Flexible Arm Manipulator Control for Industrial Tasks

Mircea Ivanescu, Nirvana Popescu, Decebal Popescu, Dorin Popescu

Abstract—This paper addresses the control problem of a class of hyper-redundant arms. In order to avoid discrepancy between the mathematical model and the actual dynamics, the dynamic model with uncertain parameters of this class of manipulators is inferred. A procedure to design a feedback controller which stabilizes the uncertain system has been proposed. A PD boundary control algorithm is used in order to control the desired position of the manipulator. This controller is easy to implement from the point of view of measuring techniques and actuation. Numerical simulations verify the effectiveness of the presented methods. In order to verify the suitability of the control algorithm, a platform with a 3D flexible manipulator has been employed for testing. Experimental tests on this platform illustrate the applications of the techniques developed in the paper.

Keywords—Distributed model, flexible manipulator, observer, robot control.

I. INTRODUCTION

THIS paper is focused on the implementation of a control system for a class of flexible manipulators with continuum components. Numerous examples of continuum robots can be found in [1]. The notion of continuum arms and the main properties of this class of systems were identified in [2], [3]. The first kinematic models were discussed in [4], [5]. The spatial curve method was used in [6] for the arm shape kinematic control. A new kinematic model by using differential geometry was derived in [7], [8]. Reference [9] introduces new formulations for determining the maximum curvature of the arm and proposes a real-time controller for continuum robots. Reference [10] presents a new nonlinear model-based control strategy for continuum robots. The approach is applicable to continuum robots that can extend/contract, as well as bend throughout their structure. Reference [11] describes a general coordinate-free energy formulation for modelling the shape of concentric tube continuum robots, known as active cannulas. Reference [12] studies the controllability problem of continuum arms. The observer control for a class of flexible manipulators is studied in [13]. The dynamic and differential kinematic models of the micro-robots are developed in [14], [15] and the orientation control is formulated as an optimization problem. Several areas of research, discussed below, inform the results on continuum arm control reported in this paper.

The dynamic model with uncertain parameters is inferred. The standard feedback control design assumes full-state feedback with measurements of the entire state. In this case, the dynamic equations are represented by a new curvature gradient lumped parameter model with uncertain components, a set of ODEs in time, instead of PDEs in time and space, and a robust decoupled algorithm is used for the shape control of the arm.

A PD boundary control algorithm is used in order to achieve a desired position of the manipulator. The constraints on the controller gains are easy to implement, and the numerical simulations and experimental tests verify the effectiveness of the presented techniques.

The paper is divided as follows: Section II presents the technological arm, Section III analyses the mathematical model, Section IV discusses the control problem, Section V verifies the control laws by computer simulation, Section VI presents experimental results and Section VII is dedicated to the conclusions.

II. TECHNOLOGICAL MANIPULATOR

The technological model basis is a 3D model (Fig. 1) consisting of a central, long and thin, highly flexible and incompressible backbone, with a distributed mass. The arm is divided in several segments, each segment having its own driving system. The motion of the arm, the bending, is determined by antagonistic Bowden cables (tendons) attached to the terminal point of each segment and a DC motor driving system. These cables develop the driven torques τ_i, i=1, 2… The driven system is a decoupled one, each torque τ_i controls the motion, bending, of its own segment.

Practical constraints require a boundary placement of the position sensors. The position measuring of the segment is obtained by an angle sensor that is placed on the surface at the terminal sub-regions of each segment. These sensors can measure the position on the boundary of the segment and allow to evaluate the arm curvature.

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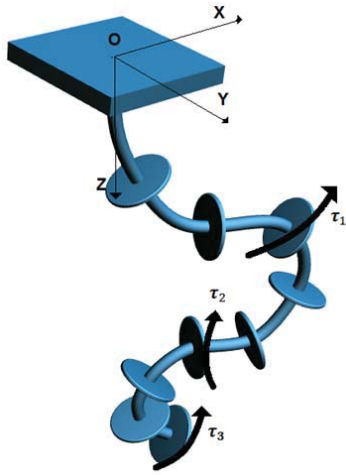
Fig. 1 Flexible technological arm

III. MATHEMATICAL MODEL

We consider a hyper-redundant arm constituted by a serial connection of a number of continuum arm segments (the backbone model), with equal lengths l (Fig. 2). For this arm, the orientation is represented by the two angles $\omega(s), q(s)$, (Fig. 3) [2] and the curvature is defined by:

$$\kappa(s) = \begin{bmatrix} \frac{d\theta^i(s)}{ds} \\ \frac{dq^i(s)}{ds} \end{bmatrix} \qquad (1)$$

In this paper, the state variable is determined by the curvature at the terminal point of the arm $\kappa(t, l)$.

