

Financial Portfolio Optimization in Turkish Electricity Market via Value at Risk

F. Gökgöz, M. E. Atmaca

Abstract—Electricity has an indispensable role in human daily life, technological development and economy. It is a special product or service that should be instantaneously generated and consumed. Sources of the world are limited so that effective and efficient use of them is very important not only for human life and environment but also for technological and economic development. Competitive electricity market is one of the important way that provides suitable platform for effective and efficient use of electricity. Besides benefits, it brings along some risks that should be carefully managed by a market player like Electricity Generation Company. Risk management is an essential part in market players' decision making. In this paper, risk management through diversification is applied with the help of Value at Risk methods for case studies. Performance of optimal electricity sale solutions are measured and the portfolio performance has been evaluated via Sharpe-Ratio, and compared with conventional approach. Biennial historical electricity price data of Turkish Day Ahead Market are used to demonstrate the approach.

Keywords—Electricity market, portfolio optimization, risk management, Sharpe ratio, value at risk.

I. INTRODUCTION

ELECTRICITY plays very important role in human life and it is an indispensable part of the daily lives of people and society. Furthermore, it has great effect on the environment depending on the type of source. Because, some of the primary energy resources have adverse effects on environment.

World primary energy demand increased by 55% between 1990 and 2013, and it is projected to grow by a further 45% to 2040 in the Current Policies Scenario study of World Energy Outlook 2015 [1]. In a different study, 36% increase between 2011 and 2030 is predicted [2]. Primary energy sources like coal, oil, natural gas, shell gas, nuclear energy, solar, wind, and hydropower are used to generate electricity needed, and electricity has a significant share within Primary energy demand.

There are many stakeholders in electricity industry: Electricity generators, consumers, transmission system operators, regulatory bodies, dispatchers, industries, traders, market operators, retailers, non-profit organizations etc., Taking into account above mentioned facts, it is understood that electricity generation is a very strategic and important industry and it is a key input element of cost management calculations of other commercial and industrial businesses.

It has been considered that competitive, transparent and liquid electricity market provides a convenient environment for effective and efficient use of electricity. On the other hand, it brings a burden that appropriate risk management techniques should be used by market players to compensate risks arising from electricity market. Risk management can be divided to two main areas: Risk control and risk assessment. Hedging and portfolio optimization have been defined as two important tools for risk control area [3]. This paper concentrates on portfolio optimization which is an effective decision tool to evaluate tradeoff between risk and revenue and it is widely used in finance.

Before demonstration of Modern Portfolio Theory (MPT) by H. M. Markowitz in 1950s, classical portfolio theory was widely accepted [4]. According to the classical portfolio theory approach, total portfolio risk of a portfolio can be decreased by diversification, moreover, it can converge the lowest risk (assumed as market risk) [5]-[8]. Diversity can be achieved by investing in different sectors or using different kinds of investments tools. The increase in the number of assets provide more diversified portfolios but this is not a systematic approach due to the fact that it does not take into consideration correlations of assets [4], [9], [10]. The presence of a positive high correlations between assets can avoid managing risk of portfolio effectively [11].

Portfolio optimization is described as the allocation of risky assets based on their relative risk and return. The main aim is whether maximization of return for a given value of risk or minimization of risk for a given value of target return [3], [5], [12], [13]. Markowitz's approach is based on mean-variance optimization and an efficient frontier that provides minimum risk for a given level of return under predefined constraints is produced to reach effective solution. Markowitz separated efficient portfolios from inefficient ones. He determined "efficient frontier" as "set of efficient mean-variance combinations" [14]. Later on, MPT demonstrated by Markowitz was improved by Sharpe and Linther respectively [10], [15]-[17]. MPT and its derivatives have been widely used for the solution of portfolio optimization problems since MPT was demonstrated by Markowitz. However, it is more prevalent in stock exchange, foreign exchange and commodity markets than power markets [1], [3]-[5], [11], [12], [18]-[23].

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Value-at-Risk is other important ancillary tool for risk management. Even though, Value-at-Risk (VaR) was widely adopted for measuring market risk in trading portfolios, it did not find itself a place in the financial lexicon until the early 1990s [24]. Its origin goes back to 1922 [25]. VaR mainly focuses on left hand side of return distribution and so unlike Markowitz it is a measure of losses resulting from “normal” market movements [27].

This paper aims to provide a theoretical background for the improvement of portfolio optimization results obtained using VaR approach. As mentioned before, VaR is one of the important risk measurement tool widely used in finance literature. VaR method is applied for three different cases for risk management through diversification. Performance of optimal electricity portfolios are measured and evaluated by using performance measurement techniques. Biennial historical working days’ electricity price data of Turkish Day Ahead Market are used to demonstrate effectiveness of VaR.

This paper is organized as follows: Section II introduces the theoretical background for MPT, VaR approach, and performance evaluation of portfolios, Section III provides a short glance into the Turkish Electricity Market. In Section IV, data, methodology, related assumptions, and the results of the study are demonstrated. In the conclusion section, important findings and future directions are listed.

II. PORTFOLIO OPTIMIZATION AND PERFORMANCE EVALUATION

A. MPT & Mean-Variance Optimization

In the financial markets, investors can manage their risks by investing in stocks from different industries, treasury bills (different maturities) or different currencies as supported by classical portfolio theory. Sufficient level of diversification helps to reduce the total risk of a portfolio to a certain degree, on the other hand, co-movements of assets will negatively affect this process [8].

MPT pays attention to co-movement/correlation of risky assets. Considering of the risky assets’ co-movements/correlation satisfies the ability to construct a portfolio that has the same expected target return and less risky portfolio than the portfolios constructed ignoring these factors [3], [5], [18], [28].

H. M. Markowitz published the article, “*Portfolio Selection*” in the Journal of Finance in 1952 [14]. That article is assumed as the first milestone of MPT [4], [10]. In this article, Markowitz proposed that only expected return and variance of return of a portfolio were enough for portfolio selection [14]. Markowitz argued that the portfolio selection process can be divided into two stages: The first stage ends with beliefs about the future performance of securities, while the second stage ends with a choice of a portfolio. His paper is mainly concern about that second stage [4]. The theory demonstrated by Markowitz was improved and amplified by Sharpe in 1964, Linther in 1965, and Mossin in 1966 respectively and independently [10], [15]–[17]. With the addition of risk-free asset by Sharpe, Linther, and Mossin to portfolio optimization

model, they improved the capital market line and developed Capital Asset Pricing Model (CAPM) [14], [29].

