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Fractional Order Controller Design for Vibration Attenuation in an Airplane Wing

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***Abstract*—**The wing is one of the most important parts of an airplane because it ensures stability, sustenance and maneuverability of the airplane. Because of its shape, the airplane wing can be simplified to a smart beam. Active vibration suppression is realized using piezoelectric actuators that are mounted on the surface of the beam. This work presents a tuning procedure of fractional order controllers based on a graphical approach of the frequency domain representation. The efficacy of the method is proven by practically testing the controller on a laboratory scale experimental stand.

***Keywords*—**fractional order controller, piezoelectric actuators, smart beam, vibration suppression.

I. Introduction

VIBRATION suppression techniques and aero elasticity allow airplanes to improve safety and flight quality. Also, the damage produced by engine vibration is minimized increasing the airplane's lifespan and minimizing the fuel consumption. Atmospheric turbulences represent an unavoidable and significant source of vibration during flight and their presence is critical in aircraft design. Also, solutions to actively suppress the effects of turbulences are taken into consideration and implemented.

The airplane wing can be seen as a cantilever beam, or a smart beam if it is equipped with sensors and/or actuators. This is the reason why most studies that tackle the problem of wing vibration prove the veracity of the solutions on smart beams , . The active vibration suppression is realized using piezoelectric materials attached to the surface of the beam. From a practical point of view, framing the piezoelectric materials on the surface is preferred, rather than embedding them since the first option facilitates access for inspection and reduce manufacturing and maintenance costs . Another advantage is the fact that piezoelectric actuators have no effect on the aerodynamics without generating auxiliary magnetic fields .

There are many studies and methods developed for vibration mitigation using a wide range of tuning methods, from fuzzy controllers to the more complex fractional order controllers. References , present the design of a fuzzy controller that suppresses bending and torsional vibrations of the airplane wing. In , a controller based on Linear Quadratic Theory is developed and implemented on a smart cantilever beam. The efficacy of using adaptive control in the case of beam vibrations is detailed in . Fractional order controller design and implementation on a smart beam is presented in .

Fractional order tuning methods and fundamentals are presented in , while presents the tuning of a fractional order Proportional Derivative controller by imposing frequency domain constraints such as phase and magnitude designed specifically for smart beam vibration suppression. Fractional order controllers represent an alternative to traditional integer order controllers. Advantages in using fractional calculus come from the increased number of tuned parameters which offers flexibility and a better characterization of the system. It is considered that fractional order $PI^{\lambda}D^{\mu}$ controllers honor more performance specifications and are more robust that integer order controllers, but are more difficult to tune since there are five parameters: k_p, k_i, k_d, λ and μ and fractional calculus is more complex than the integer one.

This paper presents the design of a fractional order $PI^{\lambda}D^{\mu}$ fractional order controller based on a graphical approach. The five parameters are determined through optimization techniques in the frequency domain.

II. Tuning Procedure

The novelty of the tuning procedure presented is directly addressing the resonant peak. Analyzing the Bode diagram of an underdamped process it can be observed a peak on the magnitude plot at the natural frequency of the process. When the system is excited with a sine wave having the frequency equal to its natural frequency, the amplitude increases with time exhibiting a behavior known as resonance. The fractional order controller further developed and tested is tuned with the purpose of lowering the resonant peak of the closed loop system.

A similar tuning procedure was used in to tune a fractional order PD controller that suppresses vibrations in civil structures using tuned mass dampers. In this paper, a fractional order $PI^{\lambda}D^{\mu}$ controller is tuned that has to successfully reject disturbances.

The fractional order transfer function of $PI^{\lambda}D^{\mu}$ is:

$$H_{FO-PID}(s) = k_p + k_i \cdot \frac{1}{s^{\lambda}} + k_d \cdot s^{\mu} \quad (1)$$

where λ is the order of the integrator and μ is the order of the differentiator. In the case of a classic PID, λ and μ are equal to 1, but if the controller is fractional the values of λ and μ usually belong to [0, 1]. However, there are cases when better results

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are obtained by expanding the interval to [0, 2]. The variables k_p, k_i, k_d represent the proportional, integral and derivative gains. When the transfer function of the controller is expressed in the frequency domain, s is replaced with $j\omega$, where j is the imaginary unit and ω is the frequency (rad/s).