Using the same procedure as in [15], the dynamic model of the system can be expressed as:

$$I_\rho \ddot{\kappa}_i(t, l) + b\dot{\kappa}_i(t, l) + EI^* \kappa_i(t, l) - \frac{2}{l} h_i(\kappa, \dot{\kappa}) = c^* \tau_i(t) \qquad (2)$$

$$i = 1, 2, \dots N$$

$$\kappa_i(0. l) = \kappa_{0i}(l) \qquad (3)$$

where I_ρ is the rotational inertial density coefficient, b, c represent the equivalent damping and elastic coefficients, EI is the equivalent stiffness coefficient,

$$EI^* = (\frac{EI}{l} - \frac{l}{2}c) \qquad (4)$$

$$c^* = \frac{l\,c}{EI} \qquad (5)$$

The state variable is defined by the vector

$$\kappa_i(t, l) = \begin{bmatrix} \kappa_{\theta i}(t, l) \\ \kappa_{qi}(t, l) \end{bmatrix} \qquad (6)$$

and the input of the system is determined by the torque applied at the terminal point of the arm

$$\tau_i(t) = \begin{bmatrix} \tau_{\theta i}(t) \\ \tau_{qi}(t) \end{bmatrix} \qquad (7)$$

and h represents the nonlinear component vector determined by gravitational components and viscous friction, $h_i = (h_1{}^i, h_2{}^i)^T$. We assume that the following constraints are satisfied [18].

$$\|h^i\| \le \eta_1 \|\kappa\| \qquad (8)$$

$$\|h^i\| \le \eta_2 \|\dot{\kappa}\| \qquad i = 1, 2, \dots, N \qquad (9)$$
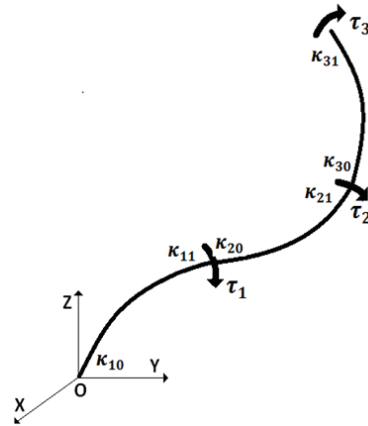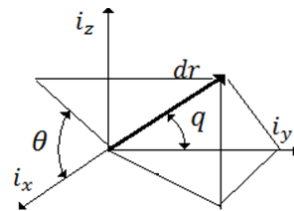

Fig. 2 Backbone model

Fig. 3 Orientation angles

IV. MANIPULATOR CONTROL

The control problem consists of finding the control law $\tau_i(t)$, on the boundary $s_i = l$, such that $(\kappa, \dot{\kappa})$ converge to desired values $(\kappa_d, \dot{\kappa}_d = 0)$ (In order to simplify the notation, the index i will be omitted).
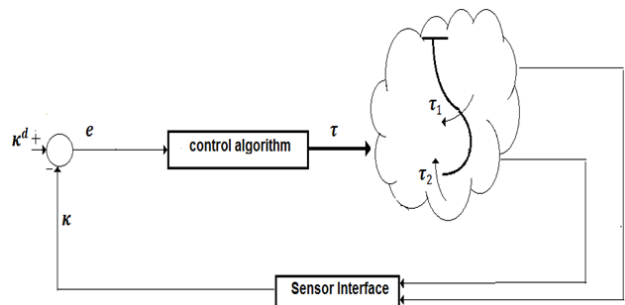