Markowitz’s approach is based on mean-variance optimization methodology and it produces an efficient frontier that provides minimum risk for a given level of return or maximum return for given level of risk under predefined constraints. Markowitz (1952) distinguished between efficient and inefficient portfolios locate on the efficient set [14]. Theory assumes that returns on risky assets have normal distribution but this is not the case that you encounter often. According to theory, investors have all the information regarding to market, understand it in the same way, and they are risk averse. One of the other important assumption is that investors consider only expected returns, deviations, and co-variances of risky assets while taking investment decisions [30].

Taking into account the normal distributions of assets’ return, return distribution of alternative portfolios can be estimated by using only their means and variances [12], [31]. The efficient frontier mentioned previously consists of efficient portfolios on it as illustrated in Fig. 1, where an efficient portfolio is the only portfolio that offers the highest return at the same level of risk. So as seen in Fig. 1, the upper part of the efficient set is known as the efficient frontier while lower part of the efficient set is called as inefficient set portfolios.

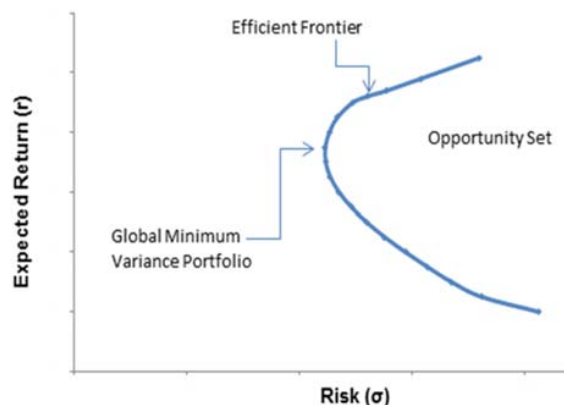


Fig. 1 Efficient frontier for mean-variance optimization model

The fundamental equation form of the Mean-variance optimization model with “n” risky assets includes three essential constraints as:

- Expected return of the portfolio will be equal to a target return.
- Non-negativity condition for assets’ portfolio weightings.
- Total weights of risky assets are equal to 1.

Object function and constraints for Mean-variance optimization are set as follows:

$$\text{Min. } (\sigma_p^2) = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_{ij} \quad (1)$$

s.t.

$$\sum_{i=1}^N X_i r_i = r_{target} \quad (2)$$

$$\sum_{i=1}^N X_i = 1 \quad (3)$$

$$X_i \geq 0, \forall X_i \in [i = 1, 2, \dots, N] \quad (4)$$

where expected return and variance of a portfolio are described as:

$$E(r_p) = \sum_{i=1}^N X_i r_i \quad (5)$$

$$\sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N X_i X_j \sigma_{ij} \quad (6)$$

where N describes a number of investable risky assets in the portfolio opportunity set, X_i denotes weights of i^{th} asset in the portfolio while r_i denotes expected return of i^{th} asset. σ_{ij} describes the covariance between i^{th} and j^{th} assets, and r_{target} describes the target portfolio return for the minimization of the portfolio's variances.

The solution of this problem produces efficient portfolios for every target return and these portfolios produce efficient set. The upper part of this set is known as the efficient frontier, as seen in Fig. 1. To reach the optimal solution, firstly the utility function that reflects an investor's risk aversion level should be determined then utility function for mean-variance optimization is determined as a quadratic function and produces indifference curves of investors. The value of utility never changes along this curve, so it is also called as the indifference curve. The tangent between the differences curve and efficient frontier determines optimum portfolio as seen in Fig. 2.

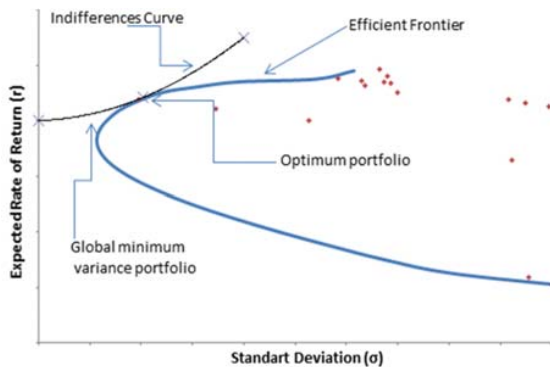


Fig. 2 Efficient frontier for mean-variance optimization model

Utility function includes the terms variance of a portfolio σ_p^2 expected return of a portfolio $E(r_p)$, and the representative risk aversion constant of an investor A [3], [5], [11], [12], [18], [19], [31], [32]. The equation set that provides the maximum utility value for the optimal portfolio solution is set up as:

$$Max. (U) = E(r_p) - 1/2A\sigma_p^2 \quad (7)$$

s.t.

$$\sum_{i=1}^N X_i = 1 \quad (8)$$

$$X_i \geq 0, \forall X_i \in [i = 1, 2, \dots, N] \quad (9)$$

where $E(r_p)$ and σ_p^2 can be seen in (5) and (6). In case of necessity a common upper investment limit like ψ_i for risky

assets can also be determined as an additional constraint in the following form:

$$X_i \leq \psi_i, \forall X_i \in [i = 1, 2, \dots, N] \quad (10)$$

Depends on the investors' need for determination of bilateral contract or market share in their portfolio or trading electricity in derivative markets; the addition of risk-free asset, customizing the upper investment constraints for each risky asset, lending and borrowing, and other different issues can easily be modelled too [5], [11], [18], [28]. Indeed, all of these additional issues does not actually included in Markowitz Mean-variance approach.

