

# Fault Tolerant $(n, k)$ - Star Power Network Topology for Multi-Agent Communication in Automated Power Distribution Systems

Ning Gong, Michael Korostelev, Qiangguo Ren, Li Bai, Saroj Biswas, Frank Ferrese

**Abstract**—This paper investigates the joint effect of the interconnected  $(n,k)$ -star network topology and Multi-Agent automated control on restoration and reconfiguration of power systems. With the increasing trend in development in Multi-Agent control technologies applied to power system reconfiguration in presence of faulty components or nodes. Fault tolerance is becoming an important challenge in the design processes of the distributed power system topology. Since the reconfiguration of a power system is performed by agent communication, the  $(n,k)$ -star interconnected network topology is studied and modeled in this paper to optimize the process of power reconfiguration. In this paper, we discuss the recently proposed  $(n,k)$ -star topology and examine its properties and advantages as compared to the traditional multi-bus power topologies. We design and simulate the topology model for distributed power system test cases. A related lemma based on the fault tolerance and conditional diagnosability properties is presented and proved both theoretically and practically. The conclusion is reached that  $(n,k)$ -star topology model has measurable advantages compared to standard bus power systems while exhibiting fault tolerance properties in power restoration, as well as showing efficiency when applied to power system route discovery.

**Keywords**— $(n, k)$ -star Topology, Fault Tolerance, Conditional Diagnosability, Multi-Agent System, Automated Power System.

## I. INTRODUCTION AND RELATED WORK

**P**OWER outages and faults are considered as significant problems occur in interconnected power systems. Technologies exist that address power system diagnosis and estimation can diagnose or estimate the fault of a power system. These, however are implemented during design process or offline. There still is a high probability for real time power outages or failures due to various reasons such as human faults, control system failures, cyber attacks or likely hardware failures.[9] Thus, proactive and reactive power system controls are introduced and applied to solve the issues in real time

Author Ning Gong is with Electrical and Computer Engineering Department, Temple University, Philadelphia, PA, USA 19122, (e-mail: ning.gong@temple.edu).

Author Michael Korostelev is with Electrical and Computer Engineering Department, Temple University, Philadelphia, PA, USA 19122, (e-mail: mike.k@temple.edu).

Author Qiangguo Ren is with Electrical and Computer Engineering Department, Temple University, Philadelphia, PA, USA 19122, (e-mail: qren@temple.edu).

Author Li Bai is with Electrical and Computer Engineering Department, Temple University, Philadelphia, PA, USA 19122, (e-mail: lbai@temple.edu).

Author Saroj Biswas is with Electrical and Computer Engineering Department, Temple University, Philadelphia, PA, USA 19122, (e-mail: sbiswas@temple.edu).

Author Frank Ferrese is with Naval Surface Warfare Center, Philadelphia, PA, USA 19112, (e-mail: frank.ferrese@gmail.com).

[15]. Many automated power system algorithms have been developed in previous literature, such as Heuristic search based methods, Artificial Intelligence (AI) based algorithms and so on. There are some solutions such as fuzzy logic approaches, belonging to the category of AI based algorithms. These prove to be a proactive approach [15]. However, it is addressed in [10] that most of the current solutions have no guarantee of real-time fault diagnosis or time efficiency in system reconfiguration or discovery when fault occurs.

Multi-Agent Systems (MAS) have been proposed by in many research works recent years. In our own studies, these have shown to offer a highly effective, flexible power restoration environments. MAS of type Belief-Desire-Intent (BDI) distributed computation system that can perform optimal decision control in real-time with minimum time and energy cost. Recently, MAS have been used to reconfigure distributed power grids and to solve the power load restoration problems in a distributed way, such as [12], [7], [5], [6], [4]. However, one major drawback of these approaches is that the performance varies when applied to different interconnected power architectures. Some topologies, especially interconnected networks such as ring and radio structures have exhibited constraints when scaled to large power grids. Later, authors in [14] proposed a fully distributed multi-agent based load restoration algorithm which is independent of system configurations, so that it can be applied to systems of any structures (radial, mesh, or mixed) [14]. However, by examining the algorithms proposed in that study, we can still notice that they lack performance in speed for real-time control a long time is requires for convergence when the algorithm is applied to a large power system. In particular, we can conclude that even the algorithm itself is independent in topology structures. The constraints related to the properties of the graph (topology) still affect the performance of the system either in the process of system establishment or the process of network reconfiguration due to a verity of faults.