Fig. 4 Manipulator control system

We define the error of the control system as:

$$e_k(t) = \kappa(t,l) - \kappa_d(l) = \begin{bmatrix} \kappa_\theta(t,l) - \kappa_{d\theta}(t,l) \\ \kappa_q(t,l) - \kappa_{dq}(t,l) \end{bmatrix} \quad (10)$$

For the desired position, the following equation holds:

$$EI^* \kappa_d(l) - \frac{2}{l} h(\kappa_d, 0) = c^* \tau_d \quad (11)$$

where τ_d represents the input variable for the steady desired position. In terms of the error (10), the dynamics of the arm segment (2), (3) can be expressed as:

$$I_\rho \ddot{e}_k(t) + b \dot{e}_k(t) + EI^* e_k(t) - \frac{2}{l}\Delta h(\kappa, \dot{\kappa}) = c^* \Delta\tau(t) \quad (12)$$

$$e_k(0) = e_{k0} \quad (13)$$

where;

$$\Delta h(\kappa, \dot{\kappa}) = h(\kappa, \dot{\kappa}) - h(\kappa_d, 0)$$

$$\Delta\tau(t) = \tau(t) - \tau_d \quad (14)$$

Theorem 1. For the system described by (2), (3) if the control law is given by:

$$\tau(t) = -K_1 e_k(t) - K_2 \dot{e}_k(t) \quad (15)$$

where $K_j = diag(k_{\theta j}, k_{qj})$, $k_{\theta j} = k_{qj} = k_j$, $j = 1,2$, are positive controller gains that satisfy the conditions

$$\left(\gamma c^* K_1 + \frac{1}{4} c^*(K_1\beta + K_2\gamma)\right) > \gamma EI^* - \frac{2\gamma}{l}\eta_1 - \frac{1}{4}(\alpha - b\gamma - \beta EI^*) \quad (16)$$

$$(\beta c^* K_2 + c^*(K_1\beta + K_2\gamma)) > \beta EI^* - \frac{2\beta}{l}\eta_2 - (\alpha - b\gamma - \beta EI^*) \quad (17)$$

and α, β, γ are positive constants that satisfy relations

$$\alpha > \frac{\gamma I_\rho}{2}, \beta > 2\gamma \quad (18)$$

The system is exponentially stable.
Proof. The following Lyapunov function is considered

$$V(t) = \frac{1}{2}(\alpha \, e_k^T(t) e_k(t) + \beta I_\rho \dot{e}_k^T(t)\dot{e}_k(t) + 2\gamma I_\rho e_k^T(t)\dot{e}_k(t) \quad (19)$$

By using Young inequality [17] (In order to simplify the notation, the dependence of the system signals from the time and space variables (t,l) will be omitted)

$$a\,b \geq -\frac{a^2}{4} - b^2 \quad (20)$$

Equation (19) can be rewritten as:

$$V \geq \frac{1}{2}\left(\left(\alpha - \frac{\gamma I_\rho}{2}\right) e_k^T e_k + I_\rho(\beta - 2\gamma)\dot{e}_k^T \dot{e}_k\right) \quad (21)$$

And, using the conditions (18), V is positive definite. Also, from (19), (20) results

$$V \leq \frac{1}{2}\left(\left(\alpha + \frac{\gamma I_\rho}{2}\right) e_k^T e_k + I_\rho(\beta + 2\gamma)\dot{e}_k^T \dot{e}_k\right) \quad (22)$$

or

$$V \leq V^* \quad (23)$$

where

$$V^* = \frac{1}{2}M^*(e_k^T e_k + \dot{e}_k^T \dot{e}_k) \quad (24)$$

and

$$M^* = \max\left(\left(\alpha + \frac{\gamma I_\rho}{2}\right), I_\rho(\beta + 2\gamma)\right) \quad (25)$$

The derivative of (19) will be

$$\dot{V} = \propto e_k^T \dot{e}_k + I_\rho\beta \, \dot{e}_k^T \ddot{e}_k + \gamma I_\rho e_k^T \ddot{e}_k + \gamma I_\rho \dot{e}_k^T \dot{e}_k \quad (26)$$

