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Exponential State Estimation for Neural Networks with Leakage, Discrete and Distributed Delays

Liyuan Wang, Shouming Zhong

Abstract—In this paper, the design problem of state estimator for neural networks with the mixed time-varying delays are investigated by constructing appropriate Lyapunov-Krasovskii functionals and using some effective mathematical techniques. In order to derive several conditions to guarantee the estimation error systems to be globally exponential stable, we transform the considered systems into the neural-type time-delay systems. Then with a set of linear inequalities(LMIs), we can obtain the stable criteria. Finally, three numerical examples are given to show the effectiveness and less conservatism of the proposed criterion.

Keywords—State estimator, Neural networks, Globally exponential stability.

I. Introduction

IN the past decades, we can easily find that neural networks have been applied in many fields such as signal processing, pattern recognition and static image processing. And the applications that introduced above strongly depend on the dynamic behavior of the network. For a long time, many investigator pay same attentions on the stability of delayed neural networks, in [1-3], the authors discussed the recurrent neural networks, the delayed stochastic genetic regulatory networks and the uncertain fuzzy system. Therefore, because of the finite switching speed of amplifiers or the finite signal propagation time, neural networks often has time delays so many sufficient conditions have been proposed for verifying the globally asymptotically stable and globally exponential stability of the neural networks [4-10] and [23,24].

In recent years,the state estimation problems of the neural networks have been obtained large amount of attention from [11-21], [25] discussed the problem of state estimation of neural networks with the leakage delays, discrete time-varying delays and distributed time-varying delays. By using convex combination technique and some analysis techniques, some stability criteria for the existence of the state estimator are derived. [30] provided some sufficient conditions for the system to be globally asymptotically stable and exponentially stable, respectively. such conditions could be easily checked by utilizing the recently developed interior-point methods and they are expressed in the form of LMIs. and not having the tuning of parameters needed in [31]. And the author of [22] used the delay departing technique to discuss the problem of the stability criteria of neural networks with the distributed time-varying delays, and it has derived the less conservation criterion to deal with the analysis problem of globally exponential stability for neural networks.

In this paper, the problem of state estimation of neural networks with leakage delays, discrete time-varying delays and distributed delays is investigated. By constructing a new class of Lyapunov function and using the delay departing technique, convex combination technique and some analysis techniques, the problem of estimating the neural states via available output measurements such that the estimation error converges to zero exponentially is investigated. bucause the system we discuss included the leakage delay, so at last we discussed the upper bounds of some variable. Note that the LMIs method can obtained less conservative criterion when deal with the analysis problem of globally exponential stability for neural networks, two numerical examples are given to illustrate the effectiveness of the propose methods.

Notation: Throughout this paper, R^n denote the n-dimensional Euclidean space and the set of all $n \times m$ real matrices are denoted by $R^{n\times m}$. And when X and Y are symmetric matrices, the notation $X \geq Y$ means that $X - Y$ is positive. The superscript T denotes transposition of matrix. $\|\cdot\|$ and $*$ denote the Euclidean norm and the symmetric block,respectively, $diag\{\cdots\}$ is a block diagonal matrix. λ_{min} and λ_{max} stand for the smallest and largest eigenvalue of a given matrix, respectively.

II. Problem statement

Consider the following neural networks with mixed time-varying delays:

$$\begin{aligned}\dot{x}(t) =& - Ax(t-\sigma) + B_1 g(x(t)) + B_2 g(x(t-\tau(t))) \\ &+ \tilde{E}\int_{t-r(t)}^{t} g(x(s))ds + I, \\ y(t) =& Cx(t) + Df(t,x(t)),\end{aligned} \tag{1}$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \cdots, x_n(\cdot)]^T \in R^n$ is neural state vector; $y(\cdot) \in R^m$ is the output of the networks; $A = diag\{a_1, \cdots, a_n\} > 0$ is a diagonal matrix with positive entries $a_i > 0$; the matrices B_1, B_2 and $\tilde{E}$ represent the connection weight matrix, the discretely delayed connection weight matrix, and distributively delayed connection weight matrix,respectively; the matrices $C \in R^{m\times n}$

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Liyuan Wang and Shouming Zhong are with the School of Mathematics Science, University Electronic Science and Technology of China, Chengdu 611731, PR China.

Shouming Zhong is with Key Laboratory for NeuroInformation of Ministry of Education, University of Electronic Science and Technology of China, Chengdu 611731, PR China.

(e-mail :wangliyuan610@126.com).

and $D \in R^{m\times n}$ are the output weighting matrices; $g(x(\cdot)) = [g_1(x_1(\cdot)), g_1(x_2(\cdot)), \cdots, g_1(x_n(\cdot))]^T, g \in R^m$ denotes the neural activation function; $I = [I_1, \cdots, I_n]^T$ is an external input vector, σ is the leakage delay and $\sigma > 0$, the discrete time-varying delays and the distributed time-varying delays are denoted by $\tau(t)$ and $r(t)$, respectively, and they satisfy

$$0 < d_1 \le \tau(t) \le d_2, 0 < r(t) \le \bar{r}, \dot{\tau}(t) \le \mu \qquad (*)$$

in above inequality $d_1, d_2, \bar{r}$ and μ are constants; and $f : R \times R^n \longrightarrow R^n$ represent the neural-dependent nonlinear disturbances. Nest, we design the full-order state estimation for the neural networks (1) as the following:

$$\begin{aligned}\dot{\bar{x}}(t) =& - A\bar{x}(t-\sigma) + B_1 g(\bar{x}(t)) + B_2 g(\bar{x}(t-\tau(t))) \\ &+ \tilde{E}\int_{t-r(t)}^{t} g(\bar{x}(s))ds + I + K(y-\bar{y}) \\ \bar{y}(t) =& C\bar{x}(t) + Df(t,\bar{x}(t)),\end{aligned} \qquad (2)$$

where $\bar{x}(t)$ and $\bar{y}(t)$ represent the estimation of the neuron state and the estimated output vector, respectively, and $K \in R^{n\times m}$ is the gain matrix of the estimator to be designed later. when the error vector is defined as $\varepsilon(t) = x(t) - \bar{x}(t)$, we can directly obtained the error dynamics from (1) and (2):

$$\begin{aligned}\dot{\varepsilon}(t) =& - A\varepsilon(t-\sigma) - KC\varepsilon(t) + B_1\varphi(t) + B_2\varphi(t-\tau(t)) \\ &+ \tilde{E}\int_{t-r(t)}^{t} \varphi(s)ds - KD\psi(t)\end{aligned} \qquad (3)$$

where $\varphi(t) = g(x(t)) - g(\bar{x}(t))$,$\psi(t) = f(t,x(t)) - f(t,\bar{x}(t))$, then the above system can be translated into the following equivalent form when the error vector is defined as $\varepsilon(t) = x(t) - \bar{x}(t)$:

$$\begin{aligned}&\frac{d}{dt}[\varepsilon(t) - A\int_{t-\sigma}^{t}\varepsilon(s)ds] = -(A+KC)\varepsilon(t) + B_1\varphi(t) \\ &+ B_2\varphi(t-\tau(t)) + \tilde{E}\int_{t-r(t)}^{t}\varphi(s)ds - KD\psi(t)\end{aligned} \qquad (4)$$

(3)and(4) satisfy $\varepsilon(t) = \Phi(t), t \in [-\tau, 0]$, and$\tau = max\{d_2, \bar{r}\}$ g and f satisfy the following Assumption.

Assumption (A): The neuron activation function $g(\cdot)$ in (1) is bounded and satisfy the following two constant matrices $\varphi^- = diag\{\phi_1^-, \phi_2^-, \cdots \phi_n^-\}$,$\varphi^+ = diag\{\phi_1^+, \phi_2^+, \cdots \phi_n^+\}$, such that:

$$\phi_l^- \le \frac{g_l(\alpha) - g_l(\beta)}{\alpha - \beta} \le \phi_l^+ \qquad (5)$$

for all $\alpha, \beta \in R, \alpha \ne \beta, l = 1, 2, \cdots, n$

Assumption (B): The neuron-dependent nonlinear disturbances f in (1) is bounded and there are a positive diagonal matrices $\hat{W}$ and two constant matrices $\omega^- = diag\{\omega_1^-, \omega_2^-, \cdots, \omega_n^-\}$, $\omega^+ = diag\{\omega_1^+, \omega_2^+, \omega_n^+\}$, such that the following inequality:

$$\begin{aligned}&[f(t,x(t)) - f(t,\bar{x}(t)) - \omega^-(x(t) - \bar{x}(t))]^T\hat{W} \\ &\times [\omega^+(x(t) - \bar{x}(t)) - (f(t,x(t)) - f(t,\bar{x}(t)))] \ge 0\end{aligned} \qquad (6)$$

In order to obtain our results,we will introduce some lemmas.

