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Exponential Stability and periodicity of a class of cellular neural networks with time-varying delays

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Abstract—The problem of exponential stability and periodicity for a class of cellular neural networks (DCNNs) with time-varying delays is investigated. By dividing the network state variables into subgroups according to the characters of the neural networks, some sufficient conditions for exponential stability and periodicity are derived via the methods of variation parameters and inequality techniques. These conditions are represented by some blocks of the interconnection matrices. Compared with some previous methods, the method used in this paper does not resort to any Lyapunov function, and the results derived in this paper improve and generalize some earlier criteria established in the literature cited therein. Two examples are discussed to illustrate the main results.

Keywords—Cellular neural networks, exponential stability, time-varying delays, partitioned matrices, periodic solution.

I. Introduction

In past few decades, cellular neural networks (CNNs)[1] and delayed cellular neural networks (DCNNs) have been well investigated since they play an important role in applications such as static image treatment [2], [3], processing of moving images, speed detection of moving objects [4], and pattern classification [5], et al,. And many stability criteria for DCNNs have been obtained (see [6]-[12]). In [6], a sufficient condition for complete stability of DCNNs with positive cell linking and dominant templates is given. In [7], it was proved that if the sum of the feedback matrix and the delayed feedback matrix is symmetrical and the length of delay is smaller than a certain value depending on the delayed feedback matrix, then the DCNNs is stable.

In this paper, by dividing the network state variables into subgroups according to the characters of the neural networks, the problem on exponential stability and periodicity for a class of cellular neural networks with time-varying delays is investigated. By using methods of variation parameters, some sufficient conditions ensuring exponential stability and the existence of periodic solution are derived. These results improve and generalize some earlier criteria obtained in the literature cited therein. Two examples are given to illustrate the improvement and effectiveness of the main results. However, the conditions obtained in [7], [9], [11], [12], [13] are not applicable to determine the stability of the system for these examples.

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II. Preliminaries

Notations. The notations are used in our paper except where otherwise specified. For $A, B \in R^n, A \leq B (A > B)$means that each pair of corresponding elements of A and B satisfies the inequality $\leq (>)$. In particular, A is called a nonnegative matrix if $A \geq 0$; $\|\cdot\|$ denotes a vector or a matrix norm and $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$, $\|A\| = \sup_{\|x\|\neq 0} \frac{\|Ax\|}{\|x\|}$; I denotes the identity matrix and $\rho(\cdot)$ denotes the spectral radius of a square matrix.

In [14], Zhong and Liu investigated the following dynamics of continuous time DCNNs model with discrete time delay

$$\frac{dx(t)}{dt} = -x(t) + Af(x(t)) + Bf(x(t-\tau)) + u, (t \geq 0)$$

In this paper, we will study the generalized dynamics of continuous time DCNNs with time-varying delays defined by the following state equations

$$\frac{dx(t)}{dt} = -x(t)+Af(x(t))+Bf(x(t-\tau(t)))+u, (t \geq 0) \quad (1)$$

where $x(\cdot) = [x_1(\cdot), x_2(\cdot), \ldots, x_n(\cdot)]^T$ is state vector; $u = [u_1, u_2, \ldots, u_n]^T$ is constant vector; $f(x(\cdot)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot)), \cdots, f_n(x_n(\cdot))]^T$ is the output; $A = (a_{ij})_{n\times n}$ is feedback matrix; $B = (b_{ij})_{n\times n}$ is delayed feedback matrix; $x(t-\tau(t)) = [x_1(t-\tau_1(t)), x_2(t-\tau_2(t)), \ldots, x_n(t-\tau_n(t))]^T$; $\tau_i(t) \geq 0 (i = 1, 2, \ldots, n)$ is delay parameter and the output equations are given by

$$f_i(x_i(\cdot)) = \frac{1}{2}(|x_i(\cdot)+1|-|x_i(\cdot)-1|), i = 1, 2, \ldots, n. \quad (2)$$

One can see that f_i is globally Lipschitz continuous with Lipschtiz constant $\mu_i = 1$ for $i = 1, 2, \ldots, n$, i.e

$$|f_i(u) - f_i(v)| \leq |u - v|, \forall u, v \in R.$$

This implies that system (1) admits a unique solution in its maximum existence interval for the initial condition given by $x(t) = \phi(t), t \in [-\tau^*, 0]$, where $\phi(t)$ is continuous on $[-\tau^*, 0]$, and $0 \leq \tau_i(t) \leq \tau^*, i = 1, 2, \ldots, n$.