B. VaR Optimization

VaR is a kind of statistical technique and an important ancillary tool for financial decision makers. Even though, VaR did not enter the financial lexicon until the early 1990s, it has found very large application area for measuring market risk in trading portfolios [24]. Its origin goes further back: New York Stock Exchange imposed the capital requirements on member firms in 1922 [25]. Thanks to RiskMetricsTM (J.P. Morgan), VaR became an important tool for the management of risk in financial industry after 1994 [26]. J.P. Morgan's attempt to establish a market standard through its RiskMetricsTM system provided a great impetus to the growth of the method [27]. Additional regulatory VaR measures were implemented for banks or securities firms: UK Securities and Futures Authority (1992), Europe's Capital Adequacy Directive (CAD-1993), Basel Committee's (1996) [24].

For a given time period and a confidence interval, a VaR measures the maximum amount of money that can be lost: VaR indicates a quantile, which is located in the left hand side of the probability distribution and it shows the loss of a portfolio for a specified time period [24]. VaR provides aggregating all of the risks of a portfolio into a single number which is suitable and understandable for decision makers (board of directors, risk managers, regulators, the other management levels etc.) [27].

VaR focuses on left hand side of return distribution. VaR is a measure of losses resulting from normal market movements [27]. There are criticisms to Markowitz for getting into account not only negative deviations but also positive deviations from expected returns while positive deviations are not describing as losses.

As to calculation of VaR, there are three basic calculation methods widely used for VaR: The Delta-normal approach (Mean-variance), Historical simulation, and Monte Carlo Simulation [24], [27], [33].

1) VaR with Delta-Normal Approach

VaR concentrates on the left tail of distribution. Standard mathematical properties of the normal distribution are used to determine the loss that will be equaled or exceeded a predefined "x" percent during "t" period [27]. The key step in this approach is computation of standard deviations of portfolio and assets. Assuming the returns of portfolio have normal distribution, characteristic distribution curve can be produced by using only two parameters: Expected return of portfolio/asset and standard

deviation of portfolio/asset. Based on these simple parameters VaR (monetary value) of an asset or portfolio are calculated as:

$$VaR_{CL} = -W(E(r_p) + Z_{1-CL}\sigma_p) \quad (11)$$

where VaR_{CL} denotes VaR under “CL” confidence interval. Z_{1-CL} describes standard score of “1-CL” and W describes initial total value of assets/portfolio. When W is taken as 1 then VaR (return) is calculated for the related period. Z_{1-CL} is calculated as:

$$Z_{1-CL} = (x - E(r_p))/\sigma_p \quad (12)$$

95% and 99% confidence levels are common in calculations and Z scores of these values are equal to -1.645 and -2.575 respectively [34]. In Fig. 3, the results of a sample calculation are graphically demonstrated for $W=1$. For instance, under normal distribution assumption with zero mean and 0.02 standard deviation, VaR(5%) is calculated as -0.0329 while VaR(1%) is calculated as -0.0515.

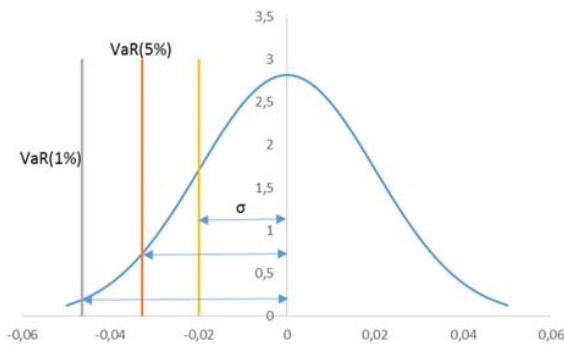


Fig. 3 VaR for a normal distribution with zero mean and 0.02 deviation

VaR (return based) for a portfolio is defined as the loss of portfolio that is exceeded with a probability of “x” percent and formulized as:

$$Max.(VaR_{95\%}) = E(r_p) + (-1.6448).(\sigma_p^2)^{1/2} \quad (13a)$$

$$Max.(VaR_{99\%}) = E(r_p) + (-2.575).(\sigma_p^2)^{1/2} \quad (13b)$$

s.t.

$$\sum_{i=1}^N X_i = 1 \quad (14)$$

$$X_i \geq 0, \forall X_i \in [i = 1, 2, \dots, N] \quad (15)$$

Here, $VaR_{95\%}$ represents, only 5 in every 100 periods, portfolio return will be under VaR (95%). $VaR_{99\%}$ represents that portfolio return will be under VaR (99%) in 1 in every 100 periods. With increase in confidence interval, probability of occurrence decreases but value of VaR in negative direction increases.

2) VaR with Historical Simulation

Historical simulation is based on observations. In delta-normal approach it is assumed that returns of portfolios or assets have normal distributions but this is not the case in real world. Historical distribution of returns doesn’t have to obey the normal distribution. According to Linsmeier and Pearson (1996) “The distribution of profits and losses is constructed by taking the current portfolio, and subjecting it to the actual changes in the key factors experienced during each of the last “a” periods... Once the hypothetical mark-to-market profit or loss for each of the last “a” periods have been calculated, the distribution of profits and losses and the value-at-risk can then be determined.” [27].

Financial institutions and banks usually prefer historical simulations or hybrid approaches that include it. Number of data is utmost important to produce effective results in this methodology, otherwise some of the results cannot be applicable especially for VaR(99%). Because, it has concentrated on only 1 event in every 100. For illustration, in case 1000 continuous historical data, VaR(95%) of portfolio is 51st lowest return of portfolio within this period. For limited number of data linear (ordinary) or polynomial interpolations can also be used to determine VaR values [24], [34].

3) VaR with Monte-Carlo Simulation

The underlying fact in Monte-Carlo simulation is that the values of financial variables can be modelled and simulated for different kind of scenarios. It is assumed that the distributions of these variables are known. Portfolio values can be recreated according to this distribution [33]. Particularly, limited data can be enlarged or so many scenarios can be created to analyze the results. The Monte-Carlo simulation has a number of similarities to historical simulation. Main difference is that, rather than carrying out the simulation using observed changes in the market factors in Monte-Carlo Simulation, one chooses a statistical distribution that is believed to adequately capture or approximates the possible changes in the market factors [27]. The designer of the analysis is free to choose any distribution that represents or approximates the distribution of past/future changes [27].