Definition 1 . The equilibrium point 0 of system (3) is said to be globally exponentially stable, if there exist scalars $k > 0$ and $\delta > 0$ such that:

$$\|\varepsilon(t)\| \le \kappa \sup_{-\tau\le s\le 0}\|\Phi(s)\|e^{-\delta t}, \forall t > 0,$$

Lemma 1 (Schur complement). For a given symmetric matrix $S = S^T = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22}\end{bmatrix}$, where $S_{11} \in R^{k\times k}$, the following conditions are equivalent:

$(1) S < 0;$
$(2) S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0;$
$(3) S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0;$

Lemma 2 [9]. For any constant matrix $Z \in R^{n\times n}, Z = Z^T > 0$, scalars $h_2 > h_1 > 0$, such that the following integrations are well defined, then:

$$\begin{aligned}&-(h_2 - h_1)\int_{t-h_2}^{t-h_1} x^T(s)Zx(s)ds \\ &\le -\int_{t-h_2}^{t-h_1} x^T(s)ds Z \int_{t-h_2}^{t-h_1} x(s)ds \\ &-\frac{1}{2}(h_2^2 - h_1^2)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x^T(s)Zx(s)dsd\theta \\ &\le \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x^T(s)dsd\theta Z \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} x(s)dsd\theta\end{aligned}$$

Lemma 3 [10]. For any scalar $h(t) \ge 0$ and any constant matrix $Q \in R^{n\times n}, Q = Q^T > 0$, the following inequalities hold:

$$\begin{aligned}&-\int_{t-h(t)}^{t}\dot{x}^T(s)Q\dot{x}(s)ds \le h(t)\zeta(t)\chi Q^{-1}\chi^T\zeta(t) + 2\zeta^T(t) \\ &\times [x(t) - x(t-h(t)],\end{aligned}$$

where $\zeta^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-\frac{h_u}{2}) \quad x^T(t-h_u) \quad \dot{x}^T(t) \quad p(t)]$,
and χ is free weighting matrix with appropriate dimensions.

III. Stability Analysis

In this section we will give sufficient conditions under which the system (3) or (4) is globally asymptotically stable. Firstly, the following notations are introduced for the representation convenience:

$$L_1 = diag\{\phi_1^-\phi_1^+, \phi_2^-\phi_2^+, \cdots, \phi_n^-\phi_n^+\},$$
$$L_2 = diag\{\frac{\phi_1^- + \phi_1^+}{2}, \frac{\phi_2^- + \phi_2^+}{2}, \frac{\phi_n^- + \phi_n^+}{2}\}.$$

From the above Assumption (A), we can obtain the following inequalities:

$$[\varphi_l(t) - \phi_l^- e_l(t)]^T[\varphi_l(t) - \phi_l^+ e_l(t)] \le 0, l = 1, 2, \cdots, n,$$

Theorem 1 For given scalars $d_1, d_2, \sigma, \bar{r}$ and μ satisfy (*) and Assumption (A) and (B) hold, then the system (3) or (4) is globally exponentially stable with the rate index k if there exist $P > 0$, $R_l > 0$, $(l = 1, 2, 3, 4, 5, 6)$, $Q_m > 0$, $(m = 1, 2, 3, 4, 5, 6, 7, 8)$, $T_n > 0$, $(n = 1, 2, 3)$, any matrices

G, U_k,V_k,W_k,X_k,Y_k, Z_k, $S_k (k = 1, 2)$, and diagonal matrices $\hat{U} > 0, \hat{V} > 0, \hat{W} > 0$ such that the following LMIs hold according to (7) and (8), then one can obtain

$$E_{16\times16} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} V \times Q_7^{-1} V^T + (d_2 - d^*) e^{-2kd_2} X Q_8^{-1} X^T < 0 \quad (7)$$

$$E_{16\times16} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} W \times Q_7^{-1} W^T + (d_2 - d^*) e^{-2kd_2} X Q_8^{-1} X^T < 0 \quad (8)$$

$$F_{16\times16} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} S \times Q_7^{-1} S^T + (d_2 - d^*) e^{-2kd_2} Y Q_8^{-1} Y^T < 0, \quad (9)$$

$$F_{16\times16} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} S \times Q_7^{-1} S^T + (d_2 - d^*) e^{-2kd_2} Z Q_8^{-1} Z^T < 0, \quad (10)$$

where

$U = [U_1^T\ 0\ -U_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$
$V = [0\ 0\ 0\ V_1^T\ 0\ V_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$
$W = [0\ 0\ W_1^T\ 0\ 0\ W_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$
$X = [0\ 0\ 0\ X_1^T\ X_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$
$Y = [0\ 0\ 0\ Y_1^T\ 0\ Y_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$
$Z = [0\ 0\ 0\ 0\ Z_1^T\ Z_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$
$S = [0\ 0\ S_1^T\ S_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^T,$

and

$E_{1,1} = 2kP - PA - A^T P - GC - C^T G^T + R_1 + R_2 + R_3 + R_4 + R_6 + \sigma^2 Q_1 + d_1 Q_3 + (d^* - d_1) Q_4 + (d_2 - d^*) Q_5 - 2e^{-2kd_1} T_1 - 2\frac{d^* - d_1}{d^* + d_1} e^{-2kd^*} T_2 - L_1 \hat{U} - L_3 \hat{W} + e^{-2kd_1} U_1 + e^{-2kd_1} U_1^T - 2\frac{d_2 - d^*}{d_2 + d^*} e^{-2kd_2} T_3,$
$E_{1,3} = e^{-2kd_1}(-U_1 + U_2^T), E_{1,7} = -C^T G^T,$
$E_{1,8} = PB_1 + L_2 \hat{U}, E_{1,9} = PB_2, E_{1,10} = -GD + L_4 \hat{W}$
$E_{1,11} = -2kPA + A^T PA + C^T G^T A, E_{1,12} = \frac{2}{d_1} e^{-2kd_1} T_1,$
$E_{1,13} = P\tilde{E}, E_{1,14} = \frac{2}{d^* + d_1} e^{-2kd^*} T_2,$
$E_{1,15} = \frac{2}{d^* + d_1} e^{-2kd^*} T_2, E_{1,16} = \frac{2}{d_2 + d^*} e^{-2kd_2} T_3,$
$E_{2,2} = -e^{-2k\sigma} R_6, E_{2,7} = -A^T P,$
$E_{3,3} = e^{-2kd_1}(U_2^T + U_2) - e^{-2kd_1} R_2 + e^{-2kd^*}(W_1 + W_1^T),$
$E_{3,6} = e^{-2kd^*}(-W_1 + W_2^T)$
$E_{4,4} = -e^{-2kd^*} R_4 + e^{-2kd^*}(-V_1 - V_1^T) + e^{-2kd_2}(X_1 + X_1^T),$
$E_{4,5} = -e^{-2kd_2}(-X_1 + X_2^T), E_{4,6} = -e^{-2kd^*}(V_1 - V_2^T),$
$E_{5,5} = -e^{-2kd_2} R_3 + e^{-2kd_2}(-X_2 - X_2^T),$
$E_{6,6} = -L_1 \hat{V} - (1 - \mu) e^{-2kd^*} R_1 + e^{-2kd^*}(V_2 + V_2^T) + e^{-2kd^*}(-W_2 - W_2^T),$
$E_{6,9} = L_2 \hat{V},$
$E_{7,7} = -2P + d_1 Q_6 + \frac{d_1^2}{2} T_1 + \frac{d^{*2} - d_1^2}{2} T_2 + \frac{d_2^2 - d^{*2}}{2} T_3 + (d^* - d_1) Q_7 + (d_2 - d^*) Q_8,$
$E_{7,8} = PB_1, \quad E_{7,9} = PB_2, \quad E_{7,10} = -GD,$
$E_{7,13} = P\tilde{E}, E_{8,8} = R_5 + \bar{r}^2 Q_2 - \hat{U}, E_{8,11} = -B_1^T PA,$
$E_{9,9} = -(1 - \mu) e^{-2kd^*} R_5 - \hat{V}, \quad E_{9,11} = -B_2^T PA,$
$E_{10,10} = -\hat{W}, \quad E_{10,11} = D^T G^T A,$
$E_{11,11} = 2kA^T PA - e^{-2k\sigma} Q_1, \quad E_{11,13} = -A^T P\tilde{E},$
$E_{12,12} = -\frac{1}{d_1} e^{-2kd_1} Q_3 - \frac{2}{d_1^2} e^{-2kd_1} T_1, E_{13,13} = -e^{-2k\bar{r}} Q_2,$
$E_{14,14} = -\frac{1}{d^* - d_1} e^{-2kd^*} Q_4 - \frac{2}{d^{*2} - d_1^2} e^{-2kd^*} T_2,$
$E_{14,15} = -\frac{2}{d^{*2} - d_1^2} e^{-2kd^*} T_2,$
$E_{15,15} = -\frac{1}{d^* - d_1} e^{-2kd^*} Q_4 - \frac{2}{d^{*2} - d_1^2} e^{-2kd^*} T_2,$
$E_{16,16} = -\frac{1}{d_2 - d^*} e^{-2kd_2} Q_5 - \frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3$
$F_{1,15} = \frac{2}{d_2 + d^*} e^{-2kd_2} T_3,$
$F_{3,3} = e^{-2kd_1} R_2 + e^{-2kd_1}(-U_2 - U_2^T) + e^{-2kd^*}(S_1 + S_1^T),$
$F_{3,4} = e^{-2kd^*}(-S_1 + S_2^T), \quad F_{3,6} = 0,$
$F_{4,4} = e^{-2kd^*} R_4 + e^{-2kd^*}(-S_2 - S_2^T) + e^{-2kd_2}(Y_1 + Y_1^T),$
$F_{4,5} = 0, \quad F_{4,6} = e^{-2kd_2}(Y_1 + Y_2^T),$
$F_{5,5} = e^{-2kd_2}(-Z_2 - Z_2^T) - e^{-2kd_2} R_3,$
$F_{5,6} = e^{-2kd_2}(Z_1 - Z_2^T),$
$F_{6,6} = -(1 - \mu) e^{-2kd_2} R_1 + e^{-2kd_2}(-Y_2 - Y_2^T) + e^{-2kd_2}(Z_2 + Z_2^T),$
$F_{9,9} = -(1 - \mu) e^{-2kd_2} R_5 - \hat{V},$
$F_{15,15} = -\frac{1}{d_2 - d^*} e^{-2kd_2} Q_5 - \frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3,$
$F_{15,16} = -\frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3,$
$F_{16,16} = -\frac{1}{d_2 - d^*} e^{-2kd_2} Q_5 - \frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3,$