If we substitute (12) in (26), use the control law (15), after simple calculations, it yields:

$$\dot{V} = -(\gamma EI^* + \gamma c^* K_1)e_k^T e_k - (b\beta - \gamma I_\rho + K_2\beta c^*)\dot{e}_k^T \dot{e}_k + (\alpha - b\gamma - \beta EI^* - c^* K_1\beta - K_2\gamma c^*)e_k^T \dot{e}_k + \frac{2\beta}{l}\dot{e}_k^T \Delta h + \frac{2\gamma}{l} e_k^T \Delta h \quad (27)$$

By using the inequalities (8), (9) and the Young inequality (20), the above relation becomes:

$$\dot{V} \leq -\delta_1 e_k^T e_k - \delta_2 \dot{e}_k^T \dot{e}_k \quad (28)$$

where

$$\delta_1 = \left(\gamma c^* K_1 + \frac{1}{4}c^*(K_1\beta + K_2\gamma)\right) + \gamma EI^* - \frac{2\gamma}{l}\eta_1 - \frac{1}{4}(\alpha - b\gamma - \beta EI^*)) \quad (29)$$

$$\delta_2 = (\beta c^* K_2 + c^*(K_1\beta + K_2\gamma)) + \beta EI^* - \frac{2\beta}{l}\eta_2 - (\alpha - b\gamma - \beta EI^*)) \quad (30)$$

Then by the conditions (16), (17) it follows that the Lyapunov function (19) is negative definite,

$$\dot{V}(t) < 0 \quad (31)$$

Also, from (27) it yields that:

$$\dot{V} < -m^*(e_k^T e_k + \dot{e}_k^T \dot{e}_k) \quad (32)$$

where;

$$m^* = \min(\delta_1, \delta_2) \quad (33)$$

From (24), (33) we obtain [16], [18]:

$$V(t) \leq V(0)e^{-2\frac{m^*}{M^*}t} \qquad \blacksquare (34)$$

V. Numerical Simulations

A 3D motion of a 2-segment arm is presented in Fig. 5. The desired position is defined by $\kappa_{1\theta d} = \kappa_{2\theta d} = -\pi/14$, $\kappa_{1qd} = \kappa_{2qd} = -\pi/24$. The control parameters were selected as $k_{1\theta}{}^i = k_{1q}{}^i = 20$, $k_{2\theta}{}^i = k_{2q}{}^i = 8$, , $i = 1, 2 \dots$ Both segments bend with the same curvature. In Fig. 6, a new motion of this arm is presented with the different desired

position of each arm segment:
$\kappa_{1\theta d}$ = -π/14, κ_{1qd} = -π/24, $\kappa_{2\theta d}$ = π/14, κ_{2qd} = -π/24

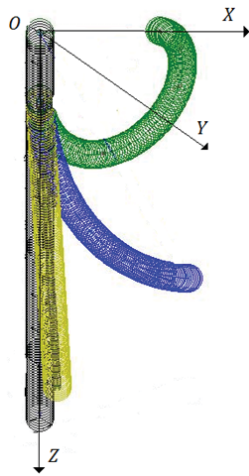The good performances of the proposed control algorithm are concluded from the graphics.

Fig. 5 3D motion: $\kappa_{1\theta d} = \kappa_{2\theta d} = -\frac{\pi}{14}$, $\kappa_{1qd} = \kappa_{2qd} = -\frac{\pi}{64}$
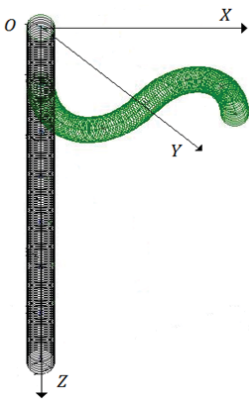
Fig. 6 3D motion $\kappa_{1\theta d} = -\frac{\pi}{14}$, $\kappa_{1qd} = -\frac{\pi}{64}$, $\kappa_{2\theta d} = \frac{\pi}{14}$, $\kappa_{2qd} = -\frac{\pi}{64}$

VI. EXPERIMENTAL RESULTS

In order to verify the suitability of the control algorithm, a platform with a flexible manipulator has been employed for testing (Fig. 7) [17]. The manipulator consists of a cable-driven continuum arm. An elastic core is the robot's backbone ($EI \approx 2\ Nm^3$, $k_{el} = 0.8\ Nm\ rad^{-1}$). It is made from homogeneous materials; the bending represents the main motion. The compressive and shear loads are neglected due to the incompressibility of the modelled elastic core compared with its bending.

