

# Fractional Order Feedback Control of a Ball and Beam System

Santosh Kr. Choudhary

**Abstract**—In this paper, fractional order feedback control of a ball beam model is investigated. The ball beam model is a particular example of the double Integrator system having strongly nonlinear characteristics and unstable dynamics which make the control of such system a challenging task. Most of the work in fractional order control systems are in theoretical nature and controller design and its implementation in practice is very small. In this work, a successful attempt has been made to design a fractional order  $PI^\lambda D^\mu$  controller for a benchmark laboratory ball and beam model. Better performance can be achieved using a fractional order PID controller and it is demonstrated through simulations results with a comparison to the classic PID controller.

**Keywords**—Fractional order calculus, fractional order controller, fractional order system, ball and beam system,  $PI^\lambda D^\mu$  controller, modelling, simulation.

## I. INTRODUCTION

THE ball and beam system is one of the most popular benchmark laboratory equipment with high non-linearity in its dynamics. The system (shown in Fig. 1) [12] is very simple but important class of second order systems as they model single degree of freedom translation and rotational systems. A ball is placed on a beam where it is allowed to roll with one degree of freedom along the length of the beam. The sensor placed on one side of the beam detects the ball roll along the beam and its position. A lever arm is attached to the beam at one end and a servo gear at the other. As the servo gear



Fig. 1. Ball and beam system

turns  $\theta$ , the lever changes the angle of the beam  $\alpha$ . When the angle is changed from the vertical position, gravity causes the ball to roll along the beam. The control job is to automatically regulate the position of the ball  $x$  on the beam by changing the angle of the beam  $\alpha$ . It is a difficult control task because the ball does not stay in one place on the beam but moves with an acceleration that is proportional to the tilt of the beam. The

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ball and beam system is an inherent open-loop unstable system [11] because the system output (the ball position) increases without limit for a fixed input (beam angle). These properties have made the ball and beam system a suitable device to test various control techniques.

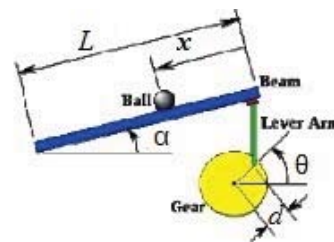


Fig. 2. Schematic diagram

Fraction order control [3], [7] is the non-conventional control technique and developed during the last few decades. Most of the work in fractional order control systems are in theoretical nature and controller design and its implementation in practice is very small but there are some limited practical application as for example, flexible spacecraft attitude control [8], temperature control [9], motion control [10], etc. The significance of fractional order control system is that it is a generalization of classical control theory which could lead to more adequate modelling and more robust control performance. Despite of this fact, the integer-order controls are still more welcome due to absence of accurate solution methods for fractional order differential equations (FODEs). But recently, many progresses in the analysis of dynamic system modelled by FODEs [1] have been made and approximation of fractional derivatives and integrals can be used in the wide area of fractional order control systems. It is also observed that PID controllers which have been modified using the notion of fractional order Integrator and differentiator applied to the integer order or fractional order plant to enhance the system control performance. References [2], [4]–[6] give the idea of simple tuning formulas for the design of PID controllers. In this work, the main objective is to apply the fractional order PID control to enhance the ball and beam system control performance. The paper presents few results and does not at all do justice to extensive real time simulation results.

Some MATLAB function files are used in this paper to simulate the fraction order dynamic system using reference [3]. The rest of the article is organized as follows: In Section II, we present a mathematical modelling of ball and beam system.

Section III illustrates a brief introduction of fractional order system and fractional calculus. Section IV gives the basic ideas and technical formulations for fractional order PID controller design. Section V deals with fractional order PID controller design for ball beam system. Section VI discuss about the results and simulation analysis and Section VII concludes the paper with some remarks and conclusion.

## II. MATHEMATICAL MODELLING OF BALL AND BEAM SYSTEM

The basic mathematical description of ball and beam system consists of (a) DC servomotor dynamic model and (b) ball on the beam model.