the other entries of E is 0 and the other entries of F is the same as E, and $d^* \in (d_1, d_2), d^* = \lambda d_1 + (1 - \lambda) d_2, \lambda \in (0, 1)$, The gain matrix of state estimator is given by

$$K = P^{-1} G \quad (11)$$

Proof: Consider a novel augmented of Lyapunov-Krasovskii functional for the system (3) or (4) as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (12)$$

where

$$V_1(t) = e^{2kt}[\varepsilon(t) - A\int_{t-\sigma}^{t} \varepsilon(s) ds]^T P[\varepsilon(t) - A\int_{t-\sigma}^{t} \varepsilon(s) ds],$$

$$\begin{aligned}V_2(t) =& \int_{t-\tau(t)}^{t} e^{2ks}\varepsilon^T(s)R_1\varepsilon(s)ds \\ &+ \int_{t-d_1}^{t} e^{2ks}\varepsilon^T(s)R_2\varepsilon(s)ds \\ &+ \int_{t-d_2}^{t} e^{2ks}\varepsilon^T(s)R_3\varepsilon(s)ds \\ &+ \int_{t-d^*}^{t} e^{2ks}\varepsilon^T(s)R_4\varepsilon(s)ds \\ &+ \int_{t-\tau(t)}^{t} e^{2ks}\varphi^T(s)R_5\varphi(s)ds \\ &+ \int_{t-\sigma}^{t} e^{2ks}\varepsilon^T(s)R_6\varepsilon(s)ds,\end{aligned}$$

$$\begin{aligned}V_3(t) =& \sigma\int_{-\sigma}^{0}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_1\varepsilon(s)dsd\theta \\ &+ \bar{r}\int_{-\bar{r}}^{0}\int_{t+\theta}^{t} e^{2ks}\varphi^T(s)Q_2\varphi(s)dsd\theta \\ &+ \int_{-d_1}^{0}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_3\varepsilon(s)dsd\theta \\ &+ \int_{-d^*}^{-d_1}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_4\varepsilon(s)dsd\theta \\ &+ \int_{-d_2}^{-d^*}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_5\varepsilon(s)dsd\theta \\ &+ \int_{-d_1}^{0}\int_{t+\theta}^{t} e^{2ks}\dot{\varepsilon}^T(s)Q_6\dot{\varepsilon}(s)dsd\theta \\ &+ \int_{-d^*}^{-d_1}\int_{t+\theta}^{t} e^{2ks}\dot{\varepsilon}^T(s)Q_7\dot{\varepsilon}(s)dsd\theta \\ &+ \int_{-d_2}^{-d^*}\int_{t+\theta}^{t} e^{2ks}\dot{\varepsilon}^T(s)Q_8\dot{\varepsilon}(s)dsd\theta,\end{aligned}$$

$$\begin{aligned}V_4(t) =& \int_{t-d_1}^{t}\int_{\theta}^{t}\int_{\lambda}^{t} e^{2ks}\dot{\varepsilon}^T(s)T_1\dot{\varepsilon}(s)dsd\lambda d\theta \\ &+ \int_{t-d^*}^{t-d_1}\int_{\theta}^{t}\int_{\lambda}^{t} e^{2ks}\dot{\varepsilon}^T(s)T_2\dot{\varepsilon}(s)dsd\lambda d\theta \\ &+ \int_{t-d_2}^{t-d^*}\int_{\theta}^{t}\int_{\lambda}^{t} e^{2ks}\dot{\varepsilon}^T(s)T_3\dot{\varepsilon}(s)dsd\lambda d\theta,\end{aligned}$$

The derivative of $V(t)$ with respect to time along the trajectory (1) is

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t), \tag{13}$$

The time derivative of $V_i(t), i = 1, 2, 3, 4$, is obtained as

$$\begin{aligned}\dot{V}_1(t) =& 2ke^{2kt}[\varepsilon(t) - A\int_{t-\sigma}^{t}\varepsilon(s)ds]^T P[\varepsilon(t) \\ &- A\int_{t-\sigma}^{t}\varepsilon(s)ds] + 2e^{2kt}[\varepsilon(t) - A\int_{t-\sigma}^{t}\varepsilon(s)ds]^T \\ &\times P[-(A+KC)\varepsilon(t) + B_1\varphi(t) + B_2\varphi(t-\tau(t)) \\ &+ E\int_{t-r(t)}^{t}\varphi(s)ds - KD\psi(t)],\end{aligned} \tag{14}$$

$$\begin{aligned}\dot{V}_2(t) \leq& e^{2kt}\varepsilon^T(t)R_1\varepsilon(t) - (1-\mu)e^{2k(t-\tau(t))}\varepsilon^T(t-\tau(t)) \\ &\times R_1\varepsilon(t-\tau(t)) + e^{2kt}\varepsilon^T(t)R_2\varepsilon(t) - e^{2k(t-d_1)} \\ &\times \varepsilon^T(t-d_1)R_2\varepsilon(t-d_1) + e^{2kt}\varepsilon^T(t)R_3\varepsilon(t) \\ &- e^{2k(t-d_2)}\varepsilon^T(t-d_2)R_3\varepsilon(t-d_2) + e^{2kt}\varepsilon^T(t)R_4 \\ &\times \varepsilon(t) - e^{2k(t-d^*)}\varepsilon^T(t-d^*)R_4\varepsilon(t-d^*) + e^{2kt} \\ &\times \varphi^T(t)R_5\varphi(t) - (1-\mu)e^{2k(t-\tau(t))}\varphi^T(t-\tau(t)) \\ &\times R_5\varphi^T(t-\tau(t)) + e^{2kt}\varepsilon^T(t)R_6\varepsilon(t) - e^{2k(t-\sigma)} \\ &\times \varepsilon^T(t-\sigma)R_6\varepsilon(t-\sigma)\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{V}_3(t) \leq& e^{2kt}\varepsilon^T(t)[\sigma^2Q_1 + d_1Q_3 + (d^*-d_1)Q_4 \\ &+ (d_2-d^*)Q_5]\varepsilon(t) + \bar{r}^2e^{2kt}\varphi^T(t)Q_2\varphi(t) \\ &+ e^{2kt}\dot{\varepsilon}^T(t)[d_1Q_6 + (d^*-d_1)Q_7 + (d_2-d^*) \\ &\times Q_8]\dot{\varepsilon}(t) - \sigma e^{2k(t-\sigma)}\int_{t-\sigma}^{t}\varepsilon^T(s)Q_1\varepsilon(s)ds \\ &- r(t)e^{2k(t-\bar{r})}\int_{t-r(t)}^{t}\varphi^T(s)Q_2\varphi(s)ds \\ &- e^{2k(t-d_1)}\int_{t-d_1}^{t}\varepsilon^T(s)dsQ_3\varepsilon(s)ds - e^{2k(t-d^*)} \\ &\times \int_{t-d^*}^{t-d_1}\varepsilon^T(s)Q_4\varepsilon(s)ds - e^{2k(t-d_2)}\int_{t-d_2}^{t-d^*}\varepsilon^T(s) \\ &\times Q_5\varepsilon(s)ds - e^{2k(t-d_1)}\int_{t-d_1}^{t}\dot{\varepsilon}^T(s)Q_6\dot{\varepsilon}(s)ds \\ &- e^{2k(t-d^*)}\int_{t-d^*}^{t-d_1}\dot{\varepsilon}^T(s)Q_7\dot{\varepsilon}(s)ds - e^{2k(t-d_2)} \\ &\times \int_{t-d_2}^{t-d^*}\dot{\varepsilon}^T(s)Q_8\dot{\varepsilon}(s)ds,\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{V}_4(t) \leq& e^{2kt}\dot{\varepsilon}^T(s)[\frac{d_1^2}{2}T_1 + \frac{d^{*2}-d_1^2}{2}T_2 + \frac{d_2^2-d^{*2}}{2}T_3]\dot{\varepsilon}(t) \\ &- e^{2k(t-d_1)}\int_{t-d_1}^{t}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_1\dot{\varepsilon}(s)dsd\lambda \\ &- e^{2k(t-d^*)}\int_{t-d^*}^{t-d_1}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_2\dot{\varepsilon}(s)dsd\lambda - e^{2k(t-d_2)} \\ &\times \int_{t-d_2}^{t-d^*}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_3\dot{\varepsilon}(s)dsd\lambda,\end{aligned} \tag{17}$$

and $\hat{U}, \hat{V}$ and $\hat{W}$ are positive diagonal matrices and from Assumption (A) and (B) we can get:

$$l_1 = \begin{bmatrix}\varepsilon(t)\\ \varphi(t)\end{bmatrix}^T\begin{bmatrix}-L_1\hat{U} & L_2\hat{U}\\ L_2\hat{U} & -\hat{U}\end{bmatrix}\begin{bmatrix}\varepsilon(t)\\ \varphi(t)\end{bmatrix} \geq 0, \tag{18}$$

$$\begin{aligned}l_2 =& \begin{bmatrix}\varepsilon(t-\tau(t))\\ \varphi(t-\tau(t))\end{bmatrix}^T\begin{bmatrix}-L_1\hat{V} & L_2\hat{V}\\ L_2\hat{V} & -\hat{V}\end{bmatrix} \\ &\begin{bmatrix}\varepsilon(t-\tau(t))\\ \varphi(t-\tau(t))\end{bmatrix} \geq 0,\end{aligned} \tag{19}$$

$$l_3 = \begin{bmatrix} \varepsilon(t) \\ \psi(t)) \end{bmatrix}^T \begin{bmatrix} -L_3\hat{W} & L_4\hat{W} \\ L_4\hat{W} & -\hat{W} \end{bmatrix} \begin{bmatrix} \varepsilon(t) \\ \psi(t) \end{bmatrix} \geq 0, \quad (20)$$

