

# Fatigue Analysis of Crack Growing Rate and Stress Intensity Factor for Stress Corrosion Cracking in a Pipeline System

A. R. Shahani, E. Mahdavi, M. Amidpour

**Abstract**—Environment-assisted cracking (EAC) is one of the most serious causes of structural failure over a broad range of industrial applications including offshore structures. In EAC condition there is not a definite relation such as Paris equation in Linear Elastic Fracture Mechanics (LEFM). According to studying and searching a lot what the researchers said either a material has contact with hydrogen or any other corrosive environment, phenomenon of electrical and chemical reactions of material with its environment will be happened. In the literature, there are many different works to consider fatigue crack growing and solve it but they are experimental works. Thus, in this paper, authors have an aim to evaluate mathematically the pervious works in LEFM. Obviously, if an environment is more sour and corrosive, the changes of stress intensity factor is more and the calculation of stress intensity factor is difficult. A mathematical relation to deal with the stress intensity factor during the diffusion of sour environment especially hydrogen in a marine pipeline is presented. By using this relation having and some experimental relation an analytical formulation will be presented which enables the fatigue crack growth and critical crack length under cyclic loading to be predicted. In addition, we can calculate  $K_{SCC}$  and stress intensity factor in the pipeline caused by EAC.

**Keywords**—Embrittlement, Fracture mechanics, Hydrogen diffusion, Stress intensity factor.

## I. INTRODUCTION

EAC is considered the most dangerous form of failure due to the presence of stress and corrosive environmental on a material [1], [2]. A pipeline stress analysis is the important part to design it but it is not sufficient. In addition to stress analysis, span analysis is another issue for designing. It is important to calculate it correctly in order to have a good and safety design of pipeline. In span analysis should be considered following items:

- Excessive yielding
- Fatigue
- Interface with human activity

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There are several reasons to induce fatigue failure of a pipe such as preventing to crack growth and so on. The fatigue is created in a pipe mostly deals with corrosion fatigue and resonance fatigue. So, for designing a pipe to protect it against the fatigue is mentioned, a using proper material in coating is essential. There are some solutions to solve the problem caused by two different fatigues. For example, if the problem is related to resonance fatigue in pipeline traditionally, using a simple model to evaluate the potential of a span to undergo resonance is based on comparison of shedding frequency and natural frequency of the span. According to researches, pipeline rested on the seabed are subject to fluid loading from both waves and steady currents. So, it is a design requirement that the pipe should be sufficient to ensure stability. In most case, a concrete weight coating on pipeline provides this weight. There is a relation to show the enough stability for pipeline that comes as follows [3]:

$$\gamma(F_D - F_L) \leq \mu(W_{sub} - F_L)t \quad (1)$$

Where:

$\gamma$ : Factor of safety.

$F_D$ : Hydrodynamic drag force per unit length.

$F_L$ : Hydrodynamic inertia force per unit length.

$\mu$ : Lateral solid friction coefficient.

$W_{sub}$ : submerged pipe weight per unit length.

$F_L$ : Hydrodynamic life force per unit length.

The fatigue sometimes is produced by corrosion. It is a biggest problem for a pipe and a very common case of this phenomenal is stress corrosion cracking (SCC). The main reasons for happening SCC are combination of pressure stress, a corrosive environment and a susceptible microstructure [4].

In order to remove SCC and at least control it, using coating is very common to prevent crack growth. Dormant SCC cracks have been shown growing by fatigue when subject to a cyclic load.

## II. CALCULATION OF CRACK LENGTH MATHEMATICALLY

For investigation this problem, we should try to estimate the time life of a pipeline in order to prevent the harmful events. In this way, the environmental conditions and different loads should be considered. These loads are as follows:

- Static load
- Cross-flow and in-line flow VIV
- Trawl gear interaction
- Direct drag

The fatigue criterion can be formulated as:

$$\eta \cdot T_{life} \geq T_{exposure} \quad (2)$$

Where:

$\eta_i$ : Allowable fatigue damage

$T_{life}$ : The fatigue design life

$T_{exposure}$ : The design life

According to fracture mechanics, the  $S$ - $N$  curve is used to calculate the number of load cycle for estimating the lifetime. This lifetime is compared with the fatigue criterion. A designer engineer can make judgment for considering either the fatigue or no. There are two crack growth mechanisms. The first is due to fatigue crack growth and the second is environmentally assisted cracking (EAC). The crack growth depends on EAC is divided into three groups as follows:

- Corrosion
- Stress-corrosion
- Fatigue corrosion

The Paris equation is related to a fatigue crack growth. If we want to involve this equation in EAC, the necessary changes made it possible. Consider the changes are done for making the Paris equation as a function of time:

$$\frac{da}{dN} = C \Delta K^m \quad (3)$$

$$\frac{dN}{dt} = Cte \quad (4)$$

According to Eqs. (3) and (4), we have:

$$\frac{da}{dt} = \frac{da}{dN} \cdot \frac{dN}{dt} = Dk^n \quad (5)$$

Where,  $D$  and  $n$  are experimental constant.