In order to discuss the exponential stability properties of DCNNs (1), the following concept of exponential stability is needed.

Definition 2.1: An equilibrium x^* of system(1) is said to be exponentially stable if there exist $\alpha \geq 1, \beta > 0$, such that for any $t \geq 0$ and $\phi \in C([-\tau^*, 0], R^n)$, $\|x(t) - x^*\| \leq \alpha\|\phi - x^*\|_\triangle e^{-\beta t}$, where $\|\phi - x^*\|_\triangle = \{\sum_{i=1}^{n} \sup_{-\tau_i\leq t\leq 0} |\phi_i(t) - x_i^*|^2\}^{\frac{1}{2}}$, $C([-\tau^*, 0], R^n)$ is the Banach space of continuous functions which map $[-\tau^*, 0]$ to R^n with the topology of uniform convergence.

For further discussion, the following lemmas are needed, which will be used in section 3.

Lemma 2.1: there exists at least one equilibrium point of system (1)

Proof. Denote $\Omega = \{x \in R^{n\times 1}, \|x-u\| \le \|A\|M_k + \|B\|M_k\}$, where $M_k = \sup\{\|f(x(t))\|\}$, since $f(x(t))$ is bounded, thus M_k exists. Define a map $F : R^n \to R^n$

$$F(x(t)) = Af(x(t)) + Bf(x(t-\tau(t))) + u \tag{3}$$

From (3), we obtain

$$\begin{aligned}\|F(x(t)) - u\| &= \|Af(x(t)) + Bf(x(t-\tau(t)))\| \\ &\le \|A\|\|f(x(t))\| + \|B\|\|f(x(t-\tau(t)))\| \quad (4)\\ &= \|A\|M_k + \|B\|M_k\end{aligned}$$

It follows that F maps Ω into itself. Since Ω is a convex compact set, then by the Brower Fixed Point theorem, we know $F : \Omega \to \Omega$ has at least one fixed point $x(t) = x^*$, which completes the proof of the lemma.

Lemma 2.2: (Holder inequality).Assume that there exist two continuous functions $f(x), g(x)$ and a set Ω, p and q satisfying $1/q + 1/p = 1$, for any $p > 0, q > 0$, if $p > 1$, then the following inequality holds.

$$\int_\Omega |f(x)g(x)|dx \le (\int_\Omega |f(x)|^p dx)^{1/p}(\int_\Omega |g(x)|^q dx)^{1/q}.$$

Lemma 2.3: [15] Assume that there exist constants $a_k \ge 0, k = 1, 2, \ldots, n, p$ and q satisfying $1/q + 1/p = 1$, for any $p > 0, q > 0$, if $p > 1$, then the following inequality holds

$$(\sum_{k=1}^n a_k)^p \le n^{p-1}\sum_{k=1}^n a_k^p.$$

Lemma 2.4: (Horn[12]).If $M \ge 0$ and $\rho(M) < 1$, then $(I - M)^{-1} \ge 0$, where I denotes the identity matrix and $\rho(M)$ denotes the spectral radius of a square matrix M.

Let x^* be an equilibrium point of system (1) and define $y(\cdot) = x(\cdot) - x^*$, then we get

$$\begin{aligned}\frac{dy(t)}{dt} &= -y(t) + A(f(y(t) + x^*) \\ &\quad - f(x^*)) + B(f(y(t-\tau(t)) + x^*) - f(x^*)). \quad (5)\end{aligned}$$

Let us divide the set $I = \{1, 2, \ldots, n\}$ into subsets I_1, I_2 and I_3,such that $I = I_1 \cup I_2 \cup I_3$ where $I_1 = \{i \in I | x_i^* > 1\}, I_2 = \{i \in I | -1 \le x_i^* \le 1\}, I_3 = \{i \in I | x_i^* < -1\}$

We may rearrange the order of $y_1, y_2, \ldots, y_n$ such that

$$\begin{aligned}I_1 &= \{1, 2, \ldots, r\},\\ I_2 &= \{r+1, r+2, \ldots, r+m\},\\ I_3 &= \{r+m+1, r+m+2, \ldots, n\},\end{aligned}$$

where $r, m, n-r-m$ are non-negative integers. The variables of system (5) are reordered, but for convenience, we may use the same symbols as those in system (5).