Expanding (1) based on the trigonometric form of a complex number gives:

$$H_{FO-PID}(j\omega) = k_p + k_i\omega^{-\lambda}cos\frac{\pi\lambda}{2} - jk_i\omega^{-\lambda}sin\frac{\pi\lambda}{2} + k_d\omega^{\mu}cos\frac{\pi\mu}{2} + jk_d\omega^{\mu}sin\frac{\pi\mu}{2} \quad (2)$$

The terms in (2) can be rearranged such that the equation is expressed as a sum of the real and imaginary parts.

The real part of the controller is noted with *ReC* and the imaginary part is noted with ImC.

$$ReC(\omega) = k_p + k_i\omega^{-\lambda}cos\frac{\pi\lambda}{2} + k_d\omega^{\mu}cos\frac{\pi\mu}{2}$$

$$ImC(\omega) = k_d\omega^{\mu}sin\frac{\pi\mu}{2} - k_i\omega^{-\lambda}sin\frac{\pi\lambda}{2}$$

$$H_{FO-PID}(j\omega) = ReC(\omega) + jImC(\omega) \quad (3)$$

The tuning procedure is exemplified on second order plant characterized by the transfer function:

$$H_p(s) = \frac{k}{s^2+2\xi\omega_n s+\omega_n{}^2} \quad (4)$$

which can also be expanded into trigonometrical form:

$$ReP(\omega) = \frac{k(\omega_n{}^2 - \omega^2)}{(\omega_n{}^2 - \omega^2)^2 - (2\xi\omega_n\omega)^2}$$

$$ImP(\omega) = \frac{-2k\xi\omega_n\,\omega}{(\omega_n{}^2 - \omega^2)^2 - (2\xi\omega_n\omega)^2}$$

$$H_p(j\omega) = ReP(\omega) + jImP(\omega) \quad (5)$$

The method is not limited to a second order process. It can be expanded for any order.

The closed loop transfer function is:

$$H_o(s) = \frac{H_p(s)\cdot H_{FO-PID}(s)}{1+H_p(s)\cdot H_{FO-PID}(s)} \quad (6)$$

from which the equation of the closed loop magnitude is obtained as:

$$|H_o(s)| = \frac{\sqrt{(ReP^2+ImP^2)(ImC^2+ReC^2)}}{\sqrt{1+(ReP^2+ImP^2)(ReC^2+ImC^2)+2\,ReC\,ReP-2\,ImC\,ImP}} \quad (7)$$

The parameters of the fractional order controller are determined by imposing the magnitude for the closed loop of the system at certain frequencies.

$$\begin{cases} |H_o(j\omega_1)| = x\,dB, & \omega_1 = a\,rad/s \\ |H_o(j\omega_r)| < y\,dB, & \omega_r = b\,rad/s \\ |H_o(j\omega_2)| = z\,dB, & \omega_2 = c\,rad/s \end{cases} \quad (8)$$

The values are imposed based on the magnitude plot of the transfer function of the process. First, the frequency range of interest should be determined and it must contain the resonant peak. There is not a minimum size for the interval, but it must include the frequencies of interest. In this method, robustness is not a parameter that influence the obtained controller and it is obtained by choosing a wider frequency of interest when imposing the constraints. The constraints from (8) are not limited to the specified equality/inequalities, but it is suggested that only the resonant peak is constrained through an inequality in order to limit the number of found solutions. Apart from the constraints in (8), the solution should be limited such that the gains k_p, k_i, $k_d > 0$ and $\lambda, \mu \in (0,2)$. Also, the chosen value for the magnitude of the resonant peak should be chosen such that the magnitude line is smoother and the peak is almost eliminated.

After determining the desired values of the closed loop magnitude, the controller tuning turns into an optimization problem which can be easily solved using MATLAB's Optimization Toolbox. One option is using the *fmincon* function which finds the minimum of a constrained nonlinear multivariable function. The obtained solutions depend on the chosen initial point. By running the optimization starting from one initial point it is not guaranteed that a satisfactory solution is found. A solution to this problem is choosing initial points for the variables close to the expected values and perform the optimization for every combination of initial points. For example, since it is known that $\lambda, \mu \in (0,2)$ a possible point to start from is $\lambda_0 = 0.05$ and $\mu_0 = 0.05$ and optimize for every combination of initial points incremented by 0.05 until $\lambda_0 = 2$ and $\mu_0 = 2$.