Fig. 7 An experimental platform

The initial position is a vertical one. Fig. 8 shows the motion of the 3rd segment with $\kappa_{3\theta}{}^d = -1.2\ m^{-1}$, $\kappa_{3q}{}^d = 1.8\ m^{-1}$.

VII. CONCLUSIONS

This paper treats the control problem of a class of flexible manipulators by using the boundary control. The dynamic model, with respect to arm curvature, is analyzed. The proposed algorithm provides the exponential convergence to the desired reference trajectory and rejects the effect of the uncertainty of the components. A constructive Lyapunov-based proof of convergence of the control algorithm is carried out. The proposed controller is easy to implement and is characterized by very simple tuning rules of the controller gains. Numerical simulations and experimental tests illustrate the proposed techniques.
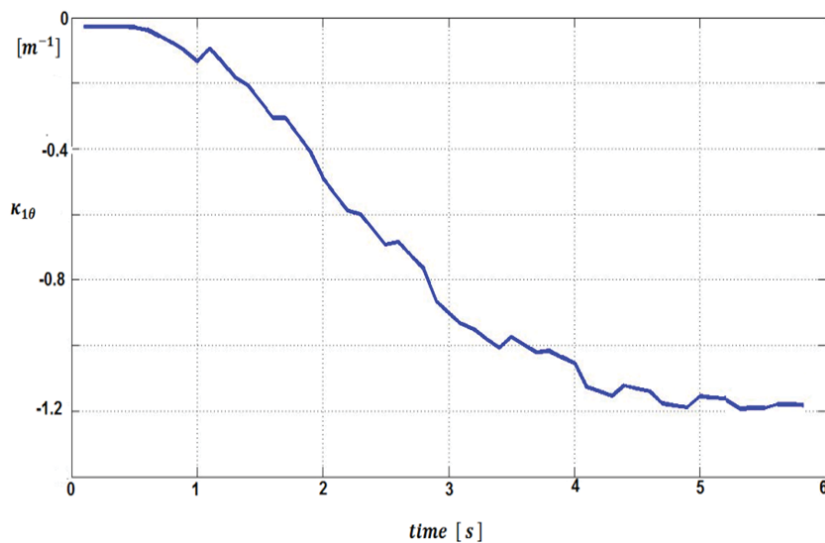
Fig. 8 Trajectory for $\kappa_{1\theta}{}^d = -1.2m^{-1}$
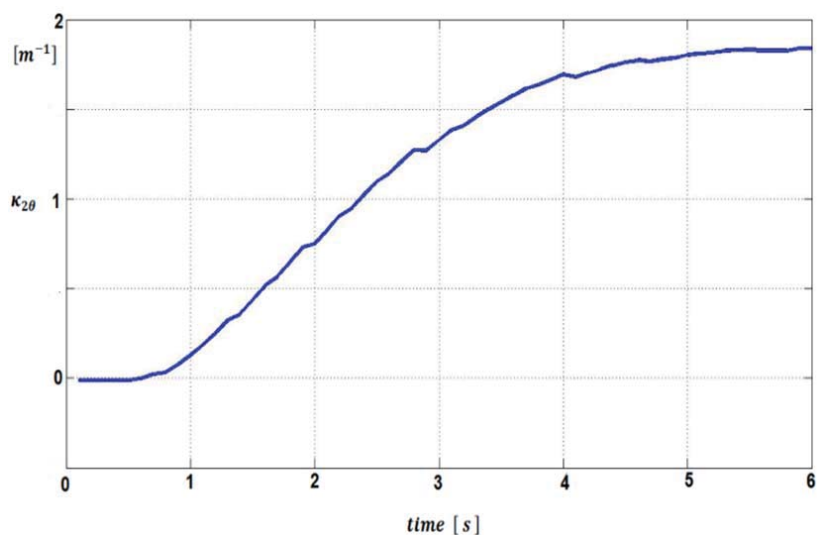
Fig. 9 Trajectory for $\kappa_{3\theta}{}^d = 1.8m^{-1}$

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