Let

$$y(t) = \begin{pmatrix} y_{(1)}(t)\\ y_{(2)}(t)\\ y_{(3)}(t)\end{pmatrix},$$

where

$$\begin{aligned}y_{(1)}(t) &= (y_1(t), y_2(t), \ldots, y_r(t))^T,\\ y_{(2)}(t) &= (y_{r+1}(t), y_{r+2}(t), \ldots, y_{r+m}(t))^T,\\ y_{(3)}(t) &= (y_{r+m+1}(t), y_{r+m+2}(t), \ldots, y_n(t))^T,\end{aligned}$$

So system (5) can be decomposed into

$$\begin{cases}\frac{dy_{(1)}(t)}{dt} = -y_{(1)}(t) + A_{11}g(y_{(1)}(t)) + A_{12}g(y_{(2)}(t))\\ \qquad + A_{13}g(y_{(3)}(t)) + B_{11}g(y_{(1)}(t-\tau(t)))\\ \qquad + B_{12}g(y_{(2)}(t-\tau(t))) + B_{13}g(y_{(3)}(t-\tau(t)))\\ \frac{dy_{(2)}(t)}{dt} = -y_{(2)}(t) + A_{21}g(y_{(1)}(t)) + A_{22}g(y_{(2)}(t))\\ \qquad + A_{23}g(y_{(3)}(t)) + B_{21}g(y_{(1)}(t-\tau(t)))\\ \qquad + B_{22}g(y_{(2)}(t-\tau(t))) + B_{23}g(y_{(3)}(t-\tau(t)))\\ \frac{dy_{(3)}(t)}{dt} = -y_{(3)}(t) + A_{31}g(y_{(1)}(t)) + A_{32}g(y_{(2)}(t))\\ \qquad + A_{33}g(y_{(3)}(t)) + B_{31}g(y_{(1)}(t-\tau(t)))\\ \qquad + B_{32}g(y_{(2)}(t-\tau(t))) + B_{33}g(y_{(3)}(t-\tau(t))),\end{cases} \tag{6}$$

where

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{pmatrix}, B = \begin{pmatrix} B_{11} & B_{12} & B_{13}\\ B_{21} & B_{22} & B_{23}\\ B_{31} & B_{32} & B_{33}\end{pmatrix},$$

$$\begin{pmatrix} g(y_{(1)}(\cdot))\\ g(y_{(2)}(\cdot))\\ g(y_{(3)}(\cdot))\end{pmatrix} = f(y(\cdot) + x^*) - f(x^*).$$

Let $k = \min\{\min_{i\in I_1}(x_i^* - 1), \min_{i\in I_3}(-1 - x_i^*)\}$, then $k > 0$. Assume that the initial function ϕ satisfied $\|\phi - x^*\|_\Delta < k$. By continuity, there exists a $T > 0$, such that for any $t \in [-\tau_i(t), T), |y_i(t)| < k$. Therefore, for $\forall t \in [0, T)$, we have

$$f(y_i(t) + x_i^*) - f(x_i^*) = 0, \forall i \in I_1 \cup I_3$$

$$f(y_i(t - \tau_i(t)) + x_i^*) - f(x_i^*) = 0, \forall i \in I_1 \cup I_3$$

Thus $g(y_{(1)}(t)) \equiv g(y_{(3)}(t)) \equiv 0, g(y_{(1)}(t - \tau(t))) \equiv g(y_{(3)}(t - \tau(t))) \equiv 0$.

It follows that, for any $t \in [0, T)$, we obtain

$$\begin{cases}\frac{dy_{(1)}(t)}{dt} = -y_{(1)}(t) + A_{12}g(y_{(2)}(t)) + B_{12}g(y_{(2)}(t-\tau(t)))\\ \frac{dy_{(2)}(t)}{dt} = -y_{(2)}(t) + A_{22}g(y_{(2)}(t)) + B_{22}g(y_{(2)}(t-\tau(t)))\\ \frac{dy_{(3)}(t)}{dt} = -y_{(3)}(t) + A_{32}g(y_{(2)}(t)) + B_{32}g(y_{(2)}(t-\tau(t)))\end{cases} \tag{7}$$