Another issue existing in MAS power control systems is, once a severe fault such as cascade outage occurs in the power system network, it is necessary sometimes to branch the whole power system into two or several sub-power systems [13]. After branching, each of the sub-power systems will be independently recovered as a balanced power system. This issue is then formulated in [16] as a topology issue because it is dependent on the network routes of the power topology. Connectivity, which is also know as *degree* in graph theories, is

defined as the number of edges that are incident to the vertices. The average degree of a power topology can represent how complex the system is when the branching process needs to be performed. Thus, to respond to the need of improving the performance of existing MAS solutions mentioned above, our goal in this paper is to study the properties of interconnected network topologies, then implement a solution with high scalability and fault tolerance to the target multi-bus power system. Also we aim to implement a distributed power topology model that has a proper average degree that can efficiently perform power branching without affecting the stability of the system.

## II. PROBLEM FORMULATION

In our research [2],  $(n,k)$ -star [17] topology has been proven to be a highly fault-tolerant topology in the area of graph theory. It is an enhanced version of  $n$ -dimensional star graph model that adds more redundancy and reliability to the infrastructure of interconnected power systems. Among all the properties of  $(n,k)$ -star graph topology, the excellent fault-tolerance it exhibits, can be especially important for power restoration networks as nodes in the distributed power network may fail and outages may occur. We define reliability as the remaining connectivity and minimum distance from node to node of the distributed power network once a power fault has occurred in part of the power system. According to [8], diagnosability is introduced to general power topologies. The diagnosability, denoted by  $t(G)$ , is described as the maximal number of faulty nodes that can be diagnosed by the system. For example, a system is  $t$ -diagnosable if all faulty nodes can be identified without a replacement, provided that the number of occurring faults does not exceed  $t$ . If all neighbors of a power node  $u$  in the system exhibit a fault simultaneously,  $u$  cannot be tested as faulty or fault-free; thus, it is impossible for traditional system diagnosability to be more than its minimum node degree. The diagnosability is studied in this paper as the major properties of  $(n,k)$ -star topology that can solve the MAS control issues addressed above.

Another outstanding property of  $(n,k)$ -star (represented by  $S_{n,k}$ ) topology is that the diameter and degree are relatively small compared to the same properties in traditional multi-bus topologies [2]. These properties contribute to performance not only theoretically but practically. It is a realistic assumption to apply this idea to modern large scale power systems. Our research focuses on the modeling and evaluation of  $(n,k)$ -star topology as implemented on agent controlled distributed power systems.

We present a multi-agent based  $(n,k)$ -star topology for power restoration control systems, which is described as follows:

- 1)  $(n,k)$ -star is a novel topology model extended from  $(n,k)$ -graph for interconnected power systems. Two different kinds of power agents are defined and assigned different types of BDI jobs accordingly.  $(n,k)$ -star mapping is performed to all of the three studied standard multi-bus power systems.
- 2) Relevant properties such as fault tolerance, conditional diagnosability, degree and diameter are studied

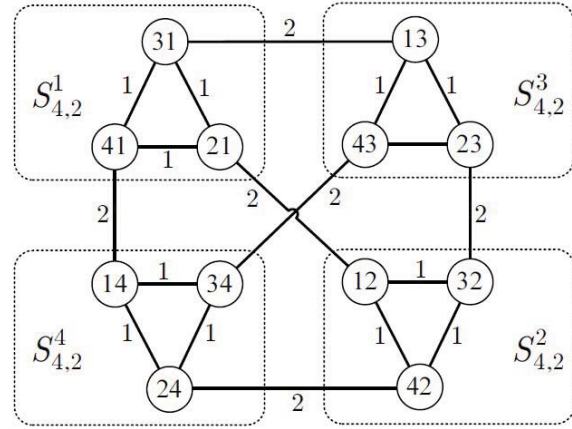


Fig. 1  $S_{4,2}$ -star topology. By applying theorem 2 the diameter is 3

accordingly. Lemmas are derived to demonstrate that  $(n,k)$ -star has advantages in distributed power route discovery, fault restoration and system branching.

- 3) A set of case-study analysis of our multi-agent based  $(n,k)$ -star control system is simulated on standard multi-bus power systems. Branching penalty, average net power and communication cost performance is evaluated and compared to related work.

The rest of this paper is organized as follows. Section III presents the original  $(n,k)$ -star topology model and the related properties contribute to the advantages of  $(n,k)$ -star power model. In Section IV we discuss the algorithms for power route discovery of MAS based power system. In Section V we present our main modeling for  $(n,k)$ -star power topology, and then we provide the related lemmas and corollaries that prove to support the benefits of  $(n,k)$ -star architecture in application to power network restoration. In Section VI some experiments that illustrate the utility of the proposed model and metrics will be discussed. Section VI presents the concluding remarks.