From the (3), the following zero equalities with symmetric matrices P is considered.

$$2e^{2kt}\dot{\varepsilon}(t)p[-\dot{\varepsilon}(t) - A\varepsilon(t-\sigma) - KC\varepsilon(t) + B_1\varphi(t) + B_2\varphi(t-\tau(t)) + E\int_{t-r(t)}^{t}\varphi(s)ds - KD\psi(t)] = 0, \quad (21)$$

and P is the same with that in V_1, from Lemma 2, we have

$$-\sigma\int_{t-\sigma}^{t}\varepsilon^T(s)Q_1\varepsilon(s)ds \leq -[\int_{t-\sigma}^{t}\varepsilon(s)ds]^TQ_1[\int_{t-\sigma}^{t}\varepsilon(s)ds],$$
$$-r(t)\int_{t-r(t)}^{t}\varphi^T(s)dsQ_2\varphi(S)ds \leq -[\int_{t-r(t)}^{t}\varphi(s)ds]^TQ_2 \times [\int_{t-r(t)}^{t}\varphi(s)ds],$$
$$-\int_{t-d_1}^{t}\varepsilon^T(s)dsQ_3\varepsilon(s)ds \leq -\frac{1}{d_1}[\int_{t-d_1}^{t}\varepsilon(s)ds]^TQ_3 \times [\int_{t-d_1}^{t}\varepsilon(s)ds],$$
$$-\int_{t-d_1}^{t}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_1\dot{\varepsilon}(s)dsd\lambda \leq -\frac{2}{d_1^2}[d_1\varepsilon(t) - \int_{t-d_1}^{t}\varepsilon(s)ds]^T \times T_1[d_1\varepsilon(t) - \int_{t-d_1}^{t}\varepsilon(s)ds], \quad (22)$$

(1) When $d_1 \leq \tau(t) \leq d^*$,

$$-\int_{t-d^*}^{t-d_1}\varepsilon^T(s)Q_4\varepsilon(s)ds \leq -\frac{1}{d^*-d_1}[\int_{t-\tau(t)}^{t-d_1}\varepsilon(s)ds]^T \times Q_4[\int_{t-\tau(t)}^{t-d_1}\varepsilon(s)ds] - \frac{1}{d^*-d_1}[\int_{t-d^*}^{t-\tau(t)}\varepsilon(s)ds]^TQ_4 \times [\int_{t-d^*}^{t-\tau(t)}\varepsilon(s)ds], \quad (23)$$

$$-\int_{t-d_2}^{t-d^*}\varepsilon^T(s)Q_5\varepsilon(s)ds \leq -\frac{1}{d_2-d^*}[\int_{t-d_2}^{t-d^*}\varepsilon(s)ds]^T \times Q_5[\int_{t-d_2}^{t-d^*}\varepsilon(s)ds], \quad (24)$$

$$-\int_{t-d^*}^{t-d_1}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_2\dot{\varepsilon}(s)dsd\lambda \leq -\frac{2}{d^{*2}-d_1^2}[(d^*-d_1) \times \varepsilon(t) - \int_{t-\tau(t)}^{t-d_1}\varepsilon(s)ds - \int_{t-d^*}^{t-\tau(t)}\varepsilon(s)ds]^TT_2[(d^*-d_1)\varepsilon(t) - \int_{t-\tau(t)}^{t-d_1}\varepsilon(s)ds - \int_{t-d^*}^{t-\tau(t)}\varepsilon(s)ds], \quad (25)$$

$$-\int_{t-d_2}^{t-d^*}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_3\dot{\varepsilon}(s)dsd\lambda \leq -\frac{2}{d_2^2-d^{*2}}[(d_2-d^*)\varepsilon(t) - \int_{t-d_2}^{t-d^*}\varepsilon(s)ds]^TT_3[(d_2-d^*)\varepsilon(t) - \int_{t-d_2}^{t-d^*}\varepsilon(s)ds], \quad (26)$$

and for any scalar $d_1 \leq \tau(t) \leq d^*$, and any constant matrix $Q_b \in R^{n\times nn}$, $Q_b = Q_b^T > 0, (b = 6, 7, 8)$, we can get the following inequalities from Lemma 3.

$$-\int_{t-d_1}^{t}\dot{\varepsilon}^T(s)Q_6\dot{\varepsilon}(s)ds \leq d_1\xi^T(t)UQ_6^{-1}U^T\xi(t) + 2\xi^T(t)U[\varepsilon(t) - \varepsilon(t-d_1)],$$
$$-\int_{t-d^*}^{t-\tau(t)}\dot{\varepsilon}^T(s)Q_7\dot{\varepsilon}(s)ds \leq (d^*-\tau(t))\xi^T(t)VQ_7^{-1}V^T\xi(t) + 2\xi^T(t)V[\varepsilon(t-\tau(t)) - \varepsilon(t-d^*)],$$
$$-\int_{t-\tau(t)}^{t-d_1}\dot{\varepsilon}^T(s)Q_7\dot{\varepsilon}(s)ds \leq (\tau(t)-d_1)\xi^T(t)WQ_7^{-1}W^T\xi(t) + 2\xi^T(t)W[\varepsilon(t-d_1) - \varepsilon(t-\tau(t))],$$
$$-\int_{t-d_2}^{t-d^*}\dot{\varepsilon}^T(s)Q_8\dot{\varepsilon}(s)ds \leq (d_2-d^*)\xi^T(t)XQ_8^{-1}X^T\xi(t) + 2\xi^T(t)X[\varepsilon(t-d^*) - \varepsilon(t-d_2)], \quad (27)$$

where $\xi^T(t)$ will defined next and U, V, W, X are free-weighting matrices with appropriate dimensions.

$$\xi^T(t) = [\varepsilon^T(t) \;\; \varepsilon^T(t-\sigma) \;\; \varepsilon^T(t-d_1) \;\; \varepsilon^T(t-d^*) \;\; \varepsilon^T(t-d_2) \;\; \varepsilon^T(t-\tau(t)) \;\; \dot{\varepsilon}^T(t) \;\; \varphi^T(t) \;\; \varphi^T(t-\tau(t)) \;\; \psi^T(t) \;\; \int_{t-\sigma}^{t}\varepsilon^T(s)ds \;\; \int_{t-d_1}^{t}\varepsilon^T(s)ds \;\; \int_{t-r(t)}^{t}\varphi^T(s)ds \;\; \int_{t-\tau(t)}^{t-d_1}\varepsilon^T(s)ds \;\; \int_{t-d^*}^{t-\tau(t)}\varepsilon^T(s)ds \;\; \int_{t-d_2}^{t-d^*}\varepsilon^T(s)ds],$$

By substituting (14)-(27) into (13) and use the relationship $PK = G$, we can obtain

$$\dot{V}(t) \leq -e^{2kt}\xi^T(t)\Sigma_1\xi(t), \quad (28)$$

where $\Sigma_1 = -\Xi_1 > 0$ with

$$\Xi_1 = E + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-\tau(t))e^{-2kd^*}VQ_7^{-1}V^T + (\tau(t)-d_1)e^{-2kd^*}WQ_7^{-1}W^T + (d_2-d^*)e^{-2kd_2}XQ_8^{-1}X^T, \quad (29)$$