### A. DC Servomotor Dynamic model

Modelling DC servomotor can be divided into electrical and mechanical two subsystems. The electrical subsystem is based on Kirchhoff's voltage law:

$$L_m \dot{I}_m + R_m I_m + K_b \dot{\theta} = U \quad (1)$$

where  $U$  is the input voltage,  $I_m$  is armature current,  $R_m$  and  $L_m$  are the resistance and inductance of the armature,  $K_b$  is back emf constant and  $\dot{\theta}$  is angular velocity.

Since compared to  $R_m I_m$  and  $K_b \dot{\theta}$ , the term  $L_m \dot{I}_m$  is very small, therefore in order to simplify the modeling, the term  $L_m \dot{I}_m$  is neglected. Equation (1) for DC motor model is reduced to

$$R_m I_m + K_b \dot{\theta} = U \quad (2)$$

The mechanical subsystem is given by

$$\frac{1}{K_g} (J_m \ddot{\theta} + B_m \dot{\theta}) = \tau_m \quad (3)$$

where  $K_g$  is the gear ratio,  $J_m$  is the effective moment of inertia,  $B_m$  is viscous friction coefficient,  $\tau_m$  is the torque produced at the motor shaft.

The electrical and mechanical subsystems are coupled to each other through an algebraic torque equation

$$\tau_m = K_m I_m \quad (4)$$

where  $K_m$  is the torque constant of motor.

Using (4), (3) can be written as

$$\begin{aligned} \frac{1}{K_g} (J_m \ddot{\theta} + B_m \dot{\theta}) &= K_m I_m \\ \Rightarrow I_m &= \frac{J_m \ddot{\theta} + B_m \dot{\theta}}{K_m K_g} \end{aligned} \quad (5)$$

Substituting the value of  $I_m$  in (2), the differential equation for DC motor model is obtained as

$$\frac{R_m J_m}{K_m K_g} \ddot{\theta} + \left( K_b + \frac{R_m B_m}{K_m K_g} \right) \dot{\theta} = U \quad (6)$$

Taking the Laplace transform of (6), the DC servomotor model for ball and beam system is obtained as

$$\begin{aligned} \frac{R_m J_m}{K_m K_g} s^2 \Theta(s) + \left( K_b + \frac{R_m B_m}{K_m K_g} \right) s \Theta(s) &= U(s) \\ \Rightarrow \frac{\Theta(s)}{U(s)} &= \frac{K_m K_g}{R_m J_m s^2 + (R_m B_m + K_b K_m K_g) s} \end{aligned} \quad (7)$$

### B. Ball on the Beam model

Consider the schematic diagram of the ball-beam model as shown in Fig. 2. The Lagrangian equation of motion of the model is written as:

$$\left( \frac{J}{R^2} + m \right) \ddot{x} + mg \sin \alpha - mx (\dot{\alpha})^2 = 0 \quad (8)$$

Equation (8) is linearized about the beam angle  $\alpha = 0$ , which gives the following linear approximation of the system:

$$\left( \frac{J}{R^2} + m \right) \ddot{x} = -mg \alpha \quad (9)$$

Since the beam angle  $\alpha$  and the angle of the gear  $\theta$  are not same, therefore Fig.3 is used to calculate them. Since the arc

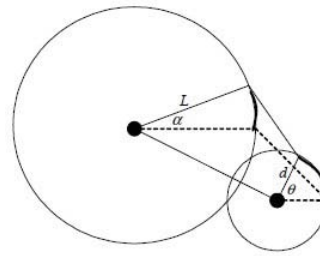


Fig. 3. Relation between motor position and beam angle

distances in the two circles are equal, therefore, the equation, which relates the beam angle  $\alpha$  to the angle of the gear  $\theta$  can be approximated by the linear relationship:

$$\begin{aligned} \alpha L &= \theta d \\ \alpha &= \frac{d}{L} \theta \end{aligned} \quad (10)$$

On substituting (10) in (9),:

$$\left( \frac{J}{R^2} + m \right) \ddot{x} = -mg \frac{d}{L} \theta \quad (11)$$