## III. $(n,k)$ -STAR GRAPH TOPOLOGY

### A. Definition

The basic definition and terminology can be found in [1], [17], [8]. An  $(n,k)$ -star graph  $S_{n,k}$  with  $1 \leq k < n$  is denoted by the two parameters  $n$  and  $k$ . For simplicity, we use  $\langle n \rangle$  to denote the set  $\{1, 2, \dots, n\}$ . The vertex set of  $S_{n,k}$  is denoted by  $\{v(S_{n,k}) = \{p_1, p_2, \dots, p_k \mid p_i \in \langle n \rangle \text{ and } p_i \neq p_j \text{ for } i \neq j\}\}$ . The edge set of  $S_{n,k}$  is defined as follows: A vertex  $p_1, p_2, \dots, p_i, \dots, p_k$  is adjacent to:

- 1) The vertex  $p_i p_2 \dots p_1 \dots p_k$  through an edge of dimension  $i$ , where  $2 \leq i \leq k$  (i.e., exchange  $p_1$  and  $p_i$ );
- 2) The vertex  $x p_2 \dots p_k$  through an edge of dimension 1, where  $x \in \langle n \rangle \setminus \{p_1, p_2, \dots, p_k\}$ .

According to the definition above, a  $S_{n,k}$ -star graph is  $(n-k)$ -regular and contains  $\frac{n!}{(n-k)!}$  vertices. The edges of type

(1) are referred to as *i-edges*, whereas the edges of type (2) are referred to as *l-edges*. Fig. 1 shows  $S_{4,2}$ . Particularly, a  $(n-1)$ -regular graph is defined as a graph that has same degrees for all the nodes. That means once  $n$  and  $k$  of a certain  $(n,k)$ -star is chosen, its degree and node number are all fixed because they can both be represented as a function of  $n$  and  $k$ . Moreover, the following two theorems demonstrated that the conditional diagnosability and diameter of a  $(n,k)$ -star graph can also be represented as a function of  $n$  and  $k$ .

The  $S_{n,k}$ , in general, where  $2 \leq k \leq n$ , can be constructed by interconnecting  $n$  number of  $S_{n-1,k-1}$  components. As is proved in [1], one of the most important properties related to scalability is that  $S_{n,k}$  can be decomposed into  $n$  vertex-disjoint  $S_{n-1,k-1}$  components over any dimension  $i$  by fixing one symbol in any position  $i$ , where  $2 \leq i \leq k$ . If the subgraph of  $S_{n,k}$  is represented by  $S_{n,k}^i$ , it has been proven that  $S_{n,k}^i \cong S_{n-1,k-1}$ . Thus, because of the property that a  $S_{n,k}$ -star graph is  $(n-1)$ -regular, we have the important property that a  $S_{n,k}$ -star graph is  $(n-1)$ -connected. So we have the assumption that conditional diagnosability of a  $S_{n,k}$ -star graph must be greater than  $n-1$ , this property makes a  $(n,k)$ -star graph competitive with other interconnected power system topologies and it is the main motivation of this paper to migrate this topology to examine the diagnosability and fault tolerant for agent communication in power network restoration.

### B. Related Properties

Particularly, the authors of [8], [2] provided us with a theorem of an  $(n,k)$ -star graph topology on fault tolerance:

*Theorem 1:* The conditional diagnosability of the  $(n,k)$ -star graph  $S_{n,k}$  is given by the following,

$$t_c(S_{n,k}) = \begin{cases} 0, & \text{if } n = 3, k = 1; \\ \lceil \frac{n}{2} \rceil - 1, & \text{if } n \geq 4, k = 1; \\ n + 2k - 5, & \text{if } n \geq 4 \text{ and } 2 \leq k \leq n - 1; \\ 3n - 7, & \text{if } n \geq 4 \text{ and } k = n, n - 1. \end{cases}$$

This theorem gives us the outstanding hardware property of a  $(n,k)$ -star power system topology. In addition to the application of fault-tolerant algorithms to a certain power restoration architectures,  $(n,k)$ -star is able to offer relatively high conditional diagnosability, which can ensure the system be resilient on hard faults of a power network agent node. Therefore, in comparison to other traditional topologies, such as rings and stars,  $(n,k)$ -star topology model is a good candidate model for practical resilient power restorations in the sense of fault tolerance. Moreover, the researchers of [3] provided us with another theorem on diameter of nodes in  $(n,k)$ -star topology, which is the longest distance between two arbitrary nodes:

