Flow Field Analysis of Submerged Horizontal Plate Type Breakwater

Ke Wang, Zhi-Qiang Zhang, Z. Chen

Abstract—A submerged horizontal plate type breakwater is pointed out as an efficient wave protection device for cage culture in marine fishery. In order to reveal the wave elimination principle of this type breakwater, boundary element method is utilized to investigate this problem. The flow field and the trajectory of water particles are studied carefully. The flow field analysis shows that: the interaction of incident wave and adverse current above the plate disturbs the water domain drastically. This can slow down the horizontal velocity and vertical velocity of the water particles.

Keywords—boundary element method, plate type breakwater, flow field analysis

I. INTRODUCTION

WITH the development of marine aquaculture, the traditional cage culture is moving from coastal to deep sea. In order to maintain the safety of sea cages, a submerged horizontal plate type breakwater is pointed as an efficient wave elimination device. This type breakwater can be constructed quickly with low cost and be little affected by water depth and geological conditions. The study of its hydrodynamic characteristics has aroused increasing attention in recent years in the field of ocean engineering ([1]-[3]). The main design concept of this type breakwater is the attenuation of part of the wave energy, which can keep the huge wave forces from acting directly on the marine structures. Besides, the free water exchanges behind the breakwater can keep the sea water clean and marine ecosystem uninfluenced.

Many mathematical derivations ([4]-[7]) and experiments ([8]-[12]) have been done in the study of the hydrodynamic characteristics of floating breakwater. Parson and Martin ([13]) established a high order singular integral equation, and studied the diffraction of the submerged horizontal plate by solving the discontinuous velocity potential on its both sides. Neelamani and Reddy ([14]), Yu and Chwang ([15]) solved the diffraction problem of the horizontal plate in finite water depth using the eigenvalue approximation in finite region. Hu, Wang and Williams ([16]) presented a two-dimensional analytical solution to study the reflection and transmission of linear water waves propagating past a submerged horizontal plate and through a vertical porous wall.

Chen, Chen and Lin *et al.*([17]) solved the scattering problem of normal incident wave passing a thin vertical and inclined barrier with rigid boundary condition which is descending from the water surface to a depth by the dual integral formulation. Usha and Gayathri ([18]) considered the scattering of surface waves by a submerged, horizontal plate or disc, by using eigenfunction expansions within the finite domain. Zheng , Shen and Ng ([19]) studied the hydrodynamic coefficients and wave exciting forces for long horizontal rectangular and circular structures by boundary element method. Liu, Li and Teng ([20]) investigated the hydrodynamic performance of a submerged two layer horizontal plate breakwater by the matched eigenfunction expansion method. Guo, Zhang and Li *et al.* ([21]) studied the wave transmission characteristics and wave induced pressures on twin plate breakwater under regular and random waves experimentally. Liu, Li and Li *et al.* ([22]) used a new approximate analytic solution for water wave scattering by a submerged horizontal porous disk in the context of the linear potential theory. However, no clear and direct elaboration has been made about wave elimination mechanism from the viewpoint of flow field analysis. Therefore, the present study adopts the boundary element method of Green function in infinite water depth to solve the diffraction problems of the submerged horizontal plate and obtain the distribution of flow filed around the plate. The numerical simulation confirms that this method has sufficient precision to be applied in calculations and analysis of the flow field around submerged horizontal plate.

In section 2, basic formulas and numerical calculation methods concerning the flow field surrounding the submerged horizontal plate are elaborated. Section 3 provides the calculation results and discussion. Finally, section 4 summarizes the flow field analysis of submerged horizontal plate under different wave conditions.

II. COORDINATE SYSTEM AND THEORETICAL FORMULAS

A. Governing Equation and Boundary Conditions

Fig.1 shows the definition of Cartesian coordinate system oxy and plate location. This coordinate system is stationary related to the undisturbed position of the free surface and floating object. The origin o is located on the static water surface, x-axis represents the horizontal coordinate, and y-axis represents the vertical coordinate with upward direction positive. The incident wave of unit amplitude is propagating from right to left along the x-axis. Besides, plate length is $B = 2a$, plate thickness is TT and plate submergence is H_S.