There are two factors to crack growth, fatigue and EAC crack growth and we can sum them to calculate the total crack growing [5], so we can write:

$$\Delta a = a_1 - a_2 = \int_{N_0}^{N_1} \left( \frac{da}{dN} \right) dN + \int_0^t \left( \frac{da}{dt} \right) dt \quad (6)$$

There are many different works to consider fatigue crack growing and solve it, but most of them are experimental. In this paper, our aim is evaluating mathematically the pervious works. As shown in Figs. 1 and 2, if an environment is more sour and corrosive, the changes of stress intensity factor is more. It should be noted that  $Zr_{412}$ ,  $Ti_{13.8}$ ,  $Cu_{12.5}$ ,  $Ni_{10}$  and  $Be_{22.5}$  (at %) are plotted as a function of the stress-intensity range for three environments: 0.5 M NaCl, deionized water, and laboratory air (relative humidity 25-35%). Also, amorphous  $Zr_{412}$ ,  $Ti_{13.8}$ ,  $Cu_{12.5}$ ,  $Ni_{10}$  and  $Be_{22.5}$  metal under sustained load, are plotted as a function of the stress intensity,  $K$ , in aerated 0.5 M NaCl. Data shown have been acquired from one sample [6].

In Fig.1, the Paris law is drawn and in Fig. 2, shown crack growth as a function of time. The Fig. 2 is the main core to discuss about it and try to evaluate it in a mathematical way. As you see, this figure shows the fact that there are not any relations between  $da/dt$  and  $K$  for especial values of stress intensity factor. It is between  $10^{-5}$  and  $10^{-4}$ .

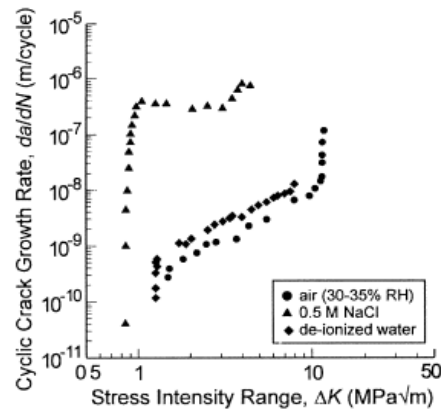


Fig. 1 Fatigue-crack rates (under sinusoidal loading) in amorphous

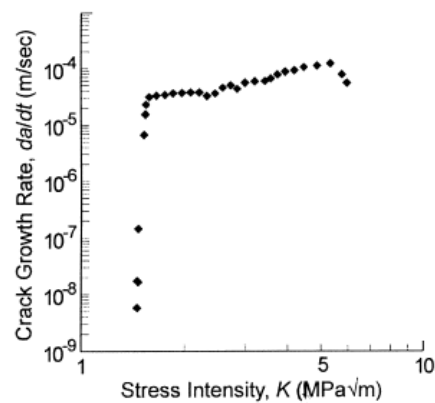


Fig. 2 Stress-corrosion cracking velocity in the

There several models proposed for fracture toughness to discuss a fatigue of a definite material. For example, there is a model for metallic glasses comes as Eq. [6]:

$$K_I = 24\pi^3 \sqrt{3} \frac{\beta \Gamma E}{\alpha} \quad (7)$$

Where:

$\beta$ : A scaling constant dependent on the work hardening behavior.

$\alpha$ : The fatigue design life.

$\Gamma$ : The design life.

This model for the fracture toughness of metallic glasses based on the resistance of blunt crack to this instability, gives an expression for  $K_{IC}$  in terms of the surface tension and Young's modulus. In NaCl at open circuit, experimentally measured crack velocities, acquired SCC, shows the crack velocities are much larger than fatigue crack growth rates in air. Prediction of fatigue crack growth rates in NaCl solution have been calculated by following equation:

$$\left( \frac{da}{dN} \right)_{eff} = \int_0^{t/v} \left( \frac{da}{dt} K(t) \right)_{SCC} dt \quad (8)$$

In addition to the experimental result, there is an important relation due to many searches on the EAC and it shows the equation comes below, obtained from fitting curve on the experimental data [7]:

$$a_c = a_0 + K_a t_i^{0.5} \quad (9)$$

Where:

$a_c$ : Critical crack length.

$a_0$ : Initial crack length.

$t_i$ : Total time of loading.

$K_a$ : Constant.

We use these experimental equations to predict a crack length without considering experimental data. In a pipe, the maximum and minimum stresses are active [8]. So, according to ASME:

$$K_{\max \text{ or } \min} = \frac{P_{\max \text{ or } \min} S}{BW^{3/2}} f\left(\frac{a}{W}\right) \quad (10)$$

Where,  $P_{\max}$  and  $P_{\min}$  are the maximum and minimum fatigue loads respectively. Also,  $W$ ,  $B$ ,  $a$  and  $S$  are geometrical parameters.

$$\Delta K = K_{\max} - K_{\min} \quad (11)$$

We put the Eq. (10) in (11), we have:

$$\Delta K = K_{\max} - K_{\min} = \frac{\Delta PS}{BW^{3/2}} f\left(\frac{a}{W}\right) \quad (12)$$