*Theorem 2:* The diameter  $D(S_{n,k})$  of  $(n,k)$ -star graph is formulated as:

$$D(S_{n,k}) = \begin{cases} 2k - 1, & \text{if } 1 \leq k \leq \lfloor \frac{n}{2} \rfloor; \\ \lfloor \frac{n-1}{2} \rfloor - 1, & \text{if } \lfloor \frac{n}{2} \rfloor \leq k < n. \end{cases}$$

It is obvious that the diameter of  $(n,k)$ -star topology is smaller than other former interconnected network topologies. Theorem 1 and 2 proved that  $(n,k)$ -star topology is suitable for power grid design in the advantages of fault tolerance and discovery cost. Overall, the  $(n,k)$ -star topology is provided in this section with:

- 1) Relatively high degree, which gives the  $(n,k)$ -star topology redundancies for power rerouting. The architecture is more fault tolerant as the degree is high.
- 2) Relatively high diagnosability. It adds hardware fault tolerance to power network restorations.
- 3) Relatively low diameters. This ensures the communication cost would be minimal for  $(n,k)$ -star agent based communication.

### IV. MAS APPROACH

Our prior work on power restoration focuses on MAS approach for power restoration. In [11] BDI MAS approach is initialized with establishing the Power Agent (Including Consumer and Producer Agents) and bus Agents, both of the agents are constructed as a single power component in the system. Details of constructing a power component are described in [10]. After agent creation, we can consider the structured Multi-Agent System as a blind marketplace to all agents. In this marketplace, we consider some agents as consumers and some as producers. Each consumer agent has no knowledge of the power supply in the market. Consumer agents start to explore the market by looking for producers in the producer set, after the bus agents determine their roles. In other words, the global goal of the market is to satisfy the total demand of power, under the condition that supply and demand are balanced. This goal should be a maintaining goal, because the power system is a dynamic system and we want maintain stability in the network at all times. *Finding a producer* plan is executed to achieve the *finding a producer goal* consequently. The plan is to access the belief set, to visit all the adjacent agents whose roles are producers, and to retrieve the net power. Then, the plan is on hold until a trading goal is achieved. A trading plan is triggered by the trading goal and it is represented for the consumer  $i$  as:

$$p_j^{k+1} = p_i^{k+1} = \max_{j \in N_i} \alpha(p_i^k + p_j^k) \text{ for } i = 1, 2, \dots, n \quad (1)$$

where  $n$  is the number of buses;  $\alpha_{ij}$  is the coefficient that indicates the connection between producer  $j$  and consumer  $i$ . If producer  $j$  and consumer  $i$  are connected as  $\alpha_{ij}$  is 1, otherwise 0;  $p_i^k$  and  $p_i^{k+1}$  are the net power of consumer  $i$  at iteration  $k$  and  $(k+1)$  respectively.

### V. MODELING AND METHODOLOGY

#### A. $(n,k)$ -Star Power Model

In our prior studies, we proposed the MAS approach for power network restoration algorithms [10], [11], in which an agent is defined as a BDI module to act as a bus for the power system. In this study, we mapped the agents in the power network system to our  $(n,k)$ -star graph model. The agents would be performing as *Power Agents* (Defined as BDI

TABLE I  
( $N, K$ )-STAR BUS SYSTEM MODELING TABLE

Bus Systems	( $n, k$ )-star Graph	Number of Components	Redundancy	Degree	Diameter
16-Bus System	$S_{5,2}$	20	4	4	3
39-Bus System	$S_{7,2}$	42	3	6	3
162-Bus System	$S_{7,3}$	210	48	6	5