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It is also supposed that the plate is rigid and very thin, and the normal vector is positively pointing out of the fluid domain.
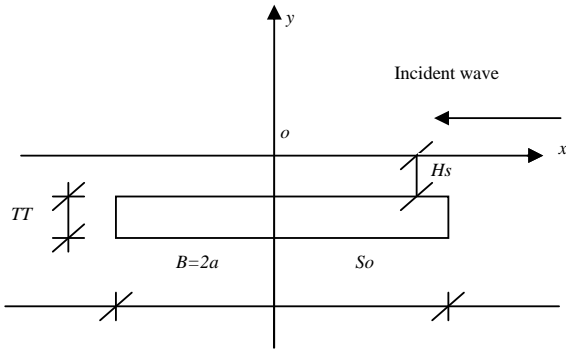

Fig. 1 Calculation sketch of submerged horizontal plate

The fluid is assumed to be non-viscous, incompressible and irrotational. And supposing the motion of the object is harmonic oscillation. Then, the fluid velocity can be expressed by the gradient of velocity potential Φ. The velocity potential Φ can be defined as $\Phi(x, y; t) = \text{Re}[\phi(x, y)e^{-i\omega t}]$, where $\text{Re}[\phi(x, y)e^{-i\omega t}]$ denotes the real part, ω is the frequency of the incident wave, t is time, $i = \sqrt{-1}$, and ϕ is the spatial complex velocity potential irrelevant to time. The following is the governing equation (Laplace equation) and boundary conditions that ϕ satisfies:

$$\nabla^2 \phi = 0 \qquad \text{in the whole fluid domain } \Omega$$

$$\frac{\partial \phi}{\partial y} - \frac{\omega^2}{g}\phi = 0 \qquad \text{on the free surface } S_F \qquad (1)$$

$$\frac{\partial \phi}{\partial n} = V_n \qquad \text{on the object surface } S_O$$

$$\frac{\partial \phi}{\partial n} = 0 \qquad \text{in infinite water depth}$$

Based on the linear assumption, complex velocity potential ϕ can be decomposed as follows:

$$\phi = \phi_I + \phi_D - i\omega \sum_{i=1}^{3} \overline{x_i}\phi_i \qquad (2)$$

where $\overline{x_i}$ is the motion amplitude of the object, ϕ_I is the incident potential, ϕ_D is the diffraction potential, and ϕ_i is the radiation potential for sway, heave and roll respectively.

The boundary condition for diffraction and radiation potentials is:

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n}, \frac{\partial \phi_i}{\partial n} = (n_1, n_2, n_3) \qquad (3)$$

Here, n_1, n_2, n_3 are the component of normal vector on the object surface and $n_1 = n_x$, $n_2 = n_y$, $n_3 = (y - y_0)n_x - (x - x_0)n_y$, where (x_0, y_0) is the rotating center of the object. The incident wave potential ϕ_I can be expressed as:

$$\phi_I = -\frac{igA}{\omega}e^{Ky - iKx} \qquad (4)$$

Here, ω is the wave circular frequency, $K = \dfrac{\omega^2}{g}$ is the wave number, g is the acceleration of gravity, and A is the amplitude of incident wave.

B. Boundary Integral Equation

The hydrodynamic of submerged horizontal plate can be solved by establishing integral equation on the object surface with Green's theorem. The following is the boundary integral equation about ϕ_i and ϕ_D :

$$C\phi(P) + \int_S \phi(P) \cdot \frac{\partial G(P, Q)}{\partial n} ds$$
$$= \int_S G(P, Q) \cdot \frac{\partial \phi(Q)}{\partial n} ds \qquad (5)$$

Where $\phi(P) = (\phi_i \text{ or } \phi_D)$, $P(x, y)$ is the field point, $Q(\xi, \eta)$ is the source point, $G(P, Q)$ is Green's function, C is the space angle. $G(P, Q)$ can be expressed as follows:

$$G(P, Q) = \log r_1 - \log r_2 - 2I_C \qquad (6)$$

Where

$$r_1^2 = (x - \xi)^2 + (y - \eta)^2 , \; r_2^2 = (x - \xi)^2 + (y + \eta)^2 \qquad (7)$$

$$I_C = \lim_{\mu \to 0} \int_0^\infty \frac{e^{u(y+\eta)} \cdot \cos u(x - \xi)}{u - K + i\mu} du \qquad (8)$$

Equation (8) can be calculated as following ([23]):

Supposing that $X = (x - \xi)$, $Y = (y + \eta)$ and $\theta_2 = \arctan \dfrac{Y}{|X|}$, then

$$I_c = \left\{ E_c \cos(K|X|) + E_S \sin(K|X|) \right\} e^{KY}$$
$$- i\pi e^{KY} \cos(K|X|)$$

$$E_c = -\log\left(r' K r_2\right) - \sum_{i=1}^{\infty} \frac{\left(K r_2\right)^n}{n \cdot n!} \cdot \cos n\left(\theta_2 - \frac{\pi}{2}\right) \tag{9}$$

$$E_s = \left(\theta_2 + \frac{\pi}{2}\right) + \sum_{n=1}^{\infty} \frac{\left(K r_2\right)^n}{n \cdot n!} \cdot \sin n\left(\theta_2 - \frac{\pi}{2}\right)$$

Where $r = \log r'$, in which r' is Euler's constant.