Assume  $K_{\min} = 0$

As you know:

$$\Delta K = Q \Delta \sigma \sqrt{\pi a} \quad (13)$$

$$\sigma_{\max} = \frac{2(1+\nu)K_{IC}}{3\sqrt{2}\pi r} \cos \frac{\theta}{2} \quad (14)$$

Now, using Eqs. (12), (13) and (14) give the Eq. (15).

$$a_{Gr} = \frac{r P_{\max}^2}{K_{IC}^2} \left( \frac{3\sqrt{2}}{2\sqrt{\pi}(1+\nu)} \cdot \frac{S}{QBW^{3/2} \cos \frac{\theta}{2}} f\left(\frac{a}{W}\right) \right)^2 \quad (15)$$

According to Eqs. (9) and (15) the following equation is obtained. We have:

$$\frac{r P_{\max}^2}{K_{IC}^2} \left( \frac{3\sqrt{2}}{2\sqrt{\pi}(1+\nu)} \cdot \frac{S}{QBW^{3/2} \cos \frac{\theta}{2}} f\left(\frac{a}{W}\right) \right)^2 = a_0 + K_a t_i^{0.5} \quad (16)$$

In (16), you can have total life time as a pipeline design or calculate it from diffusion equation and stress intensity is calculated as follows [9], [10]:

$$K_I = \sqrt{r} \ln \left( \frac{\phi}{\sqrt{P}} \right) \cdot \frac{3RT\sqrt{\pi}}{\sqrt{2(1+\nu)V_H}} \quad (17)$$

Where  $\phi = \frac{C}{S}$  and:

$\phi$ : Normalized concentration of a gas without existence of stress.

$S$ : Solubility of a gas

$P$ : Hydrostatic pressure due to diffusion of a gas into material.

$V_H$ : Ratio molar volume.

$r$ : Distance from crack tip.

This equation shows the explicit relation between toughness and pressure produced by diffusion of gas in a pipe. By the least-square fitted-line through giving value to toughness and pressure for two cases and consequently we can draw two

curves for an especial material between concentration and radial distance from the crack tip. The cross region in this curve gives us the critical value for two parameters,  $r_c$  and  $\phi_c$ . There are the critical values for maximum stress that happened on the zone at crack tip. So, we put them in (17) then, we have:

$$K_{IC} = \sqrt{r_c} \ln \left( \frac{\phi_c}{\sqrt{P}} \right) \cdot \frac{3RT\sqrt{\pi}}{\sqrt{2(1+\nu)V_H}} \quad (18)$$

From this equation, we can obtain toughness for a definite pressure and an especial material at different temperatures for a pipe. After calculation toughness and fatigue load, we have the first side of the Eq. (16). However, it is possible to draw a diagram by means of replacing different values and fit a curve in order to calculate  $a_0$  and  $K_a$ . As you see, we can get the initial crack length without having the experimental information.

### III. CALCULATION OF DYNAMIC AND FATIGUE CRACK GROWTH AND $K_{SCC}$

As it is said, some experimental equations to use for calculation mathematically the different parameters to solve the fatigue analysis in pipeline systems. In this way, we try to predict the dynamic crack growth and estimate the value to make a safety design. For achieving this purpose, consider the Eq. (6) as basic relation for our evaluation.

$$\left( \frac{da}{dN} \right)_{\text{eff}} = \int_0^{1/\nu} \left( \frac{da}{dt} K(t) \right)_{SCC} dt \quad (18)$$

Obviously, we can get the changes of  $da/dt$  through using Paris law. Substitute Eq. (10) in (8). There for, we have:

$$\frac{da}{dN} = C \Delta K^m = C \left( \frac{\Delta PS}{BW^{3/2}} f\left(\frac{a}{W}\right) \right)^m \quad (19)$$

Then, we can calculate  $da/dt$  by giving derivation from Eq. (9).

$$\frac{da}{dt} = 0.5 K_a t_i^{-0.5} \quad (20)$$

Put Eqs. (20) and (19) in (8) and then give integral from both sides of the Eq. (8).

$$C \left( \frac{\Delta PS}{BW^{3/2}} f\left(\frac{a}{W}\right) \right)^m = \int_0^{1/\nu} (0.5 K_a t_i^{-0.5} K(t))_{SCC} dt \quad (21)$$

For a specified time, the time of fracture, obtained from Eq. (19) as follows:

$$K(t_i)_{SCC} = (0.5 K_a t_i^{-0.5})_{SCC}^{-1} C \left( \frac{\Delta PS}{BW^{3/2}} f\left(\frac{a}{W}\right) \right)^m \quad (22)$$

You see the calculation of dynamic crack growth and estimate the value of  $K_{SCC}$ .

### IV. CONCLUSION

In this paper, the fatigue crack growth and stress intensity factor for stress corrosion cracking has been derived from the analytical solution obtained from the experimental equations. The extracted relation shows that the stress intensity factor in environmentally assisted cracking is a function of a geometrical pipe and its total fatigue lifetime. However, in this study, the experimental equation and data are also used to fit the best curve on the data for obtaining the relation between

the crack growth and fatigue lifetime. More over, using standard of ASME for a pipe, helps us to modify the rest of equations are needed.

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