Taking Laplace transform of (11),

$$\begin{aligned} \left( \frac{J}{R^2} + m \right) X(s) s^2 &= -mg \frac{d}{L} \Theta(s) \\ \Rightarrow \frac{X(s)}{\Theta(s)} &= \frac{-mgd}{L s^2 \left( \frac{J}{R^2} + m \right)} \end{aligned} \quad (12)$$

The physical parameters for a ball and beam system [12] are listed in Table I Finally, on substituting the value of physical

TABLE I  
PHYSICAL PARAMETERS OF BALL AND BEAM SYSTEM

Parameter's name	Unit	Value
Mass of the ball ( $m$ )	$Kg$	0.011
Radius of the ball ( $R$ )	$m$	0.015
Acceleration due to gravity ( $g$ )	$m/s^2$	-9.8
Length of the beam ( $L$ )	$m$	0.4
Radius of the gear ( $d$ )	$m$	0.04

parameters from Table I in (12) yields the ball and beam

system transfer function:

$$\begin{aligned} \frac{X(s)}{\Theta(s)} &= \frac{-0.11 \times -9.8 \times 0.4}{0.4 \times \left(\frac{2 \times 0.11}{5} + 0.11\right) \times s^2} \\ \Rightarrow P(s) &= \frac{0.7}{s^2} \end{aligned} \quad (13)$$

The obtained ball beam model (13) is a particular example of double integrator system and it indicates unstable behavior as ball position (output) increases without limit for a fixed servo gear angle (input). It is also illustrated in Fig.4.

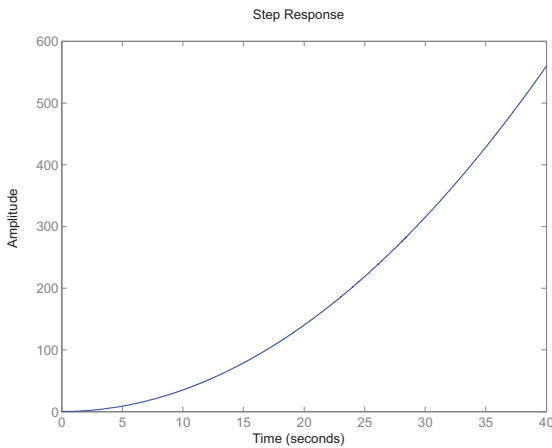


Fig. 4. Open-loop system response

### III. FUNDAMENTALS OF FRACTIONAL ORDER CONTROL SYSTEM

Fraction order control is the non-conventional way of robust control based on fractional order derivative. Most of the works in fractional order control systems are in theoretical nature and controller design and implementation in practice is very small.

#### A. A Brief Introduction to Fractional Calculus

Fractional calculus is the generalization of integration and differentiation to fractional order fundamental operator  $\alpha D_t^\beta f(t)$ , where  $\alpha$  and  $t$  are the limits and  $\beta \in R$  is the order of the operation. The continuous integro-differential operator is defined as [3]

$$\alpha D_t^\beta f(t) = \begin{cases} \frac{d^\beta}{dt^\beta} & : \beta > 0, \\ 1 & : \beta = 0, \\ \int_\alpha^t (d\tau)^{-\beta} & : \beta < 0. \end{cases} \quad (14)$$

There are several definitions of fractional integration and differentiation. The most often used are the Grunwald-Letnikov (GL) definition and the Reimann Liouville definition (RL). For a wide class of functions, the two definitions-GL and RL are equivalent.

The GL is given as [3]:

$$\alpha D_t^\beta f(t) = \lim_{h \rightarrow 0} h^{-\beta} \sum_{j=0}^{\lfloor \frac{t-\alpha}{h} \rfloor} (-1)^j \binom{\beta}{j} f(t-jh), \quad (15)$$

where  $[\cdot]$  means the integer part.