Monte-Carlo methods need more computational capacity regarding to others but the run times are dramatically improved with variance reduction techniques [24].

C. Sharpe Ratio and Treynor Ratio for Performance Evaluation

There are many performance measurement indicators in portfolio performance measurement. Depending on need of the investor, these different approaches are used separately or together in the performance analysis of portfolios. The most common ones in finance are the Sharpe Ratio (reward to variability) and the Treynor Ratio (reward to volatility) [35]. In this paper the Sharpe Ratio and Treynor Ratio together with 1 day real market data test analysis are used for performance evaluation of portfolios.

The Sharpe Ratio is very well known and widely used performance indicator in finance literature. It is a kind of one

parameter the risk/return measurement method and refers to as the reward to variability. It is calculated by the division of adjusted returns of the portfolio (residual return) to standard deviation of the portfolio itself as:

$$RVAP_p = \frac{r_p - r_f}{\sigma_p} \quad (16)$$

Treynor Ratio (reward to volatility) is the other very important performance indicator. While Sharpe Ratio measures the performance of portfolio by dividing residual return to standard deviation of portfolio, Treynor Ratio measures the performance of portfolio with the division of residual return of it to beta constant of portfolio. Treynor Ratio is calculated as:

$$RVAP_p = \frac{r_p - r_f}{\beta} \quad (17)$$

where β is calculated by the division of covariance of benchmark and optimal portfolio to benchmark portfolio's variance.

One of the important aims of this paper is the optimization and improvement of the optimal portfolio solutions, comparison of results obtained from mean-variance and VaR approaches by using performance indicators widely used in finance lexicon.

III. TURKISH ELECTRICITY MARKET

Turkey is an important emerging country, and was listed as the 18th biggest economy in the world in 2015. Turkey also has an important geopolitical position between Asia and Europe and Turkey is a unique energy corridor between Caucasasia, Middle East and Europe [36].

The electricity industry of Turkey dates back to 1902 when a 2 kW dynamo system was connected to a water mill by Italian and Swedish producer in Tarsus which lays in the south of Turkey [37]. Installed capacity in Turkey was risen only about 408 MW at the beginning of 1950s and the total amount of electricity generation had reached 789.5 GWh [37]. There was public ownership and a vertically integrated structure in the electricity industry and it continued until 1984, at which time a reform programme was initiated [18]. After the Electricity Market Law (No. 4628) entered into force in 2001, the reform programme gained momentum [12], [38]. The total installed capacity of Turkey has reached 78,072 MW as of October, 2016 [39]. Deregulation and construction process in the electricity market is in progress [13]. Authorized regulatory body of Turkey (Republic of Turkey Energy Market Regulatory-EMRA) took a decision to decrease the limit of eligible/free customers (3600 kWh) at the end of 2015 and the market openness ratio reached over 85% after this decision [40]. This limitation can be derestricted and reset but infrastructure of power dispatchers is not ready yet.

After 2001, which was the establishment date of the EMRA, many developments were put into practice. The Turkish electricity market structure consists of an ancillary services market operated by a Transmission System Operator, a

balanced market for real time balancing of load imbalances, a day-ahead market as a spot market, and an intra-day market operated by market operator and Over The Counter (OTC) for bilateral contracts. Hourly uniform marginal pricing mechanisms are used in the spot markets with daily (24 hours) settlement periods. Supply and demand lines are produced by using bids and demands offered by market players. Final clearing prices are produced by the intersection of these lines. Turkey is assumed as one region and one uniform hourly marginal price is used at clearing process. Turkey is aiming to adopt a European market model (NordPool) to itself [18].

In Turkish Spot Electricity Market, participants can tender hourly, block (4 consecutive hours) and flexible offers, but hourly and block offers have priority against flexible offers. In the day-ahead market, all offers for each hour of next day are gathered 11-35 hours before real consumption time [13]. While determining the final uniform clearing price, transmission system constraints are taken into consideration and applied to all market participants. In the intra-day market, participants can give their offers up to 90 minutes before operation hour [41].

IV. DATA, METHODOLOGY AND RESULTS OF STUDY

A. Data and Methodology

Assuming that the characteristic consumption behavior of costumers is different in weekdays and weekends, within the scope of this study, only weekdays' day-ahead electricity prices of Turkish day-ahead market are taken into account. It is assumed that investor (Generation Co.-GenCo) has a daily investment horizon. Prices for two years period between 28th of April, 2014 and 24th of April, 2016 are used as an historical database. All prices in Turkish Liras (520 data for each of the 24 hour, total 12480 data) are converted to EUR (€) by using related daily exchange rates declared by the Central Bank of the Republic of Turkey.

There is very high level of positive correlations between the hourly electricity market prices of same weekdays (see Table I). The correlations between weekdays' same hourly electricity market prices are changing between 0.6170-0.8024 and the average of them is equal to 0.7320. Representative graphic of average hourly electricity prices for a given period of time can be seen in Fig. 4. This shape is very characteristic and moreover it has a very high positive correlation with the five days consumption curve in Fig. 5 that reflects the consumption characteristics of customers.

TABLE I
CORRELATION MATRIX FOR WEEKDAYS

ρ	Mon	Tue	Wed	Thu	Fri
Mon	1	-	-	-	-
Tue	0.7974	1	-	-	-
Wed	0.7181	0.8024	1	-	-
Thu	0.6592	0.7316	0.7882	1	-
Fri	0.6170	0.7096	0.738	0.7582	1