It is known that an integrator $1/s$ introduces a slope of -20 dB/decade while s introduces a slope of +20 dB/decade. If the analysis is done for fractional order integrator $1/s^{\lambda}$, it can be observed that the slope varies between 0 and -20 dB/dec, while for the differentiator s^{μ} the slope varies between 0 and +20 dB/decade.

III. Case Study

A. Experimental Setup

The tuning procedure presented in Chapter II, was practically tested on an experimental stand. The stand consists of: smart beam, 4 PZT patches, real time controller and input and output modules.

The experimental setup can be seen in Fig. 1. The beam is made of aluminum and has the following dimensions: 250 mm length, 20 mm height, 1 mm width. It is positioned perpendicular to the ground. The fixed end allows the free end of the beam to vibrate left and right.

For the PZT patches the P-878 DuraAct Power Patch Transducers were chosen. The piezoelectric patches are glued to the fixed end of the beam, two on each size. The patches are used as actuators. The nominal operating voltages are between -20 and 120 V, while the power generation is possible up to the milliwatt range. The dimensions of one patch are 27 mm x 9.5 mm x 0.5 mm and the block force is 44 N.

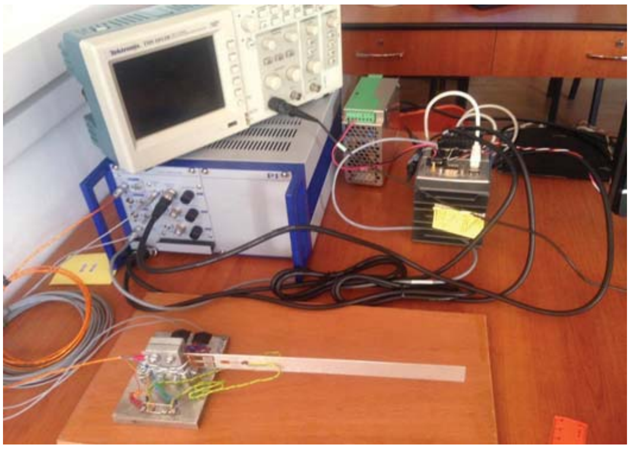

Fig. 1 Experimental setup

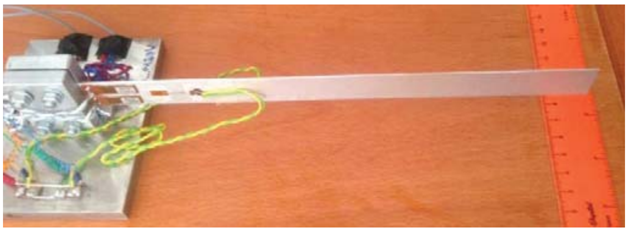

Fig. 2 The smart beam

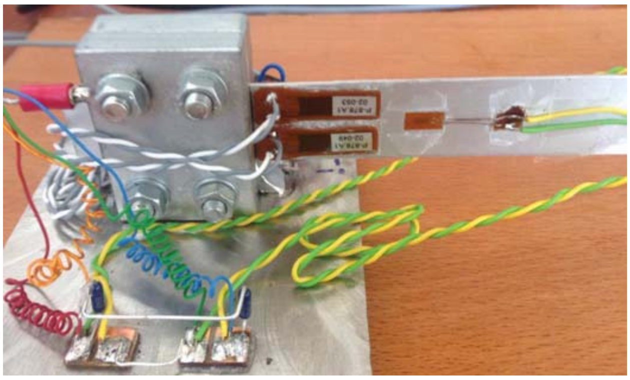

Fig. 3 PZT patches and strain gage sensor mounted on the surface of the beam

The patches are controlled with the E-500 9.5" chassis, which has E-509 PZT-Servo controller and E-503 LVPZT-Amplifier on 3 wires.

The displacement of the free end of the beam is measured by measuring the strain using 120 ohm Omega Prewired KFG-5-120-C1-11L1M2R strain gauge sensors (Fig. 3).

All 4 PZTs working together are able to produce a vibration of amplitude up to 20 mm in the free end of the smart beam. In the experiments performed only the patches from one side were used. Better results can be obtained by using all four patches for vibration attenuation.

For data acquisition and control, the CompactRIO™ 9014 real-time controller was used. The output of the strain gauge actuator is read using the NI 9215 module, while the mechanical stress is induced in the PZTs by the NI 9263 through the E-501 modular piezo controller. The control algorithm and the communication is implemented using LabVIEW ™.

For identification purposes, the NI Educational Laboratory Virtual Instrumentation Suite (NI ELVIS™) was used to generate a sine wave.