If $\bar{f}$ represents the incident potential f_I, diffraction potential f_D, radiation potential $f_R(f_1, f_2, f_3)$ and total velocity potential ϕ respectively, the velocity of flow field (u, v) induced by $\bar{f}$ can be obtained through the following equation:

$$u = \frac{\P\bar{f}}{\P x}, v = \frac{\P\bar{f}}{\P y} \tag{10}$$

Where $\bar{f}(p)$ at any point p can be solved by the solution of (11):

$$\bar{f}(p) = -\oint \bar{f}(q) \frac{\P G}{\P n} ds + \oint G(p,q) \frac{\P\bar{f}(q)}{\P n} ds \tag{11}$$

C. Boundary Element Discretization

In the numerical method, the object surface is expressed by a series of discrete linear elements with boundary element method. Thus, the coordinates and physical variables of any element can be expressed as follows:

$$x(\varsigma) = N_1(\varsigma)x_1 + N_2(\varsigma)x_2$$
$$y(\varsigma) = N_1(\varsigma)y_1 + N_2(\varsigma)y_2 \tag{12}$$
$$\phi(\varsigma) = N_1(\varsigma)\phi_1 + N_2(\varsigma)\phi_2$$

Where $N_1(\varsigma), N_2(\varsigma)$ are shape functions which can be written as:

$$N_1(\varsigma) = 1 - \frac{\varsigma}{l_N} \quad, \qquad N_2(\varsigma) = \frac{\varsigma}{l_N} \tag{13}$$

Here, $(x_1, y_1), (x_2, y_2)$ are global coordinates for the node of the element, l_N is element length, ς is local coordinates.

Substituting (12) and (13) into (5), then (5) can be changed into the following form:

$$\sum_{l=1}^{n} [H_{il}]\phi_l = \sum_{l=1}^{n} [G_{il}](\frac{\partial \phi}{\partial n})_l \ (i, l = 1, 2, 3, \cdots, n) \tag{14}$$

where

$$H_{il} = C(p_i)\delta_{il} + \int_{\Delta s} N_j(\varsigma) \frac{\partial G}{\partial n} ds$$
$$G_{il} = \int_{\Delta s} N_j(\varsigma) G ds \qquad j = 1,2; \ (i,l = 1,2,3,\cdots,n) \tag{15}$$

In (15), the item related to $\log r_1$ can be solved by analysis method, and the item related to $\log r_2$ and I_c are regular and can be directly calculated with numerical method.

D. Flow Field Velocity

Supposing the fluid domain is discritized into four-node quadrilateral elements, whose shape function is $N_i(i = 1, 2, ..., 4)$, the respective physical quantity in the domain can be expressed as:

$$x = \sum_{i=1}^{n} N_i X_i \ , \ y = \sum_{i=1}^{n} N_i Y_i \ , \ \phi = \sum_{i=1}^{n} N_i \phi_i \tag{16}$$

and the velocity of any point in the fluid domain can be calculated as:

$$u = \frac{\partial \phi}{\partial x} = \sum_{i=1}^{n} \frac{\partial N_i}{\partial x} \phi_i =$$
$$\left[\frac{\partial N_1}{\partial x} \ \frac{\partial N_2}{\partial x} \ \frac{\partial N_3}{\partial x} \ \frac{\partial N_4}{\partial x} \right] [\phi_1 \ \phi_2 \ \phi_3 \ \phi_4]^T \tag{17}$$

$$v = \frac{\partial \phi}{\partial y} = \sum_{i=1}^{n} \frac{\partial N_i}{\partial y} \phi_i =$$
$$\left[\frac{\partial N_1}{\partial y} \ \frac{\partial N_2}{\partial y} \ \frac{\partial N_3}{\partial y} \ \frac{\partial N_4}{\partial y} \right] [\phi_1 \ \phi_2 \ \phi_3 \ \phi_4]^T \tag{18}$$

Equation (17) and (18) can also be written as:

$$\begin{bmatrix} u \\ v \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{bmatrix} [\phi_i]^T \tag{19}$$

Where $[J]$ is Jacobian determinant.