According to (7) and (8), then one can obtain

$$E + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-d_1)e^{-2kd^*}VQ_7^{-1}V^T + (d_2-d^*)e^{-2kd_2}XQ_8^{-1}X^T < 0,$$
$$E + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-d_1)e^{-2kd^*}WQ_7^{-1}W^T + (d_2-d^*)e^{-2kd_2}XQ_8^{-1}X^T < 0, \quad (30)$$

then, according to the convex combination technique, we can obtain

$$\Xi_1 = E + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-\tau(t))e^{-2kd^*}VQ_7^{-1}V^T + (\tau(t)-d_1)e^{-2kd^*}WQ_7^{-1}V^T + (d_2-d^*)e^{-2kd_2}XQ_8^{-1}V^T < 0, \quad (31)$$

that is $\dot{V}(t) \leq -e^{2kt}\xi^T(t)\Sigma_1\xi(t) < 0$,
(2) When $d^* \leq \tau(t) \leq d_2$, using Lemma 2, one get

$$-\int_{t-d^*}^{t-d_1}\varepsilon^T(s)Q_4\varepsilon(s)ds \leq -\frac{1}{d^*-d_1}[\int_{t-d^*}^{t-d_1}\varepsilon(s)ds]^TQ_4 \times[\int_{t-d^*}^{t-d_1}\varepsilon(s)ds], \tag{32}$$

$$-\int_{t-d_2}^{t-d^*}\varepsilon^T(s)Q_5\varepsilon(s)ds \leq -\frac{1}{d_2-d^*}[\int_{t-\tau(t)}^{t-d^*}\varepsilon(s)ds]^TQ_5 \times[\int_{t-\tau(t)}^{t-d^*}\varepsilon(s)ds] - \frac{1}{d_2-d^*}[\int_{t-d_2}^{t-\tau(t)}\varepsilon(s)ds]^TQ_5 \times[\int_{t-d_2}^{t-\tau(t)}\varepsilon(s)ds], \tag{33}$$

$$-\int_{t-d^*}^{t-d_1}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_2\dot{\varepsilon}(s)dsd\lambda \ \leq -\frac{2}{{d^*}^2-d_1^2}[(d^*-d_1)\varepsilon(t) -\int_{t-d^*}^{t-d_1}\varepsilon(s)ds]^TT_2[(d^*-d_1)\varepsilon(t)-\int_{t-d^*}^{t-d_1}\varepsilon(s)ds], \tag{34}$$

$$-\int_{t-d_2}^{t-d^*}\int_{\lambda}^{t}\dot{\varepsilon}^T(s)T_3\dot{\varepsilon}(s)dsd\lambda \leq -\frac{2}{d_2^2-{d^*}^2}[(d_2-d^*)\varepsilon(t) -\int_{t-\tau(t)}^{t-d^*}\varepsilon(s)ds-\int_{t-d_2}^{t-\tau(t)}\varepsilon(s)ds]^TT_3[(d_2-d^*)\varepsilon(t) -\int_{t-\tau(t)}^{t-d^*}\varepsilon(s)ds-\int_{t-d_2}^{t-\tau(t)}\varepsilon(s)ds], \tag{35}$$

and for any scalar $d^* \leq \tau(t) \leq d_2$, and any constant matrix $Q_b \in R^{n\times nn}$, $Q_b = Q_b^T > 0, (b = 6, 7, 8)$, we can get the following inequalities from Lemma 3.

$$\begin{aligned}
&-\int_{t-d_1}^{t}\dot{\varepsilon}^T(s)Q_6\dot{\varepsilon}(s)ds \leq d_1\eta^T(t)UQ_6^{-1}U^T\eta(t)\\
&+2\eta^T(t)U[\varepsilon(t)-\varepsilon(t-d_1)],\\
&-\int_{t-d^*}^{t-d_1}\dot{\varepsilon}^T(s)Q_7\dot{\varepsilon}(s)ds \leq (d^*-d_1)\eta^T(t)SQ_7^{-1}S^T\eta(t)\\
&+2\eta^T(t)S[\varepsilon(t-d_1)-\varepsilon(t-d^*)],\\
&-\int_{t-\tau(t)}^{t-d^*}\dot{\varepsilon}^T(s)Q_8\dot{\varepsilon}(s)ds \leq (\tau(t)-d^*)\eta^T(t)YQ_8^{-1}Y^T\eta(t)\\
&+2\eta^T(t)Y[\varepsilon(t-d^*)-\varepsilon(t-\tau(t))],\\
&-\int_{t-d_2}^{t-\tau(t)}\dot{\varepsilon}^T(s)Q_8\dot{\varepsilon}(s)ds \leq (d_2-\tau(t))\eta^T(t)ZQ_8^{-1}Z^T\eta(t)\\
&+2\eta^T(t)Z[\varepsilon(t-\tau(t))-\varepsilon(t-d_2)],
\end{aligned} \tag{36}$$

where $\eta^T(t)$ will defined next and U, S, Y, Z are free-weighting matrices with appropriate dimensions.

$$\begin{aligned}
\eta^T(t) =&[\varepsilon^T(t)\ \ \varepsilon^T(t-\sigma)\ \ \varepsilon^T(t-d_1)\ \ \varepsilon^T(t-d^*)\ \ \varepsilon^T(t-d_2)\\
&\varepsilon^T(t-\tau(t))\ \ \dot{\varepsilon}^T(t)\ \ \varphi^T(t)\ \ \varphi^T(t-\tau(t))\ \ \psi^T(t)\\
&\int_{t-\sigma}^{t}\varepsilon^T(s)ds\ \ \int_{t-d_1}^{t}\varepsilon^T(s)ds\ \ \int_{t-r(t)}^{t}\varphi^T(s)ds\\
&\int_{t-d^*}^{t-d_1}\varepsilon^T(s)ds\ \ \int_{t-\tau(t)}^{t-d^*}\varepsilon^T(s)ds\ \ \int_{t-d_2}^{t-\tau(t)}\varepsilon^T(s)ds],
\end{aligned}$$

By substituting (14)-(22), (32)-(36) into (13) and use the relationship $PK = G$, we obtain

$$\dot{V}(t) \leq -e^{2kt}\eta^T(t)\Sigma_2\eta(t), \tag{37}$$

where $\Sigma_2 = -\Xi_2 > 0$ with

$$\begin{aligned}
\Xi_2 =&F + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-d_1)e^{-2kd^*}SQ_7^{-1}S^T\\
&+(\tau(t)-d^*)e^{-2kd_2}YQ_8^{-1}Y^T + (d_2-\tau(t))e^{-2kd_2}\\
&ZQ_8^{-1}Z^T,
\end{aligned} \tag{38}$$

According to (9) and (10), then one can obtain

$$\begin{aligned}
&F + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-d_1)e^{-2kd^*}SQ_7^{-1}S^T\\
&+(d_2-d^*)e^{-2kd_2}YQ_8^{-1}Y^T < 0,\\
&F + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-d_1)e^{-2kd^*}SQ_7^{-1}S^T\\
&+(d_2-d^*)e^{-2kd_2}ZQ_8^{-1}Z^T < 0,
\end{aligned} \tag{39}$$

then, according to the convex combination technique, we can obtain

$$\begin{aligned}
\Xi_2 =&F + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^*-d_1)e^{-2kd^*}SQ_7^{-1}S^T\\
&+(\tau(t)-d^*)e^{-2kd_2}YQ_8^{-1}Y^T + (d_2-\tau(t))e^{-2kd_2}\\
&ZQ_8^{-1}Z^T < 0,
\end{aligned} \tag{40}$$

that is $\dot{V}(t) \leq -e^{2kt}\eta^T(t)\Sigma_2\eta(t) < 0$,

Therefore, if (7)-(10) are satisfied, then (3) or (4) is guaranteed to be globally exponentially stable for $d(t) \in [d_1, d_2]$.

On the other hand, it is not difficult to obtain the following inequalities:

$$\begin{aligned}
&V_1(0) \leq \lambda_{\max}(P)\|\varepsilon(0) - A\int_{-\sigma}^{0}\varepsilon(s)ds\|^2 \leq \lambda_{\max}(P)\\
&\times(\|\Phi(0)\| - \|A\|\int_{-\sigma}^{0}\|\varepsilon(s)\|ds)^2 \leq \lambda_{\max}(P)\|\Phi(0)\|^2\\
&+\|A\|^2\int_{-\sigma}^{0}\|\varepsilon(s)\|^2ds \leq \lambda_{\max}(P)(1+\sigma\|A\|^2)\\
&\times\sup_{-\tau\leq s\leq 0}\|\Phi(s)\|^2,
\end{aligned} \tag{41}$$

$$\begin{aligned}
&V_2(0) \leq (d_2\lambda_{\max}(R_1) + d_1\lambda_{\max}(R_2) + d_2\lambda_{\max}(R_3)\\
&+d^*\lambda_{\max}(R_4 + d_2\gamma_1\lambda_{\max}(R_5) + \sigma\lambda_{\max}(R_6))\\
&\times\sup_{-\tau\leq s\leq 0}\|\Phi(s)\|^2,
\end{aligned} \tag{42}$$

$$V_3(0) \le \sigma\lambda_{\max}(Q_1)\int_{-\sigma}^{0}\int_{\theta}^{0} e^T(s)\varepsilon(s)dsd\theta + \bar{r}\lambda_{\max}(Q_2) \times \int_{-\bar{r}}^{0}\int_{\theta}^{0}\varphi^T(s)\varphi(s)dsd\theta + \lambda_{\max}(Q_3)\int_{-d_1}^{0}\int_{\theta}^{0}\varepsilon^T(s)\varepsilon(s) \times dsd\theta + \lambda_{\max}(Q_4)\int_{-d^*}^{-d_1}\int_{\theta}^{0}\varepsilon^T(s)\varepsilon(s)dsd\theta + \lambda_{\max}(Q_5) \times \int_{-d_2}^{-d^*}\int_{\theta}^{0}\varepsilon^T(s)\varepsilon(s)dsd\theta + \lambda_{\max}(Q_6)\int_{-d_1}^{0}\int_{\theta}^{0}\dot{\varepsilon}^T(s)\dot{\varepsilon}(s) \times dsd\theta + \lambda_{\max}(Q_7)\int_{-d^*}^{-d_1}\int_{\theta}^{0}\dot{\varepsilon}^T(s)\dot{\varepsilon}(s)dsd\theta + \lambda_{\max} \times (Q_8)\int_{-d_2}^{-d^*}\int_{\theta}^{0}\dot{\varepsilon}^T(s)\dot{\varepsilon}(s)dsd\theta, \tag{43}$$

$$\dot{\varepsilon}^T(s)\dot{\varepsilon}(s) \le 6[\lambda_{\max}(A^TA) + \lambda_{\max}(C^TK^TKC) + \gamma_1^2\lambda_{\max}(B_1^TB_1) + \gamma_1^2\lambda_{\max}(B_2^TB_2) + \bar{r}\gamma_1^2\lambda_{\max}(\tilde{E}^T\tilde{E}) + \gamma_2^2\lambda_{\max}(D^TK^TKD)]\sup_{-\tau\le s\le 0}\|\Phi(s)\|^2, \tag{44}$$

where
$\gamma_1 = \max_{1\le i\le n}\{|\phi_i^-|, |\phi_i^+|\}$, $\gamma_2 = \max_{1\le j\le m}\{|\omega_j^-|, |\omega_j^+|\}$.