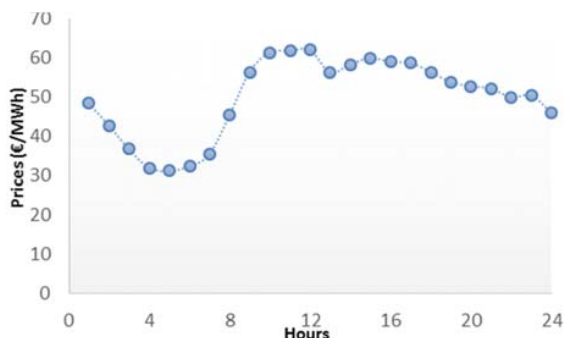


Fig. 4 Average hourly prices

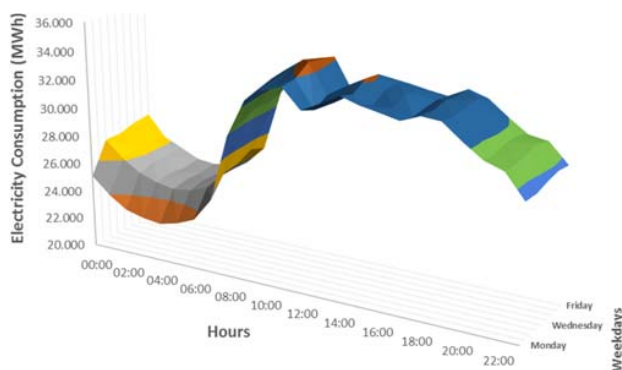


Fig. 5 Electricity consumption between 10th -14th of October, 2016

Rate of return concept of electricity markets is different from the other exchange markets. Normally rate of return for a stock market asset is calculated with division of price difference of an asset for a period to the value of asset at the beginning of this period. If investment period is short there is no need for adjustment for return but for long periods some adjustment mechanism (inflation) can be used [10]. Rate of returns in this study are calculated by using previously experienced approaches applied in this field: Market prices are normalized by the fixed generation cost for electricity [5], [11]–[13], [18], [28].

In the real life, the generation cost for electricity depends on so many factors: The age of plant, the efficiency of the power plant, technology, maintenance procedures, operation conditions, weather conditions (temperature differences, humidity, pressure etc.), and the quality of human resources, etc. Actually, the changes in generation costs can happen on a daily, weekly, and/or seasonal basis. In fact, the generation cost of a power plant is assumed strictly confidential and commercially sensitive information in electricity market environment. It cannot be disclosed by GenCos. Under the assumption of very short term investment period, generation cost of electricity is assumed constant and 35 €/MWh for the purpose of this study.

The hourly rate of returns for the day-ahead electricity market are calculated as:

$$r_{n,m} = (a_{n,m} - C_k) / C_k, (m = 1,2, \dots, 520) \tag{18}$$

$$\vec{r}_1 = \begin{bmatrix} r_{1,1} \\ r_{1,2} \\ \vdots \\ r_{1,520} \end{bmatrix}, \vec{r}_2 = \begin{bmatrix} r_{2,1} \\ r_{2,2} \\ \vdots \\ r_{2,520} \end{bmatrix}, \dots, \vec{r}_{24} = \begin{bmatrix} r_{24,1} \\ r_{24,2} \\ \vdots \\ r_{24,520} \end{bmatrix} \tag{19}$$

where $r_{n,m}$ indicates the hourly rate of returns against $a_{n,m}$ hourly weekdays' spot prices for n^{th} hour of m^{th} day for the given two years period. Here, C_k is constant and equal to 35 €/MWh. The rate of return vectors for n^{th} hour represents r_n as seen in (19). The average rate of returns and related standard deviations for risky assets are calculated as:

$$\bar{r}_n = 1/520 (\sum_{m=1}^{520} r_{n,m}) \tag{20}$$

$$\sigma_n = \sqrt{\sum_{m=1}^{520} (r_{n,m} - \bar{r}_n)^2 / (520 - 1)} \tag{21}$$

Statistical data obtained after conducting calculations in (18)–(21) are demonstrated below (see Table II).

TABLE II
HOURLY ELECTRICITY RETURNS AND DISTRIBUTION CONSTANTS

Hour	Mean	Standard Deviation	Skewness	Excess Kurtosis
1	38,27%	38,24%	-0,239	0,796
2	22,20%	42,60%	-0,616	1,191
3	5,00%	45,72%	-0,294	0,323
4	-8,73%	47,10%	-0,154	-0,304
5	-10,98%	45,50%	-0,235	-0,175
6	-7,72%	42,80%	-0,184	0,260
7	1,21%	43,84%	-0,392	0,618
8	29,46%	37,72%	-0,412	1,718
9	61,06%	36,19%	-0,426	0,152
10	75,27%	32,55%	-0,771	1,160
11	76,35%	33,65%	-0,533	2,166
12	77,08%	32,41%	-0,660	0,004
13	60,70%	40,69%	-0,447	-0,192
14	66,60%	37,47%	-0,421	-0,520
15	71,31%	35,72%	-0,461	-0,556
16	68,47%	36,22%	-0,384	-0,621
17	67,79%	37,03%	-0,354	-0,655
18	60,95%	41,67%	-0,156	-0,432
19	53,88%	38,94%	-0,134	-0,422
20	50,54%	35,95%	0,151	-0,509
21	49,13%	34,42%	0,299	-0,518
22	42,53%	37,19%	0,266	-0,473
23	43,98%	39,31%	-0,100	-0,174
24	31,41%	43,92%	-0,293	0,376

Under normal circumstances, skewness and excess kurtosis for a normal distribution should be equal to “0”. In addition, as seen the histograms of some of the hours are demonstrated in Fig. 6 and they are not only different from normal distribution but also different from each other's. Except 20, 21, and 22nd hours, the return distribution of all other hours have longer left tails (Skewness is negative) and the mass of the distributions are concentrated on the right of the figures.

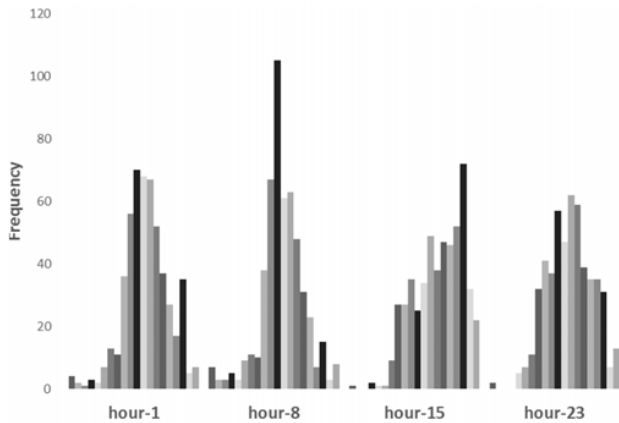


Fig. 6 Histograms for electricity returns

B. Case Studies

Day-ahead hourly weekdays' electricity market prices of the Turkish electricity market from 28th of April, 2014 to 24th of April, 2016 are assumed data interval for this study. Each of the 24 hours of a weekday is assumed as a separate risk asset [5], [12], [13], [18], [28]. Bilateral contracts (forward etc.) under the guarantee of a clearing house or other similar guarantee mechanisms are assumed as risk free assets or fixed price assets for electricity markets [13]. 3 main scenarios were formed and they are listed as Case a, Case b, and Case c.