III. Flow Field Analysis

To further reveal the wave elimination mechanism of plate type breakwater, this section analysis the flow field around the breakwater and explores the motion trajectory of water particles in the whole process of wave elimination. In the calculation, the most effective wave condition is chosen when plate location is $Hs = 0.05m$, $TT = 0.005m$ and $B = 0.4m$. Besides, three kinds of incident waves $KB/2 = 0.4, 0.8, 1.2$ are selected. In this paper, only the change of flow field velocity is given.

A. Velocity Field at $KB/2 = 0.4$

Fig.2 shows the calculation results of velocity field for submerged horizontal plate when $KB/2 = 0.4$, where transmission coefficient is 0.3 and heave wave force is the largest. The plate is symmetric about the *y*-axis, so heave flow field (u_2, v_2) is symmetric, while sway flow field and roll flow field $(u_1, v_1), (u_3, v_3)$ are anti-symmetric. It can be seen from the figure that the plate obviously disturbs the flow field under the action of waves. When the plate sways and rolls, water above the plate move rapidly in the horizontal direction, and water particles in front of and behind the plate move vertically reciprocate in the fluid regions. The different between sway and roll is that: for the sway motion, the water under the plate and above the plate have the same flow direction and the same amplitude; for the roll motion, the flow direction of the water under the plate and above the plate are opposite, and the amplitude under the plate is smaller than the amplitude above the plate. That is mainly because of the motion characteristic. When the plate heaves, water particles also move vertically reciprocate in the whole fluid region around the plate. Since the plate is thin and close to the free surface, the horizontal direction of plate length is the control direction. Because the actual radiation amplitude of the plate is very small, the diffraction velocity is the main flow field in which the water particles change. Therefore, the change of the whole flow field is basically the same with that of the diffraction velocity flow field. It can be known from the figure that water particles above the plate move vertically reciprocate, and separated into two parts along plate length direction, they flow to the fluid regions in front of and behind the plate. where fluid velocity changes dramatically. As a result, downward and forward adverse currents are generated in the head-sea area, which should be the main wave elimination principle of plate type breakwater. The interaction between adverse current above the plate and incident wave leads to the formation of wave elimination area in the head-sea of the plate,

B. Velocity Field at $KB/2 = 0.8$

Fig. 3 shows the flow field at $KB/2 = 0.8$, when wave elimination is also relatively effective and transmission coefficient is also about 0.3. Because that the wave elimination effect of this kind of submerged plate type breakwater is symmetric around $KB/2 = 0.6$. Therefore, for the same transmitted coefficient, the smallest plate length could be chosen. The results reveal that the sway, heave, roll and diffraction flow fields around the plate are basically the same with $KB/2 = 0.4$, but because of the different wave number, the flow direction is a little different about the water away from the plate.
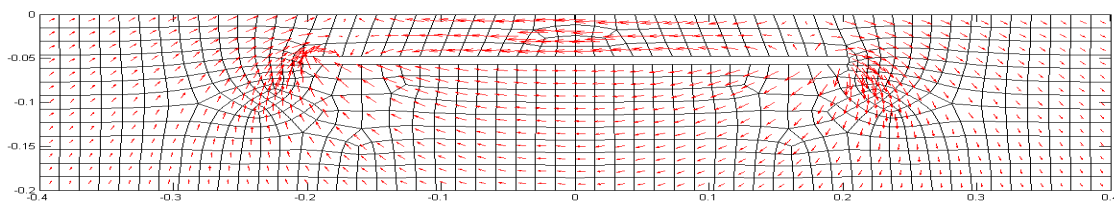
C. Velocity Field at $KB/2 = 1.2$

It can be seen from Fig. 4 that the variation of velocity field when $KB/2 = 1.2$ is basically the same with that when $KB/2 = 0.4, 0.8$. In other words, with a trend of vertical movement, water above the plate forms adverse current, and interacts with the incident wave in front of the plate. Only the flow direction of water above the plate for roll is different from $KB/2 = 0.4, 0.8$. Besides, diffraction flow field is the main flow field.

IV. Conclusion

This study explores the flow field distribution of the fluid domain around the submerged horizontal plate under different wave conditions with the boundary element method. It is found that:

(1) The heave flow field is symmetric, and the sway and roll flow fields are anti-symmetric.

(2) The diffraction velocity field is the main flow field and the changes of the whole flow field are basically the same with that of the diffraction flow field.

(3) Water particles above the plate move vertically up and down, and separated into two parts along the plate length direction.