$$\bar{r}\lambda_{\max}(Q_2)\int_{-\bar{r}}^{0}\int_{\theta}^{0}\varphi^T(s)\varphi(s)dsd\theta \le \frac{\bar{r}^3\gamma_1^2}{2}\lambda_{\max}(Q_2) \times \sup_{-\tau\le s\le 0}\|\Phi(s)\|^2, \tag{45}$$

$$V_4(0) \le [d_1^3\lambda_{\max}(T_1) + (d^{*3} - d_1^3)\lambda_{\max}(T_2) + (d_2^3 - d^{*3}) \times \lambda_{\max}(T_3)]\lambda_{\max}(A^TA) + \lambda_{\max}(C^TK^TKC) + \gamma_1^2 \times \lambda_{\max}(B_1^TB_1) + \gamma_1^2\lambda_{\max}(B_2^TB_2) + \bar{r}\gamma_1^2\lambda_{\max}(\tilde{E}^T\tilde{E}) + \gamma_2^2 \times \lambda_{\max}(D^TK^TKD)]\sup_{-\tau\le s\le 0}\|\Phi(s)\|^2, \tag{46}$$

$$\alpha = [\lambda_{\max}(P)(1+\sigma\|A\|^2) + d_2\lambda_{\max}(R_1) + d_1\lambda_{\max}(R_2) + d_2\lambda_{\max}(R_3) + d^*\lambda_{\max}(R_4 + d_2\gamma_1\lambda_{\max}(R_5) + \sigma\lambda_{\max}(R_6) + \frac{\sigma^3}{6}\lambda_{\max}(Q_1) + \frac{\bar{r}^3}{6}\gamma_1^2\lambda_{\max}(Q_2) + \frac{d_1^3}{6}\lambda_{\max}(Q_3) + \frac{d^{*2} - d_1^2}{6}\lambda_{\max}(Q_4) + \frac{d_2^2 - d^{*2}}{6} \times \lambda_{\max}(Q_5)] + [3d_1^2\lambda_{\max}(Q_6) + 3(d^{*2} - d_1^2)\lambda_{\max}(Q_7) + 3(d_2^2 - d_1^2)\lambda_{\max}(Q_8) + d_1^3\lambda_{\max}(T_1) + (d^{*3} - d_1^3)\lambda_{\max}(T_2) + (d_2^3 - d^{**})\lambda_{\max}(T_3)] \times [\lambda_{\max}(A^TA) + \lambda_{\max}(C^TK^TKC) + \gamma_1^2\lambda_{\max}(B_1^TB_1) + \gamma_1^2\lambda_{\max}(B_2^TB_2) + \bar{r}^2\gamma_1^2\lambda_{\max}(\tilde{E}^T\tilde{E}) + \gamma_2^2\gamma_1^2\lambda_{\max}(D^TK^TKD)],$$

Then we can get:

$$V(0) \le \alpha \sup_{-\tau\le s\le 0}\|\Phi(t)\|^2 < \infty, \tag{47}$$

Considering $V_1(t)$, one can easily obtain that

$$\|\varepsilon(t) - A\int_{t-\sigma}^{t}\varepsilon(s)ds\|^2 \le \frac{1}{e^{2kt}\lambda_{\min}(P)}[\varepsilon(t) - A \times \int_{t-\sigma}^{t}\varepsilon(s)ds]^TP[\varepsilon(t) - A\int_{t-\sigma}^{t}\varepsilon(s)ds] \le \frac{V(t)}{e^{2kt}\lambda_{\min}(P)} \le \frac{V(0)}{e^{2kt}\lambda_{\min}(P)}, \tag{48}$$

Which implies that

$$\|\varepsilon(t)\| \le \|A\int_{t-\sigma}^{t}\varepsilon(s)ds\| + \sqrt{\frac{V(0)}{e^{2kt}\lambda_{\min}(P)}} \le \|A\|\int_{t-\sigma}^{t}\|\varepsilon(s)\|ds + \sqrt{\frac{V(0)}{e^{2kt}\lambda_{\min}(P)}}, \tag{49}$$

Where $\|A\| = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}^2}$,
And by the well-known Gronwall inequality which can be found in [35], it yields

$$\|\varepsilon(t)\| \le \sqrt{\frac{V(0)}{e^{2kt}\lambda_{\min}(P)}}e^{\|A\|\sigma} \le \sqrt{\frac{\alpha}{\lambda_{\min}(P)}}e^{\|A\|\sigma} \times \sup_{-\tau\le s\le 0}\|\Phi(s)\|e^{-kt}, \tag{50}$$

Then from the Definition 1, the system (3) or (4) is exponentially stable with convergence rate k, and the proof is completed. ■

In particular, the model (3) becomes the well-known case when $\sigma = 0$ and the system has been directly or indirectly investigated by a lot of authors, see [26-29].

When there is no time delay in the leakage term in error-state system (3) or (4), that is $\sigma = 0$, we get:

$$\dot{\varepsilon}(t) = -(A+KC)\varepsilon(t) + B_1\varphi(t) + B_2\varphi(t-\tau(t)) + \tilde{E}\int_{t-r(t)}^{t}\varphi(s)ds - KD\psi(t), \tag{51}$$

where $\tau(t)$ and $r(t)$ are the discrete time-varying delays and the distributed time-varying delays satisfy the following inequality:

$$0 < d_1 \le \tau(t) \le d_2, \quad 0 < r(t) \le \bar{r}, \quad \dot{\tau}(t) \le \mu < \infty,$$

$d_1, d_2, \bar{r}$ and μ are constants. for system (51), we can obtain the following result.

Corollary 1 For the given scalars $d_1, d_2, \bar{r}$ and $\mu < \infty$, the equilibrium point of the error-state system (51) is globally exponentially stable with the rate index k, if there exist $P > 0$, $R_l > 0$, $(l = 1, 2, \cdots, 5)$, $Q_m > 0$, $(m = 2, 3, \cdots, 8)$, $T_n > 0$, $(n = 1, 2, 3)$, any matrices $G, U_k, V_k, W_k, X_k, Y_k, Z_k, S_k (k = 1, 2)$, and the diagonal matrices $\hat{U} > 0$, $\hat{V} > 0$, $\hat{W} > 0$ such that the following LMIs hold.

$$H_{14\times 14} + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^* - d_1)e^{-2kd^*}V \times Q_7^{-1}V^T + (d_2 - d^*)e^{-2kd_2}XQ_8^{-1}X^T < 0, \tag{52}$$

$$H_{14\times14} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} W \times Q_7^{-1} W^T + (d_2 - d^*) e^{-2kd_2} X Q_8^{-1} X^T < 0, \quad (53)$$

$$I_{14\times14} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} S \times Q_7^{-1} S^T + (d_2 - d^*) e^{-2kd_2} Y Q_8^{-1} Y^T < 0, \quad (54)$$

$$I_{14\times14} + d_1 e^{-2kd_1} U Q_6^{-1} U^T + (d^* - d_1) e^{-2kd^*} S \times Q_7^{-1} S^T + (d_2 - d^*) e^{-2kd_2} Z Q_8^{-1} Z^T < 0, \quad (55)$$

where

$U = [U_1^T \ -U_2^T \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$
$V = [0\ 0\ V_1^T\ 0\ V_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$
$W = [0\ W_1^T\ 0\ 0\ W_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$
$X = [0\ 0\ X_1^T\ X_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$
$Y = [0\ 0\ Y_1^T\ 0\ Y_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$
$Z = [0\ 0\ 0\ Z_1^T\ Z_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$
$S = [0\ S_1^T\ S_2^T\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0],$