- Case a: Portfolio optimization with 24 risky assets,
- Case b: Portfolio optimization with 24 risky assets and upper investment constraints,
- Case c: Portfolio optimization with 24 risky assets, 1 risk free bilateral contract, and upper investment constraints,

In each case, 7 portfolio optimization solutions are obtained: according to Benchmark portfolio (equally weighted portfolio), Mean-variance approach, VaR (delta-normal), VaR (historical simulation). Results of the portfolio solutions obtained via VaR are compared to the others based on their Sharpe and Treynor Ratio performances and the solution are evaluated. The assumptions of study are listed as:

- There is no transaction cost and tax.
- Investor has a one-day investment horizon (electricity selling).
- Generation cost of electricity is constant (35 €/MWh).
- Market is deep enough and it is not affected by the amount of electricity offered by the investor.
- Rate of return for a risk-free asset is assumed as 15% (approximately one third of average returns of 24 hours). so, that bilateral contract price is taken 40.25 €/MWh.
- Bids can be divided into infinitesimal parts.
- All bids will be bought by market.
- Investors are rational and prefer less risky portfolio at the same level of return, and highest return at the same level of risk.
- There is no congestion for transmission.
- Generation units have 100% availability for proposed hours.
- Rate of returns have normal distribution (it is not a necessary condition for historical simulation of VaR

application).

- Generation units are flexible to operate at every level of generation without efficiency lost.

The credentials and parameters of the empirical case studies are demonstrated in Table III.

TABLE III
CASE STUDY CREDENTIALS

Topic	Case Value
Type of Power Plant	Hydraulic
Installed capacity	250 MWe
Number of units	5x50 MW
Total available electricity energy	2500 MWh
Investment period	1 day (weekday)
Generation cost	35 €/MWh
Risk-free rate of return	15%
Bilateral contract price	40.25 €/MWh
Weekdays for electricity selling	Monday, Tuesday, Wednesday, Thursday, Friday (5 days)
Market Data	Turkish day-ahead electricity Spot market prices (from April 28, 2014 to April 24, 2016)
Number of risky assets	24
Number of risk-free assets	1
Upper investment constraint	10% for risky assets 40% for risk-free asset
Optimization Methods	Equally weighted portfolio, Mean-variance (A=3) Mean-variance (A=7) VaR (5%) VaR (1%)
Performance Method	VaR Historical (5%) VaR Historical (1%) Sharpe Ratio, Treynor Ratio

Hourly Turkish day-ahead electricity market prices are converted to the rate of return vectors. 24 risky assets have been produced as seen in Table II. A covariance matrix with dimensions 24x24 is produced by using the rate of return vectors of each asset. Covariance matrix is used in Mean-variance and VaR computations. Equally weighted portfolio is assumed as a benchmark market portfolio.

C. Results of Study

In optimization computations "MatLab" tool was used with below mentioned "options" parameter settings and 'interior-point' algorithm was used to minimize 'fmincon' object function:

```
TolCon: 1.0000000000000000e-06
TolFun: 1.0000000000000000e-06
TolX: 1.0000000000000000e-10
FinDiffRelStep: 1.490116119384766e-08
FinDiffType: 'forward'
```

MS-Excel was used for table formations and graphical demonstrations.

In Case a, there are 24 risky assets, no upper investment constraints and no risk-free asset. Weighting percentages of portfolio optimization results and performance evaluation results are demonstrated in Tables IV and V respectively.

TABLE IV
WEIGHTING PERCENTAGES OF PORTFOLIO SOLUTIONS FOR CASE A

HOUR	Equally Weight	MV A=3	MV A=7	VaR 5%	VaR 1%	HVaR 5%	HVaR 1%
1	4.17	-	-	-	-	0.11	1.22
2	4.17	-	-	-	-	0.08	0.31
3	4.17	-	-	-	-	0.04	0.17
4	4.17	-	-	-	-	0.03	0.13
5	4.17	-	-	-	-	0.03	0.12
6	4.17	-	-	-	-	0.03	0.14
7	4.17	-	-	-	-	0.04	0.18
8	4.17	-	-	-	-	0.12	0.27
9	4.17	-	-	-	-	0.07	2.09
10	4.17	24.76	38.16	34.73	39.61	29.75	9.65
11	4.17	-	-	-	-	16.30	22.88
12	4.17	75.23	61.84	65.26	60.38	37.61	10.79
13	4.17	-	-	-	-	0.30	0.88
14	4.17	-	-	-	-	0.10	2.39
15	4.17	-	-	-	-	10.17	32.11
16	4.17	-	-	-	-	0.21	2.36
17	4.17	-	-	-	-	0.17	1.55
18	4.17	-	-	-	-	0.09	0.53
19	4.17	-	-	-	-	4.23	0.70
20	4.17	-	-	-	-	0.17	2.16
21	4.17	-	-	-	-	0.14	3.50
22	4.17	-	-	-	-	0.08	1.34
23	4.17	-	-	-	-	0.07	4.01
24	4.17	-	-	-	-	0.05	0.52