They flow to the fluid domains in front of and behind the plate. The interaction between adverse current above the plate and incident wave leads to the formation of wave elimination area in the head-sea of the plate, where fluid velocity changes dramatically. As a result, downward and forward adverse currents are generated in the head-sea area, which should be the main wave elimination principle of plate type breakwater.

(4) Because of the plate, the flow velocity of the water particles changes from vertical to horizontal, the activity of wave elimination is obvious.
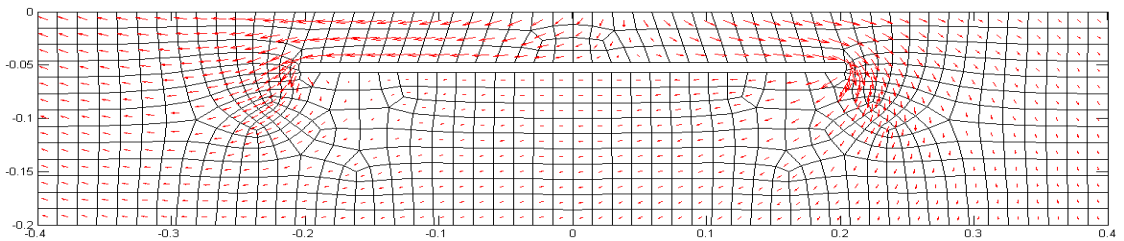

(a) Sway velocity field (u_1, v_1)
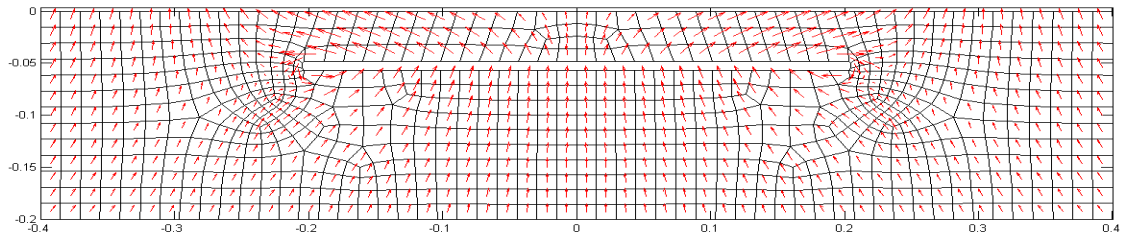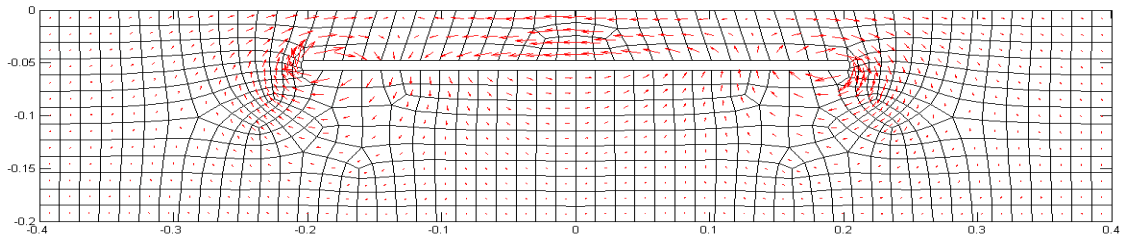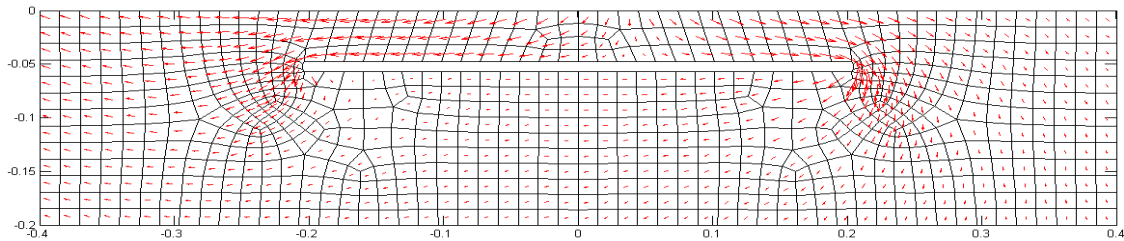
(b) Heave velocity field (u_2, v_2)