The RL definition is given as [3]:

$$\alpha D_t^\beta f(t) = \frac{1}{\Gamma(n-\beta)} \frac{d^n}{dt^n} \int_\alpha^t \frac{f(\tau)}{(t-\tau)^{\beta-n+1}} d\tau, \quad (16)$$

for  $(n-1 < \beta < n)$  and where  $\Gamma(\cdot)$  is the Gamma function. For many engineering applications, the Laplace transform are often used. The Laplace transform of the GL and RL fractional differintegral under zero initial conditions for order  $\beta$  is given by

$$\mathcal{L} \left[ \alpha D_t^{\pm\beta} f(t); s \right] = s^{\pm\beta} F(s) \quad (17)$$

#### B. Fractional Order Transfer Function

The fractional-order system is the direct extension of classical integer-order systems. It is obtained from the fractional-order differential equations. A typical n-term linear fractional order differential equation (FODE) in the time domain is given by

$$\alpha_n D_t^{\beta_n} y(t) + \dots + \alpha_1 D_t^{\beta_1} y(t) + \alpha_0 D_t^{\beta_0} y(t) = 0 \quad (18)$$

Consider the control function which acts on the FODE system (18) as follows:

$$\alpha_n D_t^{\beta_n} y(t) + \dots + \alpha_1 D_t^{\beta_1} y(t) + \alpha_0 D_t^{\beta_0} y(t) = u(t) \quad (19)$$

On taking the Laplace transform of (19), we get

$$\alpha_n s^{\beta_n} Y(s) + \dots + \alpha_1 s^{\beta_1} Y(s) + \alpha_0 s^{\beta_0} Y(s) = U(s) \quad (20)$$

From (20), we can obtain a fractional order transfer function as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{\alpha_0 s^{\beta_0} + \alpha_1 s^{\beta_1} + \dots + \alpha_n s^{\beta_n}} \quad (21)$$

In general, the fractional-order transfer function (FOTF) of a single variable dynamic system can be defined as

$$G(s) = \frac{b_0 s^{\gamma_0} + b_1 s^{\gamma_1} + \dots + b_m s^{\gamma_m}}{a_0 s^{\beta_0} + a_1 s^{\beta_1} + \dots + a_n s^{\beta_n}} \quad (22)$$

where  $b_i (i = 0, 1 \dots m)$ ,  $a_i (i = 0, 1 \dots n)$  are constants and  $\gamma_i (i = 0, 1 \dots m)$ ,  $\beta_i (i = 0, 1 \dots n)$  are arbitrary real or rational numbers and without loss of generality they can be arranged as  $\gamma_m > \gamma_{m-1} > \dots > \gamma_0$  and  $\beta_m > \beta_{m-1} > \dots > \beta_0$ .

The incommensurate fractional order system (22) can also be expressed incommensurate form by the multivalued transfer function

$$H(s) = \frac{b_0 + b_1 s^{\frac{1}{\nu}} + \dots + b_m s^{\frac{m}{\nu}}}{a_0 + a_1 s^{\frac{1}{\nu}} + \dots + a_n s^{\frac{n}{\nu}}}, \quad (\nu > 1). \quad (23)$$

Note that every fractional order system can be expressed in the form (23) and domain of the  $H(s)$  definition is a Riemann surface with  $\nu$  Riemann sheets.

C. Stability of Fractional Order System

A linear time-invariant system is stable if the roots of the characteristic polynomial are negative or have negative real parts if they are complex conjugate. It means that they are located on the left half of the complex plane. In the fractional-order LTI case, the stability is different from the integer one. The interesting point is that a stable fractional system may have roots in the right half of the complex plane (see Fig.5).

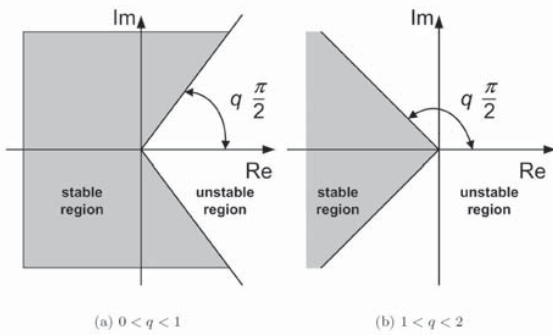


Fig. 5. Stability region of LTI fractional order systems

**Theorem 1:** (Matignon's stability theorem) [3]:The fractional transfer function  $G(s) = \frac{N(s)}{D(s)}$  is stable if and only if  $|\arg(\sigma_i)| = q\frac{\pi}{2}$ , where  $\sigma = s^q, (0 < q < 2)$  with  $\forall \sigma_i \in C, i^{th}$  root of  $D(\sigma) = 0$ .