$H_{1,1} = 2kP - PA - A^T P - GC - C^T G^T + R_1 + R_2 + R_3 + R_4 + d_1 Q_3 + (d^* - d_1) Q_4 + (d_2 - d^*) Q_5 - 2e^{-2kd_1} T_1 - 2\frac{d^* - d_1}{d^* + d_1} e^{-2kd^*} T_2 - L_1 \hat{U} - L_3 \hat{W} + e^{-2kd_1} U_1 + e^{-2kd_1} U_1^T - 2\frac{d_2 - d^*}{d_2 + d^*} e^{-2kd_2} T_3$
$H_{1,2} = e^{-2kd_1}(-U_1 + U_2^T), \quad H_{1,6} = -C^T G^T,$
$H_{1,7} = PB_1 + L_2 \hat{U}, \quad H_{1,8} = PB_2$
$H_{1,9} = -GD + L_4 \hat{W}, \ H_{1,10} = \frac{2}{d_1} e^{-2kd_1} T_1,$
$H_{1,11} = P\tilde{E}, \ H_{1,12} = \frac{2}{d^* + d_1} e^{-2kd^*} T_2,$
$H_{1,13} = \frac{2}{d^* + d_1} e^{-2kd^*} T_2, \ H_{1,14} = \frac{2}{d_2 + d^*} e^{-2kd_2} T_3,$
$H_{2,2} = e^{-2kd_1}(U_2^T + U_2) - e^{-2kd_1} R_2 + e^{-2kd^*}(W_1 + W_1^T),$
$H_{2,5} = e^{-2kd^*}(-W_1 + W_2^T)$
$H_{3,3} = -e^{-2kd^*} R_4 + e^{-2kd^*}(-V_1 - V_1)^T + e^{-2kd_2} \times (X_1 + X_1^T),$
$H_{3,4} = -e^{-2kd_2}(-X_1 + X_2^T), \quad H_{3,5} = -e^{-2kd^*}(V_1 - V_2^T),$
$H_{4,4} = -e^{-2kd_2} R_3 + e^{-2kd_2}(-X_2 - X_2^T),$
$H_{5,5} = -L_1 \hat{V} - (1 - \mu) e^{-2kd^*} R_1 + e^{-2kd^*}(V_2 + V_2^T) + e^{-2kd^*}(-W_2 - w_2^T),$
$H_{5,8} = L_2 \hat{V},$
$H_{6,6} = -2P + d_1 Q_6 + \frac{d_1^2}{2} T_1 + \frac{d^{*2} - d_1^2}{2} T_2 + \frac{d_2^2 - d^{*2}}{2} T_3 + (d^* - d_1) Q_7 + (d_2 - d^*) Q_8,$
$H_{6,7} = PB_1, \quad H_{6,8} = PB_2, \quad H_{6,9} = -GD, \quad H_{6,11} = P\tilde{E},$
$H_{7,7} = R_5 + \bar{r}^2 Q_2 - \hat{U}, \quad H_{8,8} = -(1 - \mu) e^{-2kd^*} R_5 - \hat{V},$
$H_{9,9} = -\hat{W}, \quad E_{10,11} = D^T G^T A,$
$H_{10,10} = -\frac{1}{d_1} e^{-2kd_1} Q_3 - \frac{2}{d_1^2} e^{-2kd_1} T_1, H_{11,11} = -e^{-2k\bar{r}} Q_2,$
$H_{12,12} = -\frac{1}{d^* - d_1} e^{-2kd^*} Q_4 - \frac{2}{d^{*2} - d_1^2} e^{-2kd^*} T_2,$
$H_{12,13} = -\frac{2}{d^{*2} - d_1^2} e^{-2kd^*} T_2,$
$H_{13,13} = -\frac{1}{d^* - d_1} e^{-2kd^*} Q_4 - \frac{2}{d^{*2} - d_1^2} e^{-2kd^*} T_2,$
$H_{14,14} = -\frac{1}{d_2 - d^*} e^{-2kd_2} Q_5 - \frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3$
$I_{1,13} = \frac{2}{d_2 + d^*} e^{-2kd_2} T_3$
$I_{2,2} = e^{-2kd_1} R_2 + e^{-2kd_1}(-U_2 - U_2^T) + e^{-2kd^*}(S_1 + S_1^T),$
$I_{3,4} = e^{-2kd^*}(-S_1 + S_2^T), \quad I_{3,6} = 0,$
$I_{3,3} = e^{-2kd^*} R_4 + e^{-2kd^*}(-S_2 - S_2^T) + e^{-2kd_2}(Y_1 + Y_1^T),$
$I_{4,5} = 0, \quad I_{4,6} = e^{-2kd_2}(Y_1 + Y_2^T),$
$I_{4,4} = e^{-2kd_2}(-Z_2 - Z_2^T) - e^{-2kd_2} R_3,$
$I_{5,6} = e^{-2kd_2}(Z_1 - Z_2^T),$
$I_{5,5} = -(1 - \mu) e^{-2kd_2} R_1 + e^{-2kd_2}(-Y_2 - Y_2^T) + e^{-2kd_2}(Z_2 + Z_2^T), \quad I_{8,8} = -(1 - \mu) e^{-2kd_2} R_5 + \hat{V},$
$I_{13,13} = -\frac{1}{d_2 - d^*} e^{-2kd_2} Q_5 - \frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3,$
$I_{13,14} = -\frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3,$
$I_{14,14} = -\frac{1}{d_2 - d^*} e^{-2kd_2} Q_5 - \frac{2}{d_2^2 - d^{*2}} e^{-2kd_2} T_3,$

the other entries of H is 0 and the other entries of I is the same as H, and $d^* \in (d_1, d_2), d^* = \lambda d_1 + (1 - \lambda) d_2, \lambda \in (0, 1)$, The gain matrix of state estimator is given by

$$K = P^{-1} G, \quad (56)$$

Proof: Consider a novel augmented of Lyapunov-Krasovskii functional for the system (51) as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (57)$$

where

$$V_1(t) = e^{2kt} \varepsilon(t)^T P \varepsilon(t),$$

$$V_2(t) = \int_{t-\tau(t)}^{t} e^{2ks} \varepsilon^T(s) R_1 \varepsilon(s) ds + \int_{t-d_1}^{t} e^{2ks} \varepsilon^T(s) R_2 \varepsilon(s) ds + \int_{t-d_2}^{t} e^{2ks} \varepsilon^T(s) R_3 \varepsilon(s) ds + \int_{t-d^*}^{t} e^{2ks} \varepsilon^T(s) R_4 \varepsilon(s) ds + \int_{t-\tau(t)}^{t} e^{2ks} \varphi^T(s) R_5 \varphi(s) ds,$$

$$V_3(t) = \bar{r}\int_{-\bar{r}}^{0}\int_{t+\theta}^{t} e^{2ks}\varphi^T(s)Q_2\varphi(s)dsd\theta + \int_{-d_1}^{0}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_3\varepsilon(s)dsd\theta + \int_{-d^*}^{-d_1}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_4\varepsilon(s)dsd\theta + \int_{-d_2}^{-d^*}\int_{t+\theta}^{t} e^{2ks}\varepsilon^T(s)Q_5\varepsilon(s)dsd\theta + \int_{-d_1}^{0}\int_{t+\theta}^{t} e^{2ks}\dot{\varepsilon}^T(s)Q_6\dot{\varepsilon}(s)dsd\theta + \int_{-d^*}^{-d_1}\int_{t+\theta}^{t} e^{2ks}\dot{\varepsilon}^T(s)Q_7\dot{\varepsilon}(s)dsd\theta + \int_{-d_2}^{-d^*}\int_{t+\theta}^{t} e^{2ks}\dot{\varepsilon}^T(s)Q_8\dot{\varepsilon}(s)dsd\theta,$$

$$V_4(t) = \int_{t-d_1}^{t}\int_{\theta}^{t}\int_{\lambda}^{t} e^{2ks}\dot{\varepsilon}^T(s)T_1\dot{\varepsilon}(s)dsd\lambda d\theta + \int_{t-d^*}^{t-d_1}\int_{\theta}^{t}\int_{\lambda}^{t} e^{2ks}\dot{\varepsilon}^T(s)T_2\dot{\varepsilon}(s)dsd\lambda d\theta + \int_{t-d_2}^{t-d^*}\int_{\theta}^{t}\int_{\lambda}^{t} e^{2ks}\dot{\varepsilon}^T(s)T_3\dot{\varepsilon}(s)dsd\lambda d\theta,$$

According to the similar procedure discussed in Theorem 1, we obtain

(1) When $d_1 \le \tau(t) \le d^*$,

$$\dot{V}(t) \le e^{2kt}\xi^T[H + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^* - \tau(t))e^{-2kd^*} \times VQ_7^{-1}V^T + (\tau(t) - d_1)e^{-2kd^*}WQ_7^{-1}V^T + (d_2 - d^*) \times e^{-2kd_2}XQ_8^{-1}V^T](t)\xi(t) < 0, \tag{58}$$

where

$$\bar{\xi}^T(t) = [\varepsilon^T(t) \;\; \varepsilon^T(t-d_1) \;\; \varepsilon^T(t-d^*) \;\; \varepsilon^T(t-d_2) \;\; \varepsilon^T(t-\tau(t)) \;\; \dot{\varepsilon}^T(t) \;\; \varphi^T(t) \;\; \varphi^T(t-\tau(t)) \;\; \psi^T(t) \;\; \int_{t-d_1}^{t}\varepsilon^T(s)ds \;\; \int_{t-r(t)}^{t}\varepsilon^T(s)ds \;\; \int_{t-\tau(t)}^{t-d_1}\varepsilon^T(s)ds \;\; \int_{t-d^*}^{t-\tau(t)}\varepsilon^T(s)ds \;\; \int_{t-d_2}^{t-d^*}\varepsilon^T(s)ds],$$

(2) When $d^* \le \tau(t) \le d_2$,

$$\dot{V}(t) \le e^{2kt}\xi^T[I + d_1e^{-2kd_1}UQ_6^{-1}U^T + (d^* - \tau(t))e^{-2kd^*} \times VQ_7^{-1}V^T + (\tau(t) - d_1)e^{-2kd^*}WQ_7^{-1}W^T + (d_2 - d^*) \times e^{-2kd_2}XQ_8^{-1}X^T](t)\xi(t) < 0, \tag{59}$$

where

$$\bar{\eta}^T(t) = [\varepsilon^T(t) \;\; \varepsilon^T(t-d_1) \;\; \varepsilon^T(t-d^*) \;\; \varepsilon^T(t-d_2) \;\; \varepsilon^T(t-\tau(t)) \;\; \dot{\varepsilon}^T(t) \;\; \varphi^T(t) \;\; \varphi^T(t-\tau(t)) \;\; \psi^T(t) \;\; \int_{t-d_1}^{t}\varepsilon^T(s)ds \;\; \int_{t-r(t)}^{t}\varphi^T(s)ds \;\; \int_{t-d^*}^{t-d_1}\varepsilon^T(s)ds \;\; \int_{t-\tau(t)}^{t-d^*}\varepsilon^T(s)ds \;\; \int_{t-d_2}^{t-\tau(t)}\varepsilon^T(s)ds],$$

From the (58) and (59), by using the convex combination technique and Schur complement, the (60) are equivalent to (52)-(54). This completes the proof. ■

Remark 1 It is noted that [25] discussed the error-state system (3) and (4) is globally asymptotically stable and it did not use the delay departing technique, in this paper, Theorem 1 proposes an improved exponential stability condition for neural networks with mixed time-delays. we divided $[d_1, d_2]$ into $[d_1, d^*]$ and $[d^*, d_2]$, which d^* satisfy linear combination, Each segments has a different Lyapunov matrix, which can obtain less conservative results. and the constructed Lyapunov functional $V(t)$ is much more general and desirable than that in [25].