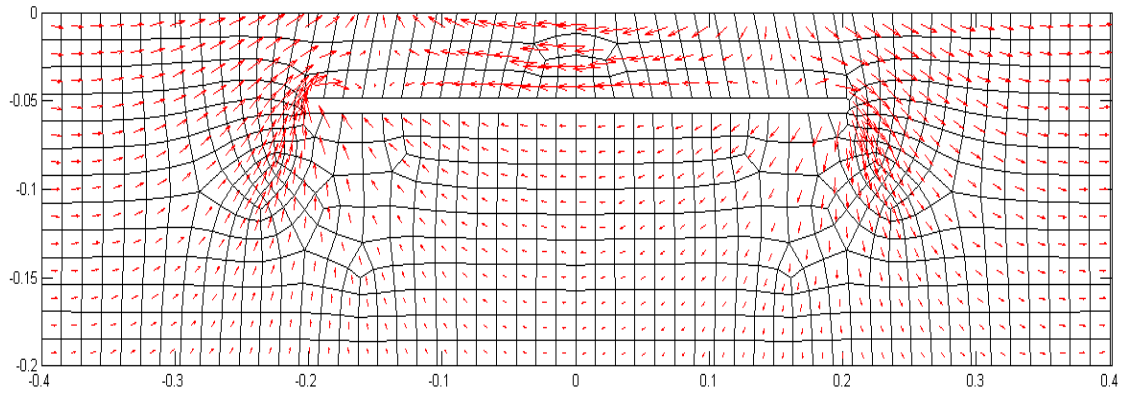(c) Roll velocity field (u_3, v_3)

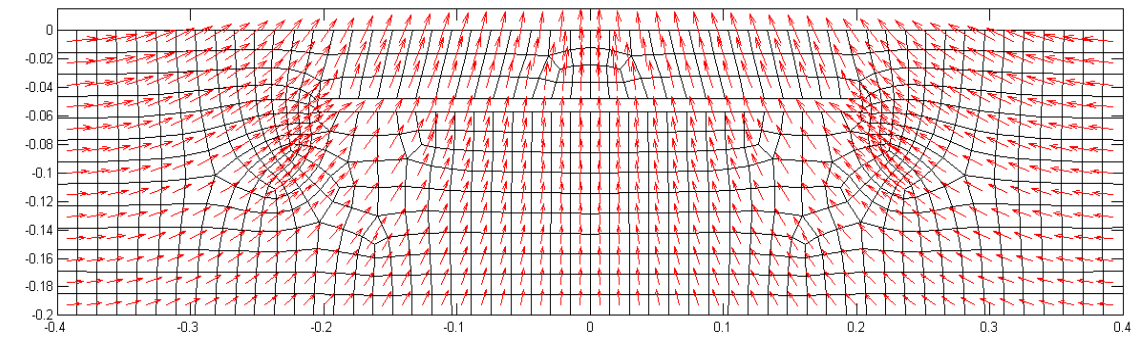(d) Diffraction velocity field (u_4, v_4)

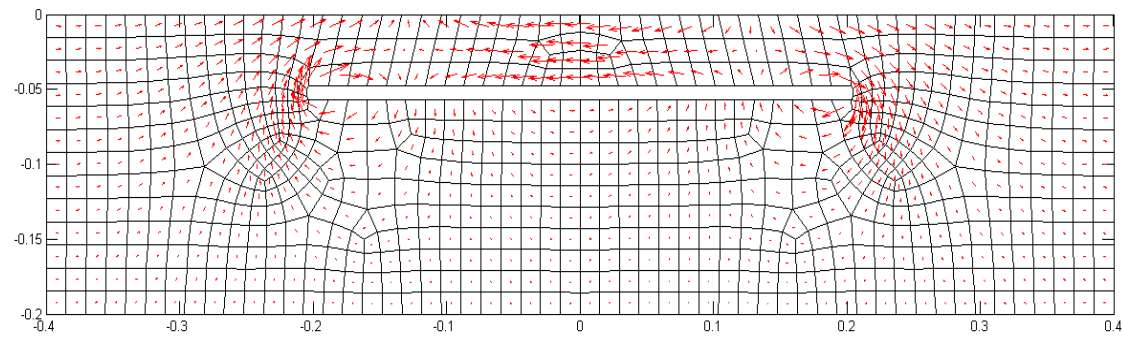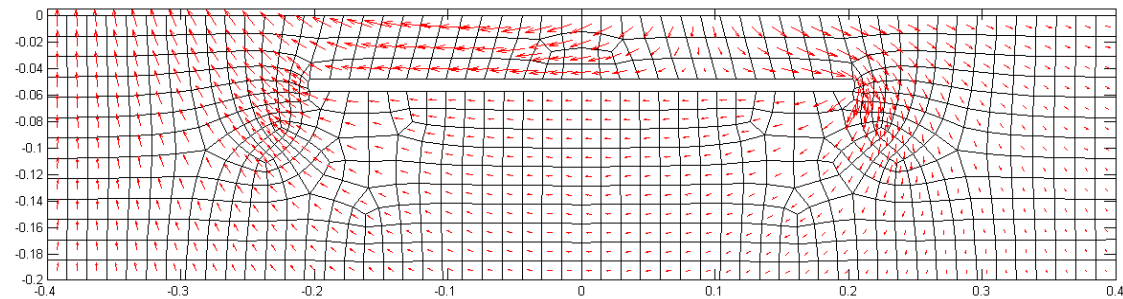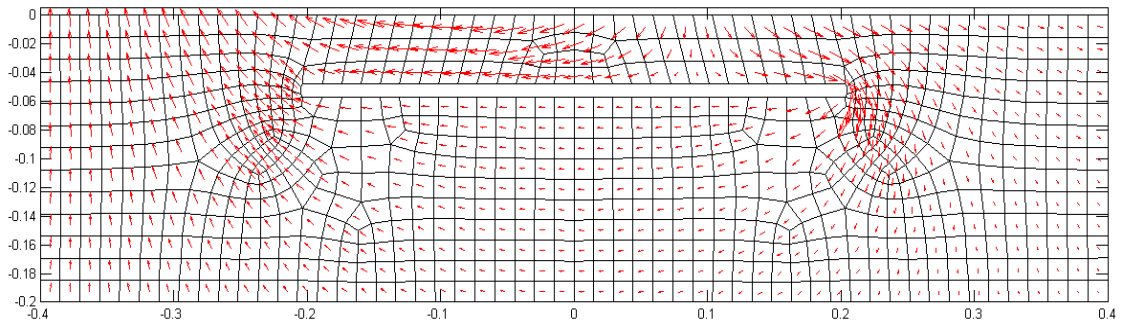(e) Whole velocity field (u_5, v_5)