**Remark 1:** When  $s=0$  is a single root of  $D(s)$ , the system cannot be stable.

For theorem 1, the stability region suggested by Fig.5 tends to the whole s-plane when  $q = 0$ , corresponds to the Routh-Hurwitz stability when  $q = 1$  and tends to the negative real axis when  $q = 2$ .

It should be noted that, only the denominator is meaningful in stability assessment and the numerator does not affect the stability of a FOTF. The stability of the fractional order system can be analyzed in another way also. Consider the characteristic equation of a general fractional order system in the form as:

$$\alpha_0 s^{\beta_0} + \alpha_1 s^{\beta_1} + \dots + \alpha_n s^{\beta_n} = \sum_{i=0}^n \alpha_i s^{\beta_i} = 0. \quad (24)$$

For  $\beta_i = \frac{\nu_i}{\nu}$ , we transform (24) into  $\sigma$ -plane:

$$\sum_{i=0}^n \alpha_i s^{\frac{\nu_i}{\nu}} = \sum_{i=0}^n \alpha_i \sigma^{\nu_i} = 0 \quad (25)$$

where  $\sigma = s^{\frac{\nu}{m}}$  and  $m$  is the least common multiple of  $\nu$ . For given  $\alpha_i$ , if the absolute phase of all roots of (18) is  $|\phi_\sigma| = |\arg(\sigma)|$ , we can summarize the following facts of stability for fractional order systems :

- 1) The condition for stability is  $\frac{\pi}{2m} < |\arg(\sigma)| < \frac{\pi}{m}$ .
- 2) The condition for oscillation is  $|\arg(\sigma)| = \frac{\pi}{2m}$ .

Otherwise the system is unstable.

IV. DESIGN PHILOSOPHY

In this section, main objective is to discuss the fractional-order  $PI^\lambda D^\mu$  controller design methodology for the system model. In theory, the control system can include both the fractional order plant to be controlled and fractional order controller. However, in control practice, more common is to consider the fractional order controller and integer order plant. Here we demonstrate the design methodology for the scenario where integer order system model is being controlled by fractional order controller.

The fractional-order  $PI^\lambda D^\mu$  generalizes the PID controller with integrator of real order  $\lambda$  and differentiator of real order  $\mu$  and expands it from point to plane. This expansion adds more flexibility to controller design and we can control our real world processes more accurately.

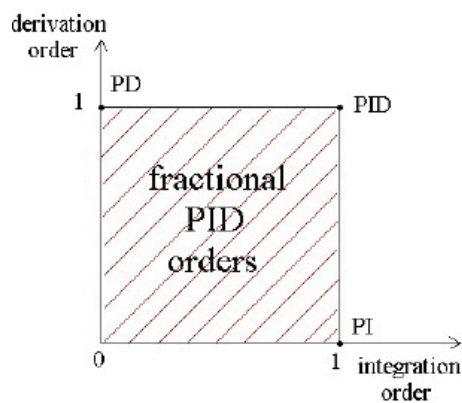


Fig. 6. Expanding from point to plane

The differential equation of fractional order PID controller is described by

$$u(t) = K_P e(t) + K_I D_t^{-\lambda} e(t) + K_D D_t^\mu e(t) \quad (26)$$

The continuous transfer function of FOPID is also obtained through Laplace transform, which is given by is given by

$$C(s) = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu, (\lambda, \mu > 0) \quad (27)$$