Remark 2 Although the system (51) has been studied in [27]-[28], the results in [27] and [28] can not tackle the estimation problems when the derivative of the time-varying is not less than one, and they did not have the leakage delay.

Remark 3 The author of [34] investigated the system with distributed delay, the system in this paper also has leakage delay σ, so [34] is a special situation of this paper's system when $d_1 = 0$ and $\sigma = 0$. that is to say this paper discuss the discrete with certain bounds.

IV. Numerical examples

In this section, three examples with be given to illustrate the usefulness of obtained results.

Example 1 Consider the error-state system (3) and (4) with the following parameters

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & -0.2 & 0.4 \\ -0.4 & 2 & 0.2 \\ 0.2 & 0.4 & -0.4 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}, \tilde{E} = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$J = \begin{bmatrix} 2sint + 0.03t^2 \\ 3cost - 0.03t^3 \\ 2cost - 0.03t^2 \end{bmatrix},$$

Where $L_1 = diag\{0,0,0\}$, $L_3 = diag\{0,0,0\}$, $L_2 = L_4 = diag\{-0.25, -0.25, 0.25\}$. If that $\tau(t) = 3sin^2t + 0.01$, $r(t) = cos^2t, \sigma = 0$, we can easily get $d_1 = 0.01, d_2 = 3.01$, $\mu = 3$ and $r = 1$. when $k = 0.01, \lambda = 0.5$, by Corollary 1 and the LMI toolbox in solving (51), we have the estimator gain matrix P and G as follows:

$$P = \begin{bmatrix} 0.0416 & 0.0257 & 0.0048 \\ 0.0257 & 0.0455 & -0.0015 \\ 0.0048 & -0.0015 & 0.0198 \end{bmatrix},$$
$$G = \begin{bmatrix} 0.0007 & 0.0007 & 0.0000 \\ 0.0014 & 0.0002 & 0.0000 \\ -0.0002 & 0.0000 & 0.0000 \end{bmatrix},$$

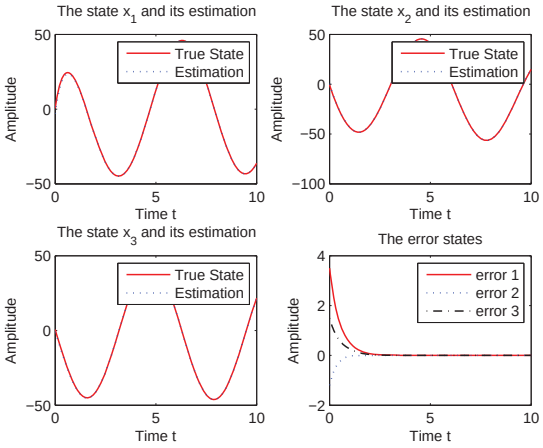

Fig. 1. Error trajectories of the state estimation in case of $\sigma = 0.01$.

Thus, the estimator gain matrix K as follows

$$K = P^{-1}G = \begin{bmatrix} -0.0006 & 0.0239 & 0.0000 \\ 0.0302 & -0.0084 & 0.0000 \\ -0.0069 & -0.0070 & 0.0000 \end{bmatrix},$$

It is easy to known that $\tau(t)$ is a time-varying one and $\mu = 3 \geq 1.5 \geq 1$, [30,32,33] fail to tackle the state estimation problem, though [34] succuss to tackle the state estimation problem when $\mu \geq 1$, the variation range of $\tau(t)$ we get is bigger than it. when $\mu = 3$ we known it satisfied the corollary 1 and when $k \leq 0.4711$ and $\lambda = 0.5$ we can always obtain a K. If that $\tau(t) = 0.9sint + 1$, we can easily get $d_1 = 0.1$, $d_2 = 1.9$, $\mu = 0.9$, if $\bar{r} = 1, \sigma = 0.01, k = 0.01, \lambda = 0.5$, by using the Theorem 1, we can get the corresponding gain matrix as follows.

$$K = P^{-1}G = \begin{bmatrix} 0.0034 & 0.0456 & 0 \\ 0.0470 & -0.0189 & 0 \\ -0.0188 & -0.0087 & 0 \end{bmatrix},$$

It can be seen that the dynamical behavior of (3) or (4) id global exponential stable when $\sigma = 0.01, k = 0.01$, the trajectories of the error system (3) or (4) converge to zero as shown in Fig 1.

Example 2 Consider the error-state system (3) and (4) with the following parameters

TABLE I
ALLOWABLE UPPER BOUND OF d_2 WITH VARIOUS d_1 AND σ

d_1	0.01	0.02	0.05	0.1
$\sigma = 0$	9.4212	8.8106	6.7741	2.8619
$\sigma = 0.01$	9.4205	8.8106	6.7678	2.8618
$\sigma = 0.1$	9.4023	8.7569	6.5624	2.3646
$\sigma = 1$	8.9460	7.7203	3.5692	infeasible

TABLE II
ALLOWABLE UPPER BOUND OF k WITH VARIOUS σ AND d_2

σ	0	0.1	0.2	0.5	1
$d_2 = 1.5$	0.6738	0.6707	0.6634	0.6344	0.5669
$d_2 = 3$	0.4722	0.4693	0.4643	0.4441	0.4008

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & -0.2 & 0.4 \\ -0.4 & 2 & 0.2 \\ 0.2 & 0.4 & -0.4 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}, \tilde{E} = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix},$$
$$C = D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, L_1 = L_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$L_2 = L_4 = \begin{bmatrix} -0.25 & 0 & 0 \\ 0 & -0.25 & 0 \\ 0 & 0 & -0.25 \end{bmatrix},$$

If $\mu = 3, r = 1, \lambda = 0.5, k = 0.01$. by Theorem 1 and the LMI toolbox in solving (3) or (4), we have estimated the allowable upper bound of d_2 when d_1 and σ are varied in Table I .

Example 3 Consider the error-state system (3) and (4) with the following parameters

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & -0.2 & 0.4 \\ -0.4 & 2 & 0.2 \\ 0.2 & 0.4 & -0.4 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \end{bmatrix}, \tilde{E} = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

If giving the parameters as $L_1 = L_3 = diag\{0, 0, 0\}, L_2 = L_4 = diag\{-0.25, -0.25, 0.25\}, d_1 = 0.01, \mu = 1.5, \lambda = 0.5$ and $r = 1$. by Theorem 1 and the LMIs toolbox in solving (3) or (4), we tackle the state estimation problem when σ and d_2 are varying, and obtain the allowable upper bound of k in Table II.

V. CONCLUSIONS

In this paper, the state estimation problem for neural networks with mixed time-varying delays has been studied, through the LMI approach and delay departing measurements,

an exponential state estimator is designed to estimate the neuron state and verify the dynamics of the estimation error is globally exponentially stable, and by dividing the discrete delay interval into multiple segments, the upper bounds of some variables are obtained. Finally, numerical examples have been given to illustrate the effectiveness of the proposed methods.

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Liyuan Wang was born in Hebei Province, China,in 1987. She received the B.S. degree from Tangshan Normal University in 2012. She currently pursuing the M.S.degree from University of Electronic Science and Technology of China. Her research interests include the stability of neural networks and the state estimator of neural networks.

Shouming Zhong was born in 1955 in Sichuan, China. He received B.S. degree in applied mathematics from UESTC, Chengdu, China, in 1982. From 1984 to 1986, he studied at the Department of Mathematics in Sun Yatsen University, Guangzhou, China. From 2005 to 2006, he was a visiting research associate with the Department of Mathematics in University of Waterloo,Waterloo,Canada.Heis currently as a full professor with School of Applied Mathematics, UESTC. His current research interests include differential equations,neural networks, biomathematics and robustcontrol.He has authored more than 80 papers in reputed journals such as the International Journal of Systems Science, Applied Mathematics and Computation, Chaos, Solitons and Fractals, Dynamics of Continuous, Discrete and Impulsive Systems, Acta Automatica Sinica, Journal of Control Theory and Applications, Acta Electronica Sinica, Control and Decision, and Journal of Engineering Mathematics.