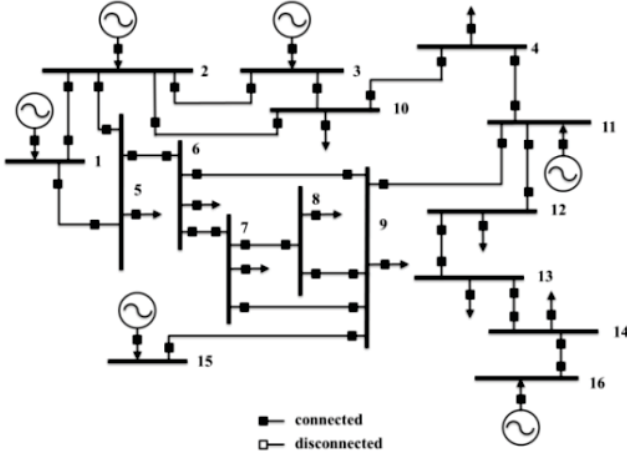


Fig. 2 IEEE Standard 16-Bus Power System test case

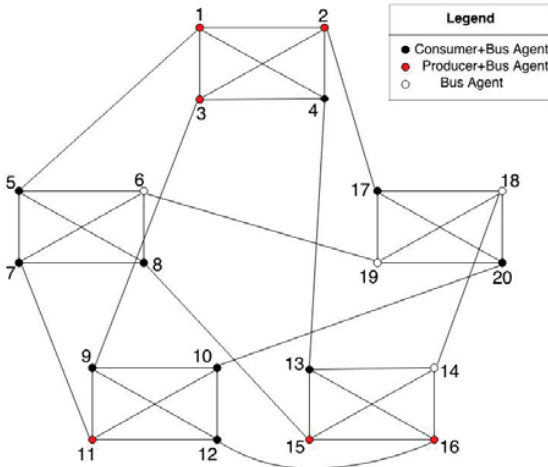


Fig. 3 MAS based  $S_{5,2}$  star topology implementing IEEE Standard 16-Bus Power System with redundant Bus Agents

consumer/producer power agents) together with *Bus Agents* in our proposed ( $n, k$ )-star power system model.

As defined in last section, the number of nodes in the ( $n, k$ )-star graph topology can be expressed linearly by  $n$  and  $k$ . Thus the number of nodes is fixed once the  $n$  and  $k$  are chosen. So for the IEEE Standard Bus System Test Cases studied in [11], [14], we selected the node number of ( $n, k$ )-star topology greater but closer to the number of nodes in Bus Systems. Thus there will be two types of agents in the system. First one is *Power Agent* modeled in our Multi-Agent System that performs as either generator or load agents in the system.

The other one is *Bus Agent* that plays the role of orchestrating switching and routing decisions when necessary. Thus in our 16-Bus Power System, as shown in Fig. 2, the *Power Agent* is always connected to a *Bus Agent* that are representing one component in ( $n, k$ )-star power topology. Moreover, since the number of components are selected greater but closet to the IEEE Standard Multi-bus Power System, there are components selected as redundant agents that are composed only of *Bus Agents* that are randomly located in the ( $n, k$ )-star topology. Table I shows the  $n$  and  $k$  value selected for our test cases which are 16-Bus Power System, 39-Bus Power System and 162-Bus Power System [12]. For example, we select  $n = 5$  and  $k = 2$  for a 16-Bus Power System. Then the number of components are calculated by the definition of ( $n, k$ )-star topology described in previous section, which is 20 in total. In this case there will be 16 components to perform as *Power* and *Bus Agents* in the architecture illustrated in Fig. 3. The other 4 components are redundant *Bus Agents* that randomly located to the topology.

#### B. ( $n, k$ )-star Power Model Properties

1) *Properties Contribute to Route Discovery*: As is discussed in the previous section, the BDI MAS control system is a distributed decision making system. In our proposed ( $n, k$ )-star power model, we implemented the same power route discovery and power fault reconfiguration (Including branching process) algorithms. According to equation (1), the consumer agent in our route discovery algorithm will be initially querying through all the neighboring agents which are producer agents for net power to fulfill the initial demand. If no neighbors are producer agents, the algorithm iterates though over all neighbors of each neighbor until it reaches the producer and get supplied with net power. Thus, if it is defined that each communication action is an iteration, then the maximum number of iterations for the whole system to get every consumer agent's goal fulfilled is represented by the *Communication Cost* of the whole distributed power system. Then we have the following lemma and corollary from theorem 1 and 2 for the *Communication Cost* of ( $n, k$ )-star power model:

**Lemma 1:** For ( $n, k$ )-star power system, the *Communication Cost* is at most the sum of the diameter and the degree. Which is  $D(S_{n,k}) + n - 1$ ,  $D(S_{n,k})$  is provided in theorem 2:

$$D(S_{n,k}) + n - 1 = \begin{cases} n + 2k - 2, & \text{if } 1 \leq k \leq \lfloor \frac{n}{2} \rfloor; \\ n + \lfloor \frac{n-1}{2} \rfloor - 2, & \text{if } \lfloor \frac{n}{2} \rfloor \leq k < n. \end{cases}$$

**Proof:** To prove the lemma, we have to find the worst case *Communication Cost* scenario that exists in the ( $n, k$ )-star



topology. Thus this case is investigated to fulfill the following two constraints:

- 1) There must exist a *consumer node* in the topology where all neighbors are not *producer agents*.
- 2) All of the neighbors of that *consumer agent* are *consumer agents* and they have the longest distance from the nearest *producer agent*.