III. Exponential stability

In this section, we consider the exponential stability for delayed neural networks (7). By the method of variation parameters, for all $t \ge 0$, we have

$$\begin{aligned}y_{(2)}(t) &= y_{(2)}(0)e^{-t} + \int_0^t e^{-(t-s)}[A_{22}g(y_{(2)}(s))\\ &\quad + B_{22}g(y_{(2)}(s-\tau(s)))]ds,\end{aligned}$$

namely

$$\begin{aligned} y_{r+i}(t) &= y_{r+i}(0)e^{-t} + \int_0^t e^{-(t-s)} \sum_{j=1}^m a_{r+i,r+j} g(y_{r+j}(s))ds \\ &\quad + \int_0^t e^{-(t-s)} \sum_{j=1}^m b_{r+i,r+j} g(y_{r+j}(s-\tau_{r+j}(s)))ds \\ &= I_{1i} + I_{2i} + I_{3i}, (\ \ i = 1,2,\ldots,m). \end{aligned}$$

Following from lemma2.3, when n=3, the following inequality holds

$$|y_{r+i}(t)|^2 \leq 3(|I_{1i}|^2 + |I_{2i}|^2 + |I_{3i}|^2),$$

then it yields for all $t \geq 0$,

$$e^{\lambda t}|y_{r+i}(t)|^2 \leq 3e^{\lambda t}(|I_{1i}|^2 + |I_{2i}|^2 + |I_{3i}|^2).$$

Here we denote $G_{r+j}(t) = \sup_{0\leq s\leq t} |y_{r+j}(s)|^2 e^{\lambda s}, j = 1,2,\ldots,m$ where $0 < \lambda < 1$. In order to get the exponential stability theorem, we first give some lemmas.

Lemma 3.1: For I_{2i}, the following inequality holds

$$e^{\lambda t}|I_{2i}|^2 \leq \frac{1}{1-\lambda} \sum_{j=1}^m |a_{r+i,r+j}|^2 \sum_{j=1}^m G_{r+j}(t).$$

Proof

$$\begin{aligned} e^{\lambda t} \ \ | \ \ I_{2i}|^2 &= e^{\lambda t}|\int_0^t e^{-(t-s)} \sum_{j=1}^m a_{r+i,r+j} g(y_{r+j}(s))ds|^2 \\ &\leq e^{\lambda t}[\int_0^t e^{-(t-s)} \sum_{j=1}^m |a_{r+i,r+j}||g(y_{r+j}(s))|ds]^2 \\ &= e^{\lambda t}[\int_0^t e^{\frac{-(t-s)}{2}} e^{\frac{-(t-s)}{2}} \sum_{j=1}^m |a_{r+i,r+j}||g(y_{r+j}(s))|ds]^2 \\ &\leq e^{\lambda t}\{[\int_0^t e^{-(t-s)}ds] \cdot [\int_0^t e^{-(t-s)} \\ &\times \ (\sum_{j=1}^m |a_{r+i,r+j}||g(y_{r+j}(s))|)^2 ds]\} \\ &\leq e^{\lambda t}\{[\int_0^t e^{-(t-s)}ds] \cdot [\int_0^t e^{-(t-s)} \\ &\times \ (\sum_{j=1}^m |a_{r+i,r+j}||y_{r+j}(s)|)^2 ds]\} \\ &= e^{\lambda t}(1-e^{-t})[\int_0^t e^{-(t-s)}(\sum_{j=1}^m |a_{r+i,r+j}||y_{r+j}(s)|)^2 ds]\} \\ &\leq e^{\lambda t}[\int_0^t e^{-(t-s)}(\sum_{j=1}^m |a_{r+i,r+j}||y_{r+j}(s)|)^2 ds]\} \\ &\leq e^{\lambda t} \int_0^t e^{-(t-s)}[\sum_{j=1}^m |a_{r+i,r+j}|^2 \sum_{j=1}^m |y_{r+j}(s)|^2]ds\} \\ &= e^{\lambda t} \sum_{j=1}^m |a_{r+i,r+j}|^2 (\int_0^t e^{-(t-s)} \sum_{j=1}^m |y_{r+j}(s)|^2 ds) \\ &= \sum_{j=1}^m |a_{r+i,r+j}|^2 (\int_0^t e^{-(t-s)} \sum_{j=1}^m e^{\lambda t}|y_{r+j}(s)|^2 ds) \\ &= \sum_{j=1}^m |a_{r+i,r+j}|^2 (\int_0^t e^{-(1-\lambda)(t-s)} \sum_{j=1}^m e^{\lambda s}|y_{r+j}(s)|^2 ds) \\ &\leq \sum_{j=1}^m |a_{r+i,r+j}|^2 \sum_{j=1}^m G_{r+j}(t) \int_0^t e^{-(1-\lambda)(t-s)} ds \\ &= \sum_{j=1}^m |a_{r+i,r+j}|^2 \sum_{j=1}^m G_{r+j}(t) \cdot \frac{1-e^{-(1-\lambda)t}}{1-\lambda} \\ &\leq \frac{1}{1-\lambda} \sum_{j=1}^m |a_{r+i,r+j}|^2 \sum_{j=1}^m G_{r+j}(t), \end{aligned}$$

which complete the proof.