B. System Identification

The system identification was performed experimentally based on the response of the system to a sine wave of amplitude 2 and frequency 14.75 Hz. The transfer function that describes the system is:

$$H_f(s) = \frac{49.62}{s^2+1.286s+5857} \tag{9}$$

The fit of the sinus response of the second order transfer function and the experimental data is greater than 90%.

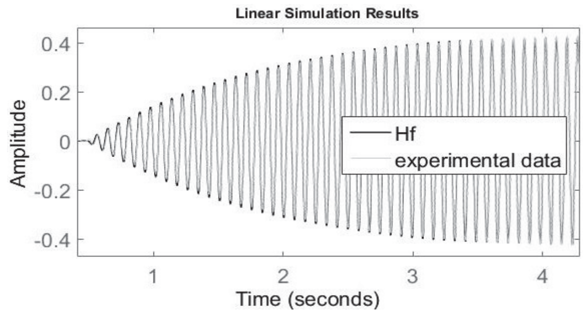

Fig. 4 Sinus response of Hf compared to the experimental data

Since a second order transfer function is able to accurately describe de system, there is no need to determine a higher order approximation. Using a higher order, the volume of calculus increases without considerable improvements in the closed loop response.

C. Fractional Order Controller Tuning

Since the transfer function of the process is identified, the real and imaginary parts expressed in (5) are known.

The next step is to determine the constraints that have to be imposed. Since the $PI^{\lambda}D^{\mu}$ controller has 5 parameters, 5 constraints will be imposed. These are determined by analyzing the Bode magnitude plot and determining the frequencies and corresponding values in the frequency range of interest.

$$\begin{cases} |H_o(j\omega)| = -41\,dB,\ \omega = 10\,rad.s \\ |H_o(j\omega)| = -41.5dB,\ \omega = 23\,rad/s \\ |H_o(j\omega)| < -42\,dB,\ \omega_r = 76.5\,rad/s \\ |H_o(j\omega)| = -86\,dB,\ \omega = 10^3 rad/s \\ |H_o(j\omega)| = -126\,dB,\ \omega = 10^4\,rad/s \end{cases} \tag{10}$$

As can be seen in (10), only the resonant peak has an inequality constraint, while the rest are equality constraints. These are not the only possible values; any combination of constraints should achieve satisfactory results for the imposed range.

The frequency domain plot of the open loop system and the coordinates of the imposed constraints can be seen in Fig. 5.

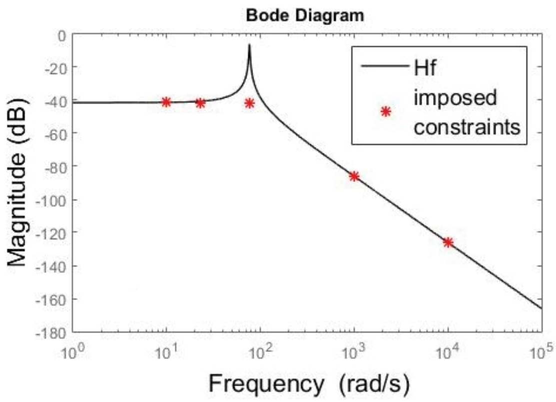

Fig. 5 Open loop magnitude plot and imposed constraints

The initial conditions for which the optimization was performed were chosen k_p, k_i, k_d, $\mu \in [0,1]$ and $\lambda \in [-1,0]$. The optimization algorithm was applied for every combination of initial parameters in the previously mentioned intervals with a step of 0.1.

One of the controllers that honors the constraints has the transfer function:

$$H_{FO-PID}(s) = 2 \cdot \frac{1}{s^{-0.96}} + 0.001 \cdot s^{0.696} \quad (11)$$

The values found for the proportional gain was 0 which is not surprising since we do not want to move the plot up or down, we only want to lower the peak. Even if the derivative gain has a very small value, it makes a considerable difference in the closed loop Bode plot.