Once these constraints are fulfilled, it is obvious that exemplifies the highest communication cost for this node to communicate to the nearest *producer agent*. Therefore, respectively, if the first constraint is fulfilled, the communication cost is exactly how many neighbors each node has, which is also the degree of the  $(n, k)$ -star topology; if the second constraint is fulfilled, the communication cost is the number of agents included in the longest path of the topology, which is identical to the definition of diameter. ■

Moreover, if the degree is represented as  $d(S_{n,k})$  and diameter is  $D(S_{n,k})$ , we have the corollary for the time complexity of route discovery algorithm of the MAS approach:

*Corollary 1:* Time complexity of the route discovery algorithm of the MAS approach on the  $(n, k)$ -star distributed power network topology is  $O(d(S_{n,k}) + D(S_{n,k}))$ .

2) *Property Contributes to Power Restoration:* It is given in the definition of  $(n, k)$ -star topology that the degree is  $(n - 1)$  and in theorem 1 that the *Conditional Diagnosability* is in terms of selected  $n$  and  $k$ , we can conclude that the relatively high degree gives redundancy to the system so that the branching may happen less frequently compared to the original *Standard Bus Architecture*. The relatively high *Conditional Diagnosability* would lead the proposed  $(n, k)$ -star topology to a more stable architecture when multiple faults occur.

In next section, a set of simulation scenarios will be executed to prove the properties discussed in this section that contribute to  $(n, k)$ -star topology route discovery and restoration. More specifically, the properties of optimized communication cost during route discovery and advantages when power branching will be illustrated by the experiments described in the following section. The benefit of *Conditional Diagnosability* and its contribution to multiple fault restoration will be studied in the future.

## VI. SIMULATION RESULTS

In order to evaluate the performance of the proposed power topology, a simulation system is developed in MATLAB. Similar to the simulation studies in [10], [14], we simulate the 16-bus power system, IEEE 39-bus power system, and IEEE 162-bus power system, and compare the results to theirs in order to understand the properties of the proposed topology model to different scale power systems.

### A. Route Discovery Result

First, a set of experiments are executed on all of the three  $(n, k)$ -star power model for the 16-bus power system, IEEE 39-bus power system, and the IEEE 162-bus power system. The experiment for the 16-bus system is performed 20 times,

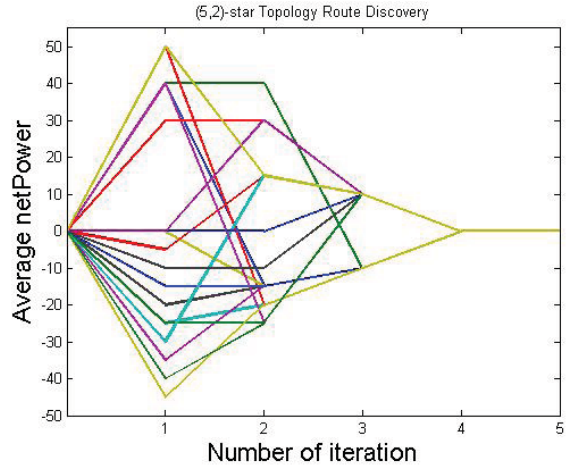


Fig. 4 MAS based  $S_{5,2}$ -star topology implementing IEEE Standard 16-Bus Power System with redundant Bus Agents

the one for the IEEE 39-bus power system is performed 42 times and the one for the IEEE 162-bus power system is performed 210 times. In each case a set of redundant bus agents is selected according to Table I. They are randomly located in the  $(n, k)$ -star power system. Accordingly, the number of iterations for each bus system is recorded as a measurement of the *Communication Cost*. Table II illustrates the average number of iterations, which is the identical metric to the communication cost. We can see from the table that the average number of iterations of each test case is similar to the test cases of our prior work [10], [11]. And also we can address that the average number of iterations is always smaller than the estimated communication cost, which is the sum of *Diameter* and *Degree* of the related  $(n, k)$ -star power system.

Also in Fig. 4, 5, 6 we can observe that the average net power converges to zero for most of the nodes (different color indicates different agent number). It means most of the agents' goal has been fulfilled once the algorithm is finished. We observed as well that some of the agents' net power is not zero at the end. This is mainly because our proposed  $(n, k)$ -star power topology model added redundant agents to perform as a bus, whose goal can not be fulfilled in the end.