Lemma 3.2: For I_{3i}, the following inequality holds

$$\begin{aligned} e^{\lambda t}|I_{3i}|^2 &\leq \frac{1}{1-\lambda} \sum_{j=1}^m |b_{r+i,r+j}|^2 e^{\lambda\tau^*} \\ &\quad \times(\sum_{j=1}^m \sup_{-\tau^*\leq\theta\leq 0} |y_{r+j}(\theta)|^2 + \sum_{j=1}^m G_{r+j}(t)) \end{aligned}$$

Proof

$$\begin{aligned} e^{\lambda t}|I_{3i}|^2 &= e^{\lambda t}|\int_0^t e^{-(t-s)} \sum_{j=1}^m b_{r+i,r+j} g(y_{r+j}(s-\tau_{r+j}(s)))ds|^2 \\ &\leq e^{\lambda t}\{\int_0^t e^{-(t-s)} \sum_{j=1}^m |b_{r+i,r+j}| \\ &\times \ \ |g(y_{r+j}(s-\tau_{r+j}(s)))|ds\}^2 \\ &= e^{\lambda t}\{\int_0^t e^{\frac{-(t-s)}{2}} e^{\frac{-(t-s)}{2}} \sum_{j=1}^m |b_{r+i,r+j}| \\ &\times \ \ |g(y_{r+j}(s-\tau_{r+j}(s)))|ds\}^2 \\ &\leq e^{\lambda t}\{[\int_0^t e^{-(t-s)}ds][\int_0^t e^{-(t-s)}(\sum_{j=1}^m |b_{r+i,r+j}| \\ &\times \ \ |g(y_{r+j}(s-\tau_{r+j}(s)))|)^2 ds]\} \\ &= e^{\lambda t}\{(1-e^{-t})[\int_0^t e^{-(t-s)}(\sum_{j=1}^m |b_{r+i,r+j}| \\ &\times \ \ |g(y_{r+j}(s-\tau_{r+j}(s)))|)^2 ds]\} \\ &\leq e^{\lambda t}[\int_0^t e^{-(t-s)}(\sum_{j=1}^m |b_{r+i,r+j}| \\ &\times \ \ |g(y_{r+j}(s-\tau_{r+j}(s)))|)^2 ds] \\ &\leq e^{\lambda t}\{\int_0^t e^{-(t-s)}[\sum_{j=1}^m |b_{r+i,r+j}|^2] \\ &\times \ \ [\sum_{j=1}^m |g(y_{r+j}(s-\tau_{r+j}(s)))|^2]ds\} \\ &\leq e^{\lambda t} \sum_{j=1}^m |b_{r+i,r+j}|^2 \cdot \{\int_0^t e^{-(t-s)} \\ &\times \ \ \sum_{j=1}^m |y_{r+j}(s-\tau_{r+j}(s))|^2 ds\} \\ &= \sum_{j=1}^m |b_{r+i,r+j}|^2 \cdot \{\int_0^t e^{-(t-s)(1-\lambda)} \\ &\times \ \ \sum_{j=1}^m e^{\lambda\tau_{r+j}(t)} e^{\lambda(s-\tau_{r+j}(t))}|y_{r+j}(s-\tau_{r+j}(s))|^2 ds\} \\ &\leq e^{\lambda\tau^*} \sum_{j=1}^m |b_{r+i,r+j}|^2 \cdot (\int_0^t e^{-(t-s)(1-\lambda)} ds) \end{aligned}$$