Fig. 2 Velocity field for submerged horizontal plate at $KB/2 = 0.4$

(a) Sway velocity field (u_1, v_1)

(b) Heave velocity field (u_2, v_2)

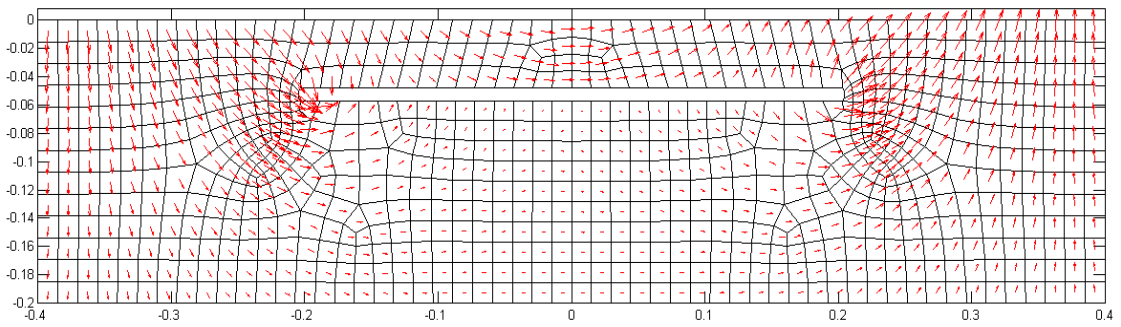(c) Roll velocity field (u_3, v_3)
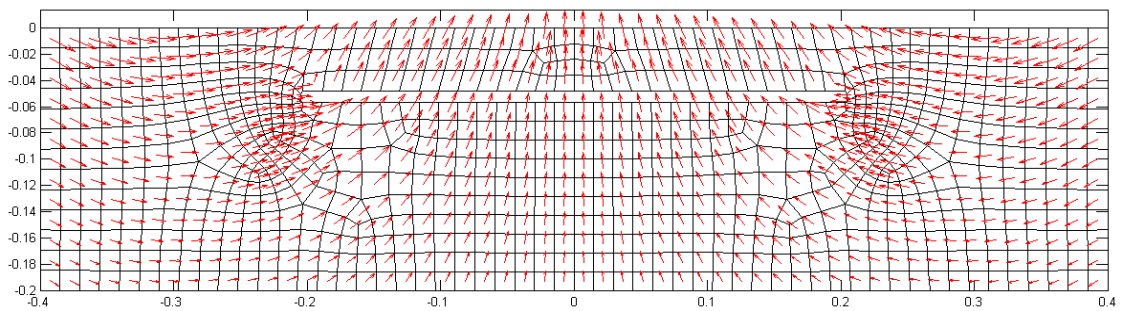
(d) Diffraction velocity field (u_4, v_4)