### B. Power Branching Result

In the second set of experiments, we simulated the single fault power restoration and branching for each test cases. And we compared the result to the IEEE Standard Bus Power Systems. For each of the three  $(n, k)$ -star test scenarios, we set one single fault to the power system when the route discovery is finished. We recursively disabled the agent one by one for different rounds of the test case. Thus for  $(5, 2)$ -star topology model, 20 tests are simulated for all the possible single faults. 42 tests and 210 tests are simulated respectively to  $(7, 2)$ -star topology model and  $(7, 3)$ -star topology model. From Table III we can observe that the average number of shutdown agents due to power branching is listed for both  $(n, k)$ -star and Standard Power Bus Systems. We can conclude that for

TABLE II  
ROUTE DISCOVERY AVERAGE COMMUNICATION COST COMPARED TO ESTIMATED LIMIT

Bus Systems	$(n, k)$ -star Graph	Simulation Iterations	Estimated Cost Limit
16-Bus System	$S_{5,2}$	4	7
39-Bus System	$S_{7,2}$	4	9
162-Bus System	$S_{7,3}$	5	11

TABLE III  
NUMBER OF AVERAGE SHUTDOWN NODE DUE TO SINGLE FAULT POWER BRANCHING  
(COMPARED TO STANDARD BUS SYSTEM)

Shutdown Agents	$(5, 2)$ -star	16-Bus System	$(7, 2)$ -star	39-Bus System	$(7, 3)$ -star	162-Bus System
Average Number	1.45	1.5	1.2143	1.2821	1.0381	1.0617

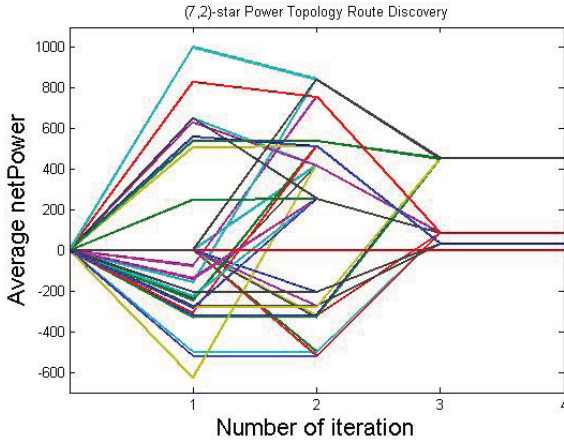


Fig. 5 MAS based  $S_{7,2}$ -star topology implementing IEEE Standard 39-Bus Power System with redundant Bus Agents

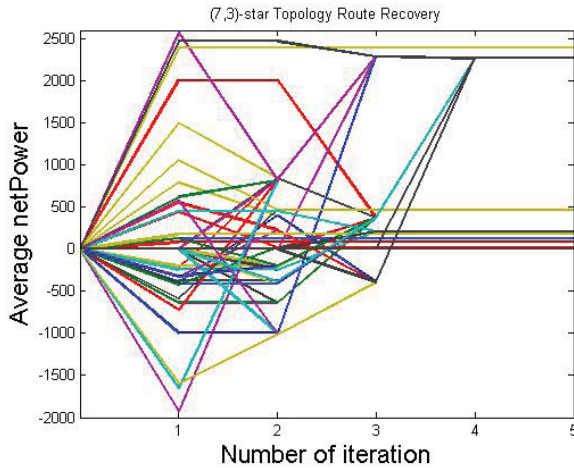


Fig. 6 MAS based  $S_{7,3}$ -star topology implementing IEEE Standard 162-Bus Power System with redundant Bus Agents

single fault power branching.  $(n, k)$ -star has lower number of shutdown nodes due to power branching than the Standard Bus System. Note that for  $(7, 3)$ -star and 162 bus system, the numbers are close to 1 because both of them have sparse number of agents and the redundancies are both high.

## VII. IMPACT AND CONCLUSIONS

In this paper, we presented the  $(n, k)$ -star topology for Multi-Agent based distributed power control system. We derived and proved lemmas and corollaries that provide result in beneficial properties of the  $(n, k)$ -star topology, such as *Fault Tolerance*, *Conditional Diagnosability*, *Diameter* and *Degrees*, which contribute to the study of power network route recovery, power restoration and power branching. We also provided a topology model for the IEEE Standard Multi-Bus Power Systems and analyzed the critical factors that influence the entire power restoration performance. Our approach provides not only aggregate performance metrics such as communication cost and net power, but also feedback about the topology characteristics at a fine-level of granularity.