TABLE V
PERFORMANCE EVALUATION OF PORTFOLIOS IN CASE A

HOUR	Return	Std.Dev.	VaR	Sharpe Ratio	Treynor Ratio	1d Test
Eq. Weighted Portfolio	0.4274	0.3338	-	0.8310	0.2774	0
Mean-variance Port. (A=3)	0.7663	0.3167	-	1.9460	0.7386	115
Mean-variance Port. (A=7)	0.7639	0.3148	-	1.9501	0.7369	9
VaR Portfolio (5%)	0.7645	0.3152	0.2461	1.9496	0.7373	8
VaR Portfolio (1%)	0.7636	0.3147	-0.047	1.9498	0.7366	15
Historical Sim. VaR (5%)	0.7430	0.3171	0.2345	1.8701	0.6860	2
Historical Sim. VaR (1%)	0.6861	0.3263	0.0446	1.6430	0.5782	1

According to the optimal portfolio solutions of 7 methodological approach: Mean-variance (for A=3 and A=7) and VaR (5% and 1% based on normal distributions) are produced very similar results and their solutions include only 10th and 12th hours in their portfolios with different weighting ratios. Historical VaR approach is not based on normal distribution assumption and it takes into account all historical combinations of possible portfolios. There are 520 historical data so that HVaR (5%) tries to maximize 27th lowest historical value of portfolio while HVaR (1%) tries to maximize 6th lowest historical value of portfolio. As seen from Table IV, the increment in confidence interval causes dramatic change in optimal portfolio solutions. On the other hand, 10th, 11th, 12th, and 15th hours have very significant weights in HVaR solutions.

By applying integer programming weighting percentages less than 1% can be adjusted to get more clear solution.

As to performance of portfolios, Sharpe and Treynor ratios are used to measure backward performances of optimal portfolios and real market data set between 25th of April and 26th of October (this is the test period that is not included in the analysis but it covers 124 business days after two years historical data used in analysis). According to performance results Sharpe and Treynor ratio performances of Mean-variance and VaR (based on normal distribution) are very close and better than others. Mean-variance with A=7 has the best Sharpe performance ratio while Mean-variance with A=3 has the largest Treynor performance ratio and the highest rate of return. Performance of portfolios are also compared by using real market data that belongs to following 124 business days period. The best performing results of each day was determined and methods were scored based on this. It is seen that mean-variance (with A=3) has obtained the best in 115 of 124 days. Even though there is not normal distribution in historical data, the methods based on this assumption are found very successful.

In Case b, there are 24 risky assets with an upper investment constraints and no risk-free asset. Weighting percentages of portfolio optimization results and performance evaluation results are demonstrated in Tables VI and VII respectively.

TABLE VI
WEIGHTING PERCENTAGES OF PORTFOLIO SOLUTIONS FOR CASE B

HOUR	Equally Weight	MV A=3	MV A=7	VaR 5%	VaR 1%	HVaR 5%	HVaR 1%
1	4.17	-	-	-	-	0.51	3.55
2	4.17	-	-	-	-	0.29	0.21
3	4.17	-	-	-	-	0.19	0.18
4	4.17	-	-	-	-	0.14	0.14
5	4.17	-	-	-	-	0.14	0.14
6	4.17	-	-	-	-	0.14	0.21
7	4.17	-	-	-	-	0.24	0.21
8	4.17	-	-	-	-	0.75	0.50
9	4.17	10.00	10.00	10.00	10.00	8.56	4.42
10	4.17	10.00	10.00	10.00	10.00	9.41	7.75
11	4.17	10.00	10.00	10.00	10.00	8.76	9.08
12	4.17	10.00	10.00	10.00	10.00	9.43	9.31
13	4.17	10.00	-	-	-	2.00	1.13
14	4.17	10.00	10.00	10.00	10.00	8.58	8.73
15	4.17	10.00	10.00	10.00	10.00	7.85	9.30
16	4.17	10.00	10.00	10.00	10.00	8.73	8.74
17	4.17	10.00	10.00	10.00	10.00	8.81	8.89
18	4.17	10.00	-	8.29	-	6.20	2.71
19	4.17	-	-	-	-	3.23	4.99
20	4.17	-	10.00	1.74	10.00	3.04	8.41
21	4.17	-	10.00	9.96	10.00	7.70	8.46
22	4.17	-	-	-	-	2.99	1.39
23	4.17	-	-	-	-	1.67	1.30
24	4.17	-	-	-	-	0.64	0.26

In Case b, very similar portfolio results are obtained as in Case a. When portfolio solution combinations are analyzed, it is noticed that 9, 10, 11, 12, 14, 15, 16, and 17th hours are common in all solutions even in HVaR results. Depending on

the methodology used in optimization, it is seen that some diversified solutions are emerging especially in HVaR optimizations. Apparently, normal distribution based portfolio optimization methods have significant performance over historical based ones again. This shows us, even though normal distribution is an assumption it works very well.

TABLE VII
PERFORMANCE EVALUATION OF PORTFOLIOS IN CASE B

HOURLY	Return	Std.Dev.	VaR	Sharpe Ratio	Treynor Ratio	1d Test
Eq. Weighted Portfolio	0.4274	0.3338	-	0.8310	0.2774	0
Mean-variance Port. (A=3)	0.6856	0.3402	-	1.5744	0.5555	79
Mean-variance Port. (A=7)	0.6636	0.3259	-	1.5759	0.5508	29
VaR Portfolio (5%)	0.6722	0.3307	0.1283	1.5791	0.5533	15
VaR Portfolio (1%)	0.6636	0.3259	-0.094	1.5761	0.5508	29
Historical Sim. VaR (5%)	0.6424	0.3301	0.1610	1.4917	0.5169	0
Historical Sim. VaR (1%)	0.6329	0.3284	0.0112	1.4705	0.5082	3