It is obvious to note that the FOPID controller not only needs to design the three parameters  $K_P, K_I$  and  $K_D$  but also need to design two orders  $\lambda$  and  $\mu$  of integral and derivative controllers. The orders  $\lambda$  and  $\mu$  are not necessarily integers. It can be any real numbers. Taking  $\lambda = 1$  and  $\mu = 1$ , we obtain a classical PID controller. If  $\mu = 0$ , we can obtain  $PI^\lambda$  controller, etc. All these type of controllers are particular case of the  $PI^\lambda D^\mu$  controller, which is more flexible and gives an opportunity to better adjust the dynamical properties of the fractional-order control system. In this article, we have used particle swarm optimization (PSO) [13] technique to design fractional order  $PI^\lambda D^\mu$  controller.

The Particle swarm optimization algorithm attempts to mimic the natural process of group communication of individual knowledge, which occurs when a social swarm elements flock, migrate, forage, etc. in order to achieve some optimum property such as configuration or location. The swarm is

initialized with a population of random solutions. Each particle in the swarm is a different possible set of the unknown parameters to be optimized. Representing a point in the solution space, each particle adjusts its flying toward a potential area according to its own flying experience and shares social information among particles. The goal is to efficiently search the solution space by swarming the particles toward the best fitting solution encountered in previous iterations with the intent of encountering better solutions through the course of the process and eventually converging on a single minimum error solution. The investigated PSO-based method for finding a solution to the FOPID controller design problem is described as follows:

*Step-01:* Create a uniformly distributed population of particles.

*Step-02:* Evaluate each particle's position according to the objective function.

*Step-03:* If a particle's current position is better than its previous best position, update it.

*Step-04:* Determine the best particle (according to the particle's previous best positions).

*Step-05:* Update particle's velocities.

*Step-06:* Move particles to their new positions.

*Step-07:* Go to step 2 until stopping criteria are satisfied.

*Step-08:* Latest position vector of the best particle is the optimized values of controller design parameters.

#### V. FRACTIONAL ORDER CONTROLLER DESIGN FOR BALL AND BEAM SYSTEM

Let transfer function of fractional order feedback control system is given by

$$P_{cl}(s) = \frac{P(s)C(s)F(s)}{1 + P(s)C(s)F(s)} \quad (28)$$

Let  $\phi_m$  is required phase margin of the system. In order to achieve required phase margin, our controller  $C(s)$  must satisfy

$$\begin{aligned} |P(j\omega)C(j\omega)F(j\omega)| &= 1 \\ \Rightarrow C(j\omega) &= \frac{1}{|P(j\omega)F(j\omega)|} e^{j\phi_m} \end{aligned} \quad (29)$$

If  $\frac{1}{|P(j\omega)F(j\omega)|} = K$ , we can write (29) as

$$\begin{aligned} C(j\omega) &= K e^{j\phi_m} \\ \Rightarrow C(j\omega) &= K \cos(\phi_m) + jK \sin(\phi_m) \end{aligned} \quad (30)$$

Now on substituting  $s = j\omega$  in (27), we can get

$$\begin{aligned} C(j\omega) &= K_P + \frac{K_I}{(j\omega)^\lambda} + K_D(j\omega)^\mu \\ \Rightarrow C(j\omega) &= K_P + K_I \omega^{-\lambda} \cos\left(\frac{\pi}{2}\lambda\right) + K_D \omega^\mu \cos\left(\frac{\pi}{2}\mu\right) \\ &\quad + j \left( -K_I \omega^{-\lambda} \sin\left(\frac{\pi}{2}\lambda\right) + K_D \omega^\mu \sin\left(\frac{\pi}{2}\mu\right) \right) \end{aligned} \quad (31)$$

On equating real and imaginary parts of (30) and (31) for  $C(j\omega)$  we get,

$$\begin{aligned} f_1(\lambda, \mu) = R &= K_P + K_I \omega^{-\lambda} \cos\left(\frac{\pi}{2}\lambda\right) + K_D \omega^\mu \cos\left(\frac{\pi}{2}\mu\right) \\ &\quad - K \cos(\phi_m) = 0 \end{aligned} \quad (32)$$

$$\begin{aligned} f_2(\lambda, \mu) = I &= -K_I \omega^{-\lambda} \sin\left(\frac{\pi}{2}\lambda\right) + K_D \omega^\mu \sin\left(\frac{\pi}{2}\mu\right) \\ &\quad - K \sin(\phi_m) = 0 \end{aligned} \quad (33)$$

$$P = \tan^{-1}\left(\frac{I}{R}\right) \quad (34)$$

where R=real part of the complex expression, I=imaginary part of the complex expression and P=phase(=  $\tan^{-1}\left(\frac{I}{R}\right)$ ).