(e) Whole velocity field (u_5, v_5)
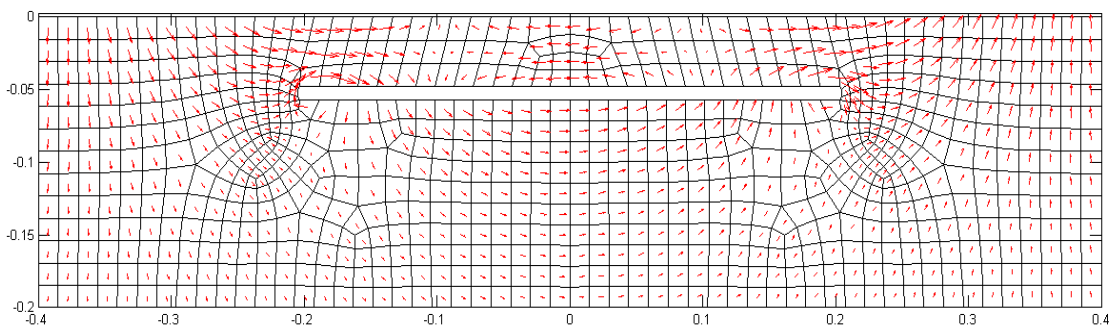
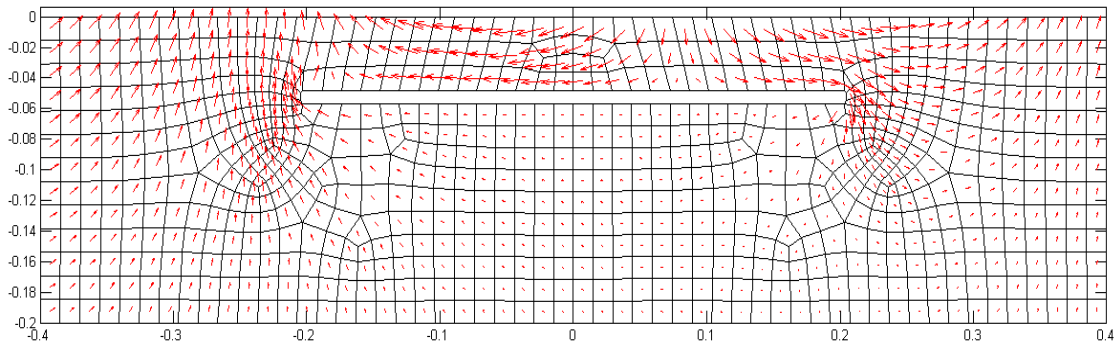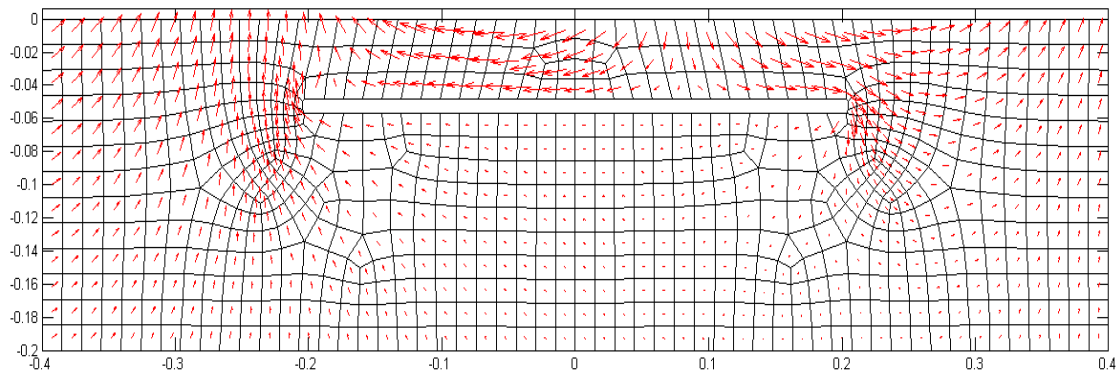Fig. 3 Velocity field for submerged horizontal plate at $KB/2 = 0.8$

(a) Sway velocity field (u_1, v_1)

(b) Heave velocity field (u_2, v_2)

(c) Roll velocity field (u_3, v_3)

(d) Diffraction velocity field (u_4, v_4)

(e) Whole velocity field (u_5, v_5)

Fig. 4 Velocity field for submerged horizontal plate at $KB/2 = 1.2$

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