TABLE VIII
WEIGHTING PERCENTAGES OF PORTFOLIO SOLUTIONS FOR CASE C

HOURLY	Equally Weight	MV A=3	MV A=7	VaR 5%	VaR 1%	HVaR 5%	HVaR 1%
1	4.17	-	-	-	-	0.22	1.18
2	4.17	-	-	-	-	0.18	0.18
3	4.17	-	-	-	-	0.14	0.16
4	4.17	-	-	-	-	0.10	0.13
5	4.17	-	-	-	-	0.09	0.13
6	4.17	-	-	-	-	0.14	0.19
7	4.17	-	-	-	-	0.21	0.19
8	4.17	-	-	-	-	0.56	0.38
9	4.17	10.00	4.03	0.09	5.45	1.25	1.19
10	4.17	10.00	10.00	10.00	10.00	9.23	2.13
11	4.17	10.00	10.00	10.00	10.00	9.01	7.75
12	4.17	10.00	10.00	10.00	10.00	9.66	8.81
13	4.17	9.99	-	-	-	0.53	0.62
14	4.17	10.00	-	-	-	2.32	4.99
15	4.17	10.00	10.00	10.00	10.00	7.07	8.68
16	4.17	10.00	10.00	9.99	5.56	3.78	4.98
17	4.17	10.00	10.00	9.92	8.98	4.57	6.77
18	4.17	10.00	-	-	-	3.03	1.12
19	4.17	-	-	-	-	2.50	1.49
20	4.17	-	-	-	-	5.74	4.10
21	4.17	-	-	-	-	3.31	3.96
22	4.17	-	-	-	-	1.12	0.74
23	4.17	-	-	-	-	0.42	0.69
24	4.17	-	-	-	-	0.35	0.22
Rf	-	-	35.97	40.00	40.00	34.56	39.23

As to Sharpe and Treynor ratio performance of portfolios, VaR (5%) has the best Sharpe ratio while Mean-variance (for A=3) has the best Treynor ratio. Performance of portfolios are also compared by using real market data that belongs to following 124 business day period. The best performing results of each day was determined and methods were scored based on this period. It is seen that mean-variance (with A=3) has

obtained the best in 79 of 124 days. Average rate of returns of optimal portfolios are located within a few percentages. Standard deviations of optimal solutions are also very close to each other. HVaR (1%) has the second lowest performance in this case same in previous case.

TABLE IX
PERFORMANCE EVALUATION OF PORTFOLIOS IN CASE C

HOURLY	Return	Std.Dev.	VaR	Sharpe Ratio	Treynor Ratio	1d Test
Eq. Weighted Portfolio	0.4274	0.3338	-	0.8310	0.2774	0
Mean-variance Port. (A=3)	0.6856	0.3402	-	1.5744	0.5555	105
Mean-variance Port. (A=7)	0.5148	0.2095	-	1.7413	0.6226	2
VaR Portfolio (5%)	0.4962	0.1970	0.1722	1.7574	0.6315	13
VaR Portfolio (1%)	0.4923	0.1951	-0.101	1.7545	0.6277	3
Historical Sim. VaR (5%)	0.4854	0.2119	0.1725	1.5828	0.5522	2
Historical Sim. VaR (1%)	0.4558	0.1999	0.0730	1.5298	0.5326	0

In Case c, there are 24 risky assets with upper investment constraints (40% for risk-free asset and 10% for risky assets) and a risk-free asset (40.25 €/MWh bilateral contract). Weighting percentages of portfolio optimization results and performance evaluation results are demonstrated in Tables VIII and IX respectively.

In the last case study (Case c), again common results are obtained with previous cases. Hour 10, 11, 12, 15, 16, and 17 together with risk-free asset have significant weights in the portfolio optimization solutions. Except mean-variance (for a=3) solution, all the solutions give place to risk-free asset in their optimal portfolio solutions. VaR(5%) has the best Sharpe and Treynor ratio. On the other hand, 1 day test results obtained using consecutive 124 business days show that Mean-variance (for A=3) has the best in 105 of 124 days. Again, normal distribution based portfolio optimization methods have significant performance over historical based ones. Average rate of return for Mean-variance (for A=3) is very high respect to other solutions, on the other hand, the standard deviation value of it is very high mostly because of absence of risk-free asset in the portfolio.

V. CONCLUSION

In this paper, Mean-variance, VaR (delta-normal), and VaR (Historical Simulations) are applied to Turkish Day-ahead electricity market, performance of optimal portfolios have measured, and then comparison of methodologies have been conducted successfully.

Two main methodologies widely used in finance literature are used in the study. Two year data of Turkish Day-ahead market were used for application. Using these data sets, risky assets have been created and determined according to an empirical electricity cost value (see Table II). A benchmark portfolio is determined as an equally weighted portfolio. For each of three different cases, 7 portfolio solutions are obtained and total 19 optimization codes are run with MATLAB tool. 4

of 7 optimal solutions are based on normal distribution assumption: Mean-variance ($A=3$ and $A=7$) and VaR (5% and 1%) while 2 of 7 are based on historical simulation, and there is one benchmark portfolio.

In all cases Mean-variance (for $A=3$) is produced the best performance results according to one day test period. On the other hand, with increase in A from 3 to 7 (to a more risk aversion level) performance of Mean-variance optimal portfolio decrease dramatically even though Mean-variance ($A=7$) has better Sharpe Ratio performance. In Case a, and b, Mean-variance ($A=3$) has the highest Treynor Ratio scores.

As seen from Table IX, adding a risk free asset to portfolio changes the deviation of portfolio based on the portion of risk free asset. With the decrease of average standard deviation of overall portfolio, σ and β values of portfolio solutions decrease and the value of Sharpe and Treynor Ratios start to increase. Financial decision makers should be careful about using these performance indicators for the comparison of performances of portfolios including risk free assets. It is understood that using these indicators for comparison of similar portfolio structure produces better comparison results.

During application of HVaR, it is noticed that number of data is very important. Increase in number of data or decrease in confidence integral will provide more accurate and clear results.

By using VaR, in two of the cases better Sharpe Ratio and in one of the cases better Treynor Ratio results have been obtained; however, Mean-variance (for $A=3$) has produced better results in 1 day tests in all cases.

As a result, as seen from Table IV, VI, and VIII; normal distribution based methodologies are produced very similar and close optimal portfolio solution results (there are some differences). According to the all of the three performance measures, normal distribution assumption based methods have an obvious superiority over historical simulation based VaR in all cases. This shows that even though returns of risky assets do not have normal distribution, normal distribution assumptions are working very well in our cases.

As to future directions for VaR application on electricity markets; effect of different confidence level on the performance of VaR optimization for electricity markets can be further evaluated to improve the performance methodology against other methods.

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