Now define an objective function

$$f = |R| + |I| + |P| \quad (35)$$

and minimize  $f$  using particle swarm optimization (PSO) technique to find out the optimum solution set  $\{K_P, K_I, K_D, \lambda, \mu\}$  for which  $f = 0$ .

The solution space is five-dimensional, the five dimensions being  $K_P, K_I, K_D, \lambda$  and  $\mu$ . So each particle has a five dimensional position and velocity vectors.

The limits on the position vectors of the particles (i.e. the controller design parameters) are set by us as follows.

As a practical assumption, we allow  $K_P$  to vary between 1 and 1000,  $K_I$  and  $K_D$  between 1 and 500,  $\lambda$  and  $\mu$  between 0 and 2. Initializations of the five variables are also done in the above mentioned ranges. For unity feedback loop, we also consider the gain margin for our plant model  $K = \frac{1}{|P(j\omega)F(j\omega)|} = 1$  at phase crossover frequency  $\omega = 0.8367$ .

After running the PSO algorithm in MATLAB, we obtained the position vector of the best particle, i.e. the optimized values of controller design parameters  $\{K_P, K_I, K_D, \lambda, \mu\}$  as follows:

$$\begin{cases} K_P = 20.4501 \\ K_I = 1.43 \\ K_D = 10.233 \\ \lambda = 0.9191 \\ \mu = 0.8845 \end{cases} \quad (36)$$

On substituting the value of design parameters from (36) into (27), we got the following fractional order PID controller for ball and beam system:

$$C(s) = 20.4501 + \frac{1.43}{s^{0.9191}} + 10.233s^{0.8845} \quad (37)$$

#### VI. SIMULATION AND RESULTS ANALYSIS

The transfer function of unity feedback control loop with fractional order controller (37) and the ball beam system (13) has the following form:

$$\begin{aligned} G_{cl} &= \frac{G_o(s)}{1 + G_o(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)} \\ &= \frac{7.1631s^{1.8036} + 14.3151s^{0.9191} + 1.001}{s^{2.9191} + 7.1631s^{1.8036} + 14.3151s^{0.9191} + 1.001} \end{aligned} \quad (38)$$



where  $G_o(s)$  is the transfer function of the open control loop with

$$G_o(s) = \frac{7.1631s^{1.8036} + 14.3151s^{0.9191} + 1.001}{s^{2.9191}} \quad (39)$$

The fractional order ball beam feedback control system

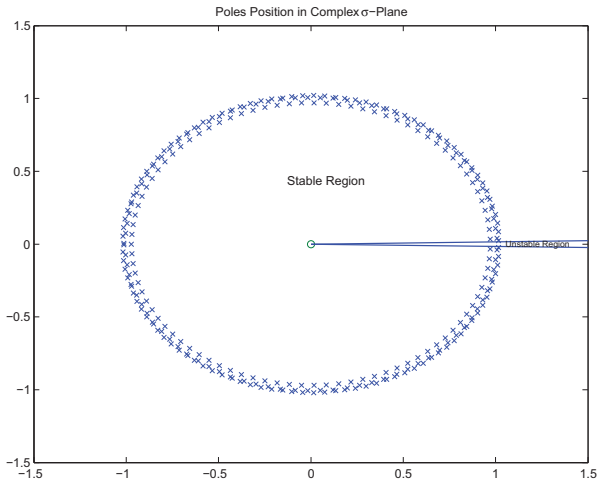


Fig. 7. Stability region of ball beam system controlled by fractional order  $PI^\lambda D^\mu$  controller

is simulated in MATLAB environment under `fotf` class using reference [3]. The stability region of ball beam system controlled by fractional order  $PI^\lambda D^\mu$  controller is shown in Fig.7.