$$
\begin{aligned}
&\times \sum_{j=1}^{m} \sup_{0\le s\le t} e^{\lambda(s-\tau_{r+j}(t))}|y_{r+j}(s-\tau_{r+j}(s))|^2\\
&= e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\cdot(\frac{1-e^{-(1-\lambda)t}}{1-\lambda})\\
&\times \sum_{j=1}^{m} \sup_{0\le s\le t} e^{\lambda(s-\tau_{r+j}(t))}|y_{r+j}(s-\tau_{r+j}(s))|^2\\
&\le \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\\
&\times \sum_{j=1}^{m} \sup_{0\le s\le t} e^{\lambda(s-\tau_{r+j}(t))}|y_{r+j}(s-\tau_{r+j}(s))|^2\\
&\le \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\cdot\sum_{j=1}^{m}\sup_{-\tau^*\le\theta\le t}e^{\theta}|y_{r+j}(\theta)|^2\\
&= \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\\
&\times \sum_{j=1}^{m}(\sup_{-\tau^*\le\theta\le 0}e^{\theta}|y_{r+j}(\theta)|^2+\sup_{0\le\theta\le t}e^{\theta}|y_{r+j}(\theta)|^2)\\
&\le \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\\
&\times (\sum_{j=1}^{m}\sup_{-\tau^*\le\theta\le 0}|y_{r+j}(\theta)|^2+\sum_{j=1}^{m}G_{r+j}(t)),
\end{aligned}
$$

which complete the proof.

Lemma 3.3: If $\rho(MK+NK)<1$ then $y_{(2)}(t)$ satisfied the following inequality

$$\|y_{(2)}(t)\|^2 \le 3[\sum_{i=r+1}^{r+m}R_{ii}][\sum_{i=r+1}^{r+m}\sum_{j=r+1}^{r+m}M_{ij}(\alpha)]\|y_{(2)}(0)\|^2e^{-\lambda t},$$

where

$$M=diag\{a_1,a_2,\ldots,a_m\}, a_i=3\sum_{j=1}^{m}|a_{r+i,r+j}|^2,$$

$$N=diag\{b_1,b_2,\ldots,b_m\}, b_i=3\sum_{j=1}^{m}|b_{r+i,r+j}|^2,$$

$R=diag\{1+r_1,1+r_2,\ldots,1+r_m\}$, r_j satisfy the inequality $\frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\cdot(\sum_{j=1}^{m}\sup_{-\tau^*\le\theta\le 0}|y_{r+j}(\theta)|^2 \le r_j\cdot|y_{i+r}(0)|^2$ and $M(\alpha)=(I-(I-\alpha I)^{-1}(MK+e^{\alpha\tau}NK)), K=(k_{ij})_{m\times m}, k_{ij}=1, 0<\alpha\le\lambda$.

Proof According to lemma3.1, lemma3.2, we can obtain the following inequality for all $t\ge 0$

$$
\begin{aligned}
e^{\lambda t}|y_{r+i}(t)|^2 &\le 3\{\frac{1}{1-\lambda}\sum_{j=1}^{m}|a_{r+i,r+j}|^2\sum_{j=1}^{m}G_{r+j}(t)\\
&+ \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\sum_{j=1}^{m}G_{r+j}(t))\\
&+ |y_{r+i}(0)|^2+\frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\\
&\times (\sum_{j=1}^{m}\sup_{-\tau^*\le\theta\le 0}|y_{r+j}(\theta)|^2)\}
\end{aligned}
$$

it can be found that there must exist some positive constants r_j, such that the following inequality hold

$$\frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2\cdot(\sum_{j=1}^{m}\sup_{-\tau^*\le\theta\le 0}|y_{r+j}(\theta)|^2)\le r_j\cdot|y_{i+r}(0)|^2.$$

Thus, for all $t\ge 0$

$$
\begin{aligned}
G_{r+i}(t)\le 3\{(1+r_j)|y_{r+i}(0)|^2+\frac{1}{1-\lambda}[\sum_{j=1}^{m}|a_{r+i,r+j}|^2\\
+e^{\lambda\tau^*}\sum_{j=1}^{m}|b_{r+i,r+j}|^2]\sum_{j=1}^{m}G_{r+j}(t)\}.
\end{aligned}
$$

Namely,

$$G_{(2)}(t)\le 3Ry_{(2)}^2(0)+(I-\lambda I)^{-1}(MK+e^{\lambda\tau^*}NK)G_{(2)}(t),$$

where $G_{(2)}(t)=(G_{r+1}(t),G_{r+2}(t),\ldots,G_{r+m}(t))^T, y_{(2)}^2(0)=(y_{r+1}^2(0),y_{r+2}^2(0),\ldots,y_{r+m}^2(0))^T$.

