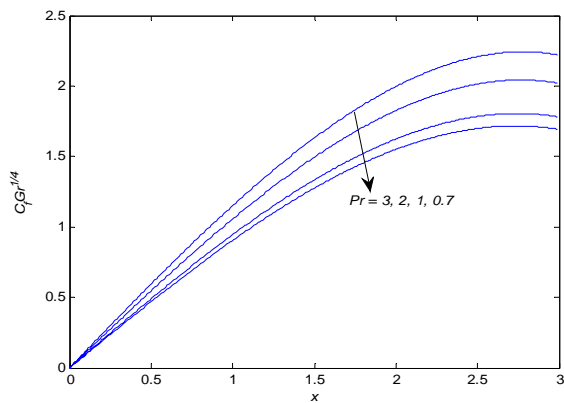Fig. 7 (b) Temperature distribution for different values of
K at $\gamma = 1$ and Pr = 1

Fig. 8 (a) Variation of the local skin friction for different
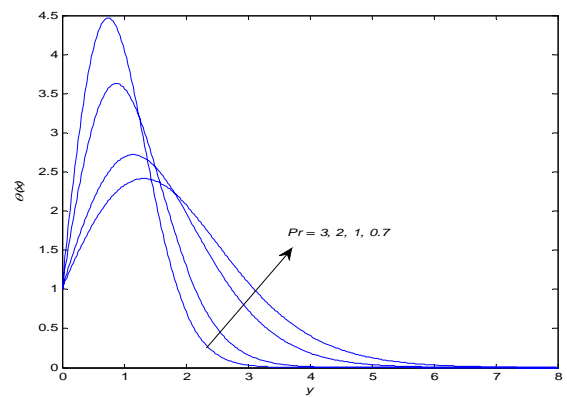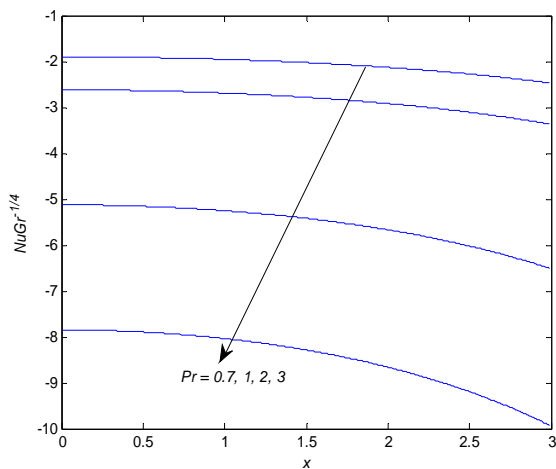values of Pr at $\gamma = 1$ and K=1

Fig. 8 (b) Variation of the local heat transfer for different values of
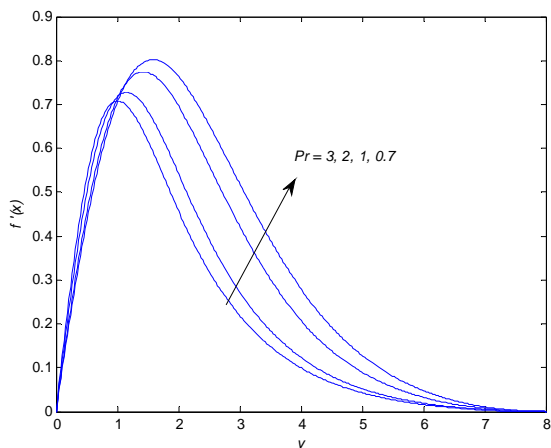Pr = 1 at $\gamma = 1$ and K=1

Fig. 9 (a) Velocity distribution for different values of Pr
at $\gamma = 1$ and K=1

Fig. 9 (b) Temperature distribution for different values of
Pr at $\gamma = 1$ and K=1

V. CONCLUSION

The steady free convection boundary layer flow of an incompressible viscoelastic fluid past an isothermal horizontal circular cylinder has been investigated numerically in this paper. The governing boundary layer equations were transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations was solved numerically using the Keller-box method.

This paper has revealed how the parameter K, and the Prandtl number, Pr affect the flow and heat transfer characteristics. From the present investigation, the following conclusions can be drawn:

1. An increase in the value of Pr leads to a decrease of both the wall temperature distribution $\theta_w(x)$ and the velocity distribution $f'(x)$

2. An increase in the value of Prandtl number leads to an increase in the value of the rate of heat transfer, while the opposite effect applies for the local skin-friction.

3. An increase in heat generation parameter, γ, leads to an increase in the local skin-friction coefficient and decrease on the local Nusselt number.

4. In the presence of heat generation, the velocity distributions decrease when the value of viscoelastic parameter, K increases. The values of these profiles are lower for a viscoelastic fluid than for a Newtonian fluid (K = 0).

5. Velocity distributions decrease when the value of viscoelastic parameter, K is increased while the opposite behaviour is observed for the temperature profiles.

6. When the viscoelastics parameter, K increases it reduces both the values of skin friction and heat transfer (local Nusselt number).

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