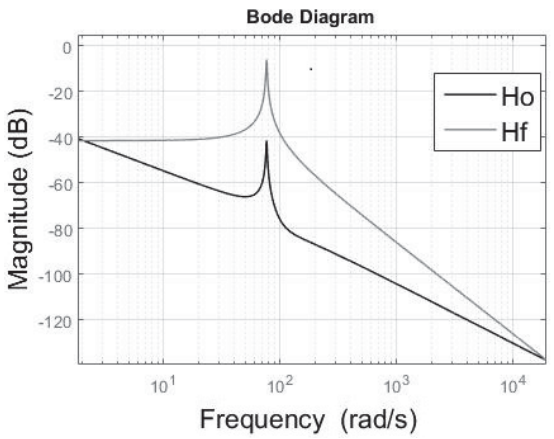

Fig. 6 Magnitude plots of the open and closed loop system

As it can be seen in Fig. 5, all the equality constraints imposed in (10) are honored. The resonant peak has not disappeared, but the peak magnitude has lowered.

Since fractional order transfer functions cannot be directly implemented practically, they have to be approximated to integer order transfer functions. Several approximation methods have been developed among the years such as Continued Fraction Expansion, Oustaloup Filter Approximation, Modified Oustaloup Filter, etc., [13].

The method chosen for approximation is a novel digital implementation that maps the discrete time operator z^{-1} to the Laplace operator s and evaluates the discrete frequency response of the fractional order system [14].

Since the purpose of real life active vibration suppression techniques is to reject unwanted vibrations, the performance of the controller was tested based on impulse type disturbances. In the case of an airplane wing, the vibration suppression algorithm has the purpose of eliminating all disturbances caused by atmospheric turbulences.

The amplitude of the impulse disturbance is 1 cm and it was applied to the free end of the beam.

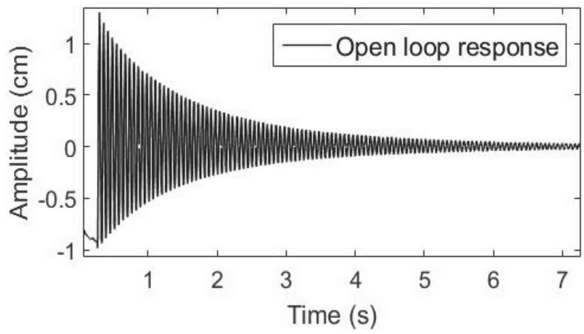

Fig. 7 Impulse type disturbance response of the uncompensated system

Fig. 6 shows the disturbance response of the open loop system with a settling time between 5 and 6 seconds.

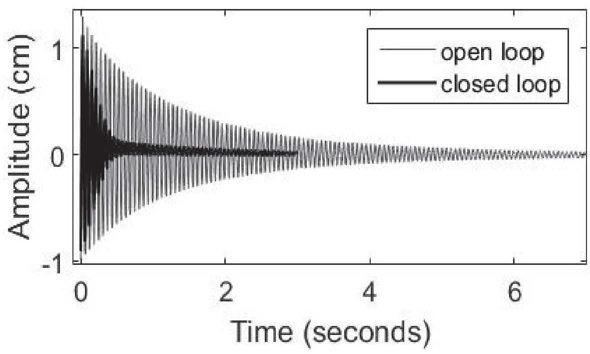

Fig. 8 Closed loop impulse disturbance response

The closed loop response to a disturbance equal in amplitude is shown in Fig. 7 overlaid on the uncompensated system response. In Fig. 6, it can be see that the oscillations of the beam are mitigated in approx. 0.4 seconds, improving the open loop settling time by 92%.

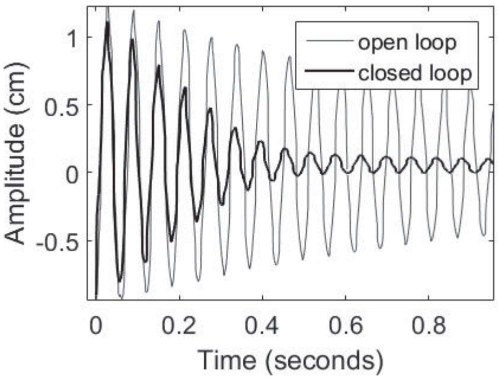

Fig. 9 Zoomed disturbance response of the closed loop

An important remark is the fact that the disturbance response of the closed loop is obtained by using only 2 patches from the same side. If all 4 patches would be used the response would be improved even further.

IV. Conclusion

Airplane wings can be modeled as smart beams due to their shape and vibration characteristics. Piezoelectric actuators are ideal in vibration mitigation of the airplane wing.

The graphical frequency domain method presented based on lowering the resonant peak can be successfully used to tune fractional order controllers.

The obtained controller successfully rejects impulse type disturbances of the free end of the smart beam, bringing improvements of the settling time with up to 92%.

Acknowledgment

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