Since $\rho(MK+NK)<1$ and $MK+NK\ge 0$, from Lemma 2.4, it deduces

$$[I-I^{-1}(MK+NK)]^{-1}\ge 0.$$

Hence, there exists a sufficiently small positive constant $\alpha\le\lambda$ such that

$$[I-(I-\alpha I)^{-1}(MK+e^{\alpha\tau^*}NK)]^{-1}\ge 0$$

One can derive that

$$\sum_{i=r+1}^{r+m}|y_i(t)|^2\le 3[\sum_{i=r+1}^{r+m}R_{ii}][\sum_{i=r+1}^{r+m}\sum_{j=r+1}^{r+m}M_{ij}(\alpha)][\sum_{i=r+1}^{r+m}|y_i(0)|^2]e^{-\lambda t}$$

That is

$$\|y_{(2)}(t)\|^2\le 3[\sum_{i=r+1}^{r+m}R_{ii}][\sum_{i=r+1}^{r+m}\sum_{j=r+1}^{r+m}M_{ij}(\alpha)]\|y_{(2)}(0)\|^2e^{-\lambda t},$$

which complete the proof.

Theorem 3.1: The equilibrium of system(7) is exponential stability if $\rho(MK+NK)<1$

Proof Set $M_2^2(\alpha)=3[\sum_{i=r+1}^{r+m}R_{ii}][\sum_{i=r+1}^{r+m}\sum_{j=r+1}^{r+m}M_{ij}(\alpha)]$, thus we have

$$\|y_{(2)}(t)\|\le M_2(\alpha)\|y_{(2)}(0)\|e^{-\frac{\lambda t}{2}}\le M_2(\alpha)\|\phi-x^*\|_\Delta e^{-\frac{\lambda t}{2}}$$

For the first and the third equations of system (7), by using the method of variation of parameters, for $i=1,3$, we have

$$y_{(i)}(t)=y_{(i)}(0)e^{-t}+\int_0^t e^{-(t-s)}[A_{i2}g(y_{(2)}(s))+B_{i2}g(y_{(2)}(s-\tau(t)))]ds,$$

then, we can obtain

$$
\begin{aligned}
\|y_{(i)}(t)\| &\le \|y_{(i)}(0)\|e^{-t}+\int_0^t e^{-(t-s)}[\|A_{i2}\|\|g(y_{(2)}(s))\|\\
&+ \|B_{i2}\|\|g(y_{(2)}(s-\tau(t)))\|]ds\\
&\le \|y_{(i)}(0)\|e^{-t}+\int_0^t e^{-(t-s)}[\|A_{i2}\|\|y_{(2)}(s)\|\\
&+ \|B_{i2}\|\|y_{(2)}(s-\tau(t))\|]ds\\
&\le \|\phi-x^*\|_\Delta[e^{-t}+M_2(\alpha)(\|A_{i2}\|\\
&+ e^{\frac{\lambda\tau^*}{2}}\|B_{i2}\|)\int_0^t e^{-t+s-\frac{\varepsilon}{2}s}ds]
\end{aligned}
$$

$$\begin{aligned} &\leq [1+\frac{2M_2(\alpha)}{2-\alpha}(\|A_{i2}\|+e^{\frac{\lambda\tau^*}{2}}\|B_{i2}\|)]\|\phi-x^*\|_\Delta e^{-\frac{\lambda}{2}t} \\ &= M_i(\lambda)\|\phi-x^*\|_\Delta e^{-\frac{\lambda}{2}t}, (i=1,3) \end{aligned}$$

where $M_i(\alpha)=1+\frac{2M_2(\alpha)}{2-\alpha}(\|A_{i2}\|+e^{\frac{\lambda\tau^*}{2}}\|B_{i2}\|)$.

Let $M = \max M_1(1), M_2(1), M_3(1)$, then we have $M_i(\alpha) \leq M$ for $i=1,2,3$. Since $\alpha\in(0,1)$, and if choose the initial function ϕ such that $\|\phi-x^*\|_\Delta < \frac{k}{M}$, $\forall t\in[0,T), (i=1,2,3)$, it yields

$$|y_{(i)}(t)\| \leq M_i(\alpha)\|\phi-x^*\|_\Delta e^{-\frac{\lambda}{2}t} \leq M\|\phi-x^*\|_\Delta e^{-\frac{\lambda}{2}t} < k,$$

By repeating these procedures, we can ensure that the same result holds for $t\in[T,T_1),[T_1,T_2),\ldots,[T_{n-1},T_n)$ with $T_n\to\infty$ when $n\to\infty$. So under the condition of the theorem, the existing interval of solution of system(5) is $[0,+\infty)$ and zero solution of system(5) is exponential stable, thus, the equilibrium $x=x^*$ of system (1) is exponentially stable, which complete the proof.