We noted that, compared to traditional bus systems, the power system with  $(n, k)$ -star topology has relatively high conditional diagnosability and low diameter. It gives the  $(n, k)$ -star topology good performance in power route discovery and restoration. In our future work we will explore similar measurements and comparisons involving the multi-fault test scenarios in order to be able to fully analyze various performance trade-offs for  $(n, k)$ -star topology on power systems.

## ACKNOWLEDGMENT

The authors would like to acknowledge ONR for funding this work.

## REFERENCES

- [1] EDDIE CHENG and MARC J. LIPMAN. Unidirectional  $(n, k)$ -star graphs. *Journal of Interconnection Networks*, 03(01n02):19–34, 2002.
- [2] Ning Gong, Michael Korostelev, Li Bai, Saroj K. Biswas, and Frank Ferrese. Evaluation of highly conditionally diagnosable  $(n, k)$ -star topology for applications in resilient network on chip. In *Resilient Control Systems (ISRCS), 2014 7th International Symposium on*, pages 1–6, Aug 2014.
- [3] L. He and Brock University (Canada). *Properties and Algorithms of the  $(n, K)$ -star Graphs*. Canadian theses. Brock University (Canada), 2009.
- [4] K. Huang, D.A Cartes, and S.K. Srivastava. A multiagent-based algorithm for ring-structured shipboard power system reconfiguration. *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on*, 37(5):1016–1021, Sept 2007.
- [5] W. Khamphanchai, S. Pisanupoj, W. Ongsakul, and M. Pipattanasomporn. A multi-agent based power system restoration approach in distributed smart grid. In *Utility Exhibition on Power and Energy Systems: Issues Prospects for Asia (ICUE), 2011 International Conference and*, pages 1–7, Sept 2011.

- [6] Y. L. Lo, C. H. Wang, and C.N. Lu. A multi-agent based service restoration in distribution network with distributed generations. In *Intelligent System Applications to Power Systems, 2009. ISAP '09. 15th International Conference on*, pages 1–5, Nov 2009.
- [7] J.A Momoh and Ousmane S. Diouf. Optimal reconfiguration of the navy ship power system using agents. In *Transmission and Distribution Conference and Exhibition, 2005/2006 IEEE PES*, pages 562–567, May 2006.
- [8] Wei-Hao Deng Nai-Wen Chang and Sun-Yuan Hsieh. Conditional diagnosability of (n,k)-star networks under the comparison diagnosis model, unpublished manuscript.
- [9] Fenghui Ren, Minjie Zhang, D. Soetanto, and XiaoDong Su. Conceptual design of a multi-agent system for interconnected power systems restoration. *Power Systems, IEEE Transactions on*, 27(2):732–740, May 2012.
- [10] Qiangguo Ren and Li Bai. A bdi agent-based approach for power restoration. In *Collaboration Technologies and Systems (CTS), 2014 International Conference on*, pages 652–656, May 2014.
- [11] Qiangguo Ren, Li Bai, Saroj Biswas, Frank Ferrese, and Qing Dong. A bdi multi-agent approach for power restoration. In *Resilient Control Systems (ISRCs), 2014 7th International Symposium on*, pages 1–6, Aug 2014.
- [12] J.M. Solanki, S. Khushalani, and N.N. Schulz. A multi-agent solution to distribution systems restoration. *Power Systems, IEEE Transactions on*, 22(3):1026–1034, Aug 2007.
- [13] Kai Sun, Da-Zhong Zheng, and Qiang Lu. Splitting strategies for islanding operation of large-scale power systems using obdd-based methods. *Power Systems, IEEE Transactions on*, 18(2):912–923, May 2003.
- [14] Yinliang Xu and Wenxin Liu. Novel multiagent based load restoration algorithm for microgrids. *Smart Grid, IEEE Transactions on*, 2(1):152–161, March 2011.
- [15] Mohammad Abdullah Al Faruque Qiangguo Ren Wei Zhang Paul Rosendall Yan Lu, Ram Kuruganty and David Scheidt. Risk based multi-agent chilled water control system for a more survivable naval ship. volume 17, pages 102–112, Dec 2012.
- [16] Qianchuan Zhao, Kai Sun, Da-Zhong Zheng, Jin Ma, and Qiang Lu. A study of system splitting strategies for island operation of power system: a two-phase method based on obdds. *Power Systems, IEEE Transactions on*, 18(4):1556–1565, Nov 2003.
- [17] Shuming Zhou and Lanxiang Chen. Fault tolerance of (n, k)-star graphs. In *Computer Science and Education (ICCSE), 2010 5th International Conference on*, pages 239–243, Aug 2010.