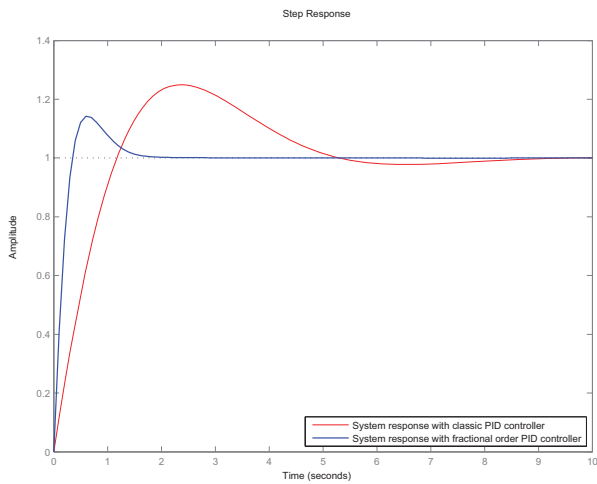


Fig. 8. Comparison of unity step response of ball beam feedback system with integer order PID controller and with fractional order PID controller

To compare the system performance, we also consider the integer order classic PID controller for our plant model. Using MATLAB `PID tool`, we got the following PID controller:

$$C(s) = 1.36 + \frac{0.032}{s} + 1.73s \quad (40)$$

In Fig.8, comparison of the unit step response of the ball beam feedback system controlled by fractional order PID

controller and integer order classic PID controller is given. The fractional order PID controller design exhibits a very negligible overshoot and effectively achieves its steady state within 2 second only, whereas classic PID controller exhibits a large overshoot and achieves its steady state after 8 second. The conclusion is that the use of the fractional order controller leads to an improvement of performance of the system (see Fig.8). We also find that the fractional order controller increases the stability region of the system (see Fig.7). The bode diagram of the controlled model is also presented in Fig.9. It can be seen that phase margin  $\phi_m \approx 60^\circ$  which satisfy our desired specifications.

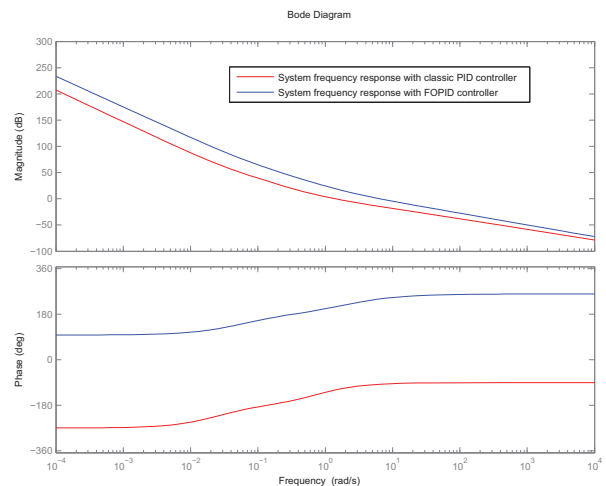


Fig. 9. Comparison of frequency response of ball beam feedback system with integer order PID controller and with fractional order PID controller

## VII. CONCLUSION

In this paper, a case study of fractional order feedback control of the ball and beam system is presented. Stability and performance analysis of fractional order control for the ball and beam system is investigated. The basic ideas and technical formulations for the analysis of fractional order control systems are also briefly illustrated. The design algorithm for fractional order  $PI^\lambda D^\mu$  parameters uses a phase margin specification of open control loop. Simulation results show that fractional order  $PI^\lambda D^\mu$  controller outperforms in ball beam plant model. The major purpose of this paper is to draw attention to the non-conventional way of system analysis and its control. We believe that fractional order control can benefit control engineering practitioners in a number of ways.

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