Theorem 3.2: The equilibrium of system (7) is exponential stability if

$$\|A_{22}\|_2^2+\|B_{22}\|_2^2<\frac{1}{3}$$

Where $\|\cdot\|_2$ is Frobenius norm, namely $\|A\|_2=(\sum_{i,j}a_{ij}^2)^{1/2}$.

Proof. From lemma3.1, lemma3.2, we have

$$\begin{aligned} e^{\lambda t}|y_{r+i}(t)|^2 \leq\ & 3\{\frac{1}{1-\lambda}\sum_{j=1}^m|a_{r+i,r+j}|^2\sum_{j=1}^m G_{r+j}(t) \\ &+ \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^m|b_{r+i,r+j}|^2\sum_{j=1}^m G_{r+j}(t)) \\ &+ |y_{r+i}(0)|^2+\frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^m|b_{r+i,r+j}|^2 \\ &\times (\sum_{j=1}^m\sup_{-\tau^*\leq\theta\leq0}|y_{r+j}(\theta)|^2)\} \\ \leq\ & 3\{\frac{1}{1-\lambda}\sum_{j=1}^m|a_{r+i,r+j}|^2\sum_{j=1}^m G_{r+j}(t) \\ &+ \frac{1}{1-\lambda}e^{\lambda\tau^*}\sum_{j=1}^m|b_{r+i,r+j}|^2\sum_{j=1}^m G_{r+j}(t)) \\ &+ (1+\frac{e\lambda\tau^*}{1-\lambda}\sum_{j=1}^m|b_{r+i,r+j}|^2) \\ &\times \sum_{j=1}^m\sup_{-\tau^*\leq\theta\leq0}|y_{r+j}(\theta)|^2)\}. \end{aligned}$$

Let $k_i=1+\frac{e\lambda\tau^*}{1-\lambda}\sum_{j=1}^m|b_{r+i,r+j}|^2$, then we have

$$\begin{aligned} G_{r+i}(t) \leq 3(k_i\|\phi-x^*\|_\Delta^2+\frac{1}{1-\lambda}[\sum_{j=1}^m|a_{r+i,r+j}|^2 \\ +e^{\lambda\tau^*}\sum_{j=1}^m|b_{r+i,r+j}|^2]\sum_{j=1}^m G_{r+j}(t)), \end{aligned}$$

namely,

$$\begin{aligned} (1-\frac{3}{1-\lambda}\sum_{i=1}^m\sum_{j=1}^m(|a_{r+i,r+j}|^2+e^{\lambda\tau^*}|b_{r+i,r+j}|^2)) \\ \times\sum_{i=1}^m G_{r+i}(t)\leq 3\sum_{i=1}^m k_i\|\phi-x^*\|_\Delta^2 \end{aligned}$$

If $3(\|A_{22}\|_2^2+\|B_{22}\|_2^2)<1$, from lemma2.4, it deduces

$$[1-3(\|A_{22}\|_2^2+\|B_{22}\|_2^2)]^{-1}>0$$

Hence there exists sufficiently small positive constant $\alpha\leq\lambda<1$ such that

$$[1-(1-\alpha)^{-1}3(\sum_{i=1}^m\sum_{j=1}^m(|a_{r+i,r+j}|^2+e^{\alpha\tau^*}|b_{r+i,r+j}|^2))]^{-1}>0$$

It can be derived that

$$\begin{aligned} \sum_{i=1}^m G_{r+i}(t) &\leq \frac{3\sum_{i=1}^m k_i\|\phi-x^*\|_\Delta^2}{\beta} \\ &= M_2'(\alpha)3k_i\|\phi-x^*\|, \end{aligned}$$

where $\beta=1-(1-\alpha)^{-1}3(\sum_{i=1}^m\sum_{j=1}^m(|a_{r+i,r+j}|^2+e^{\alpha\tau^*}|b_{r+i,r+j}|^2))$, $M_2'(\alpha)=[1-(1-\alpha)^{-1}3(\sum_{i=1}^m\sum_{j=1}^m(|a_{r+i,r+j}|^2+e^{\alpha\tau^*}|b_{r+i,r+j}|^2))]^{-1}$, hence

$$\|y_{(2)}(t)\|^2\leq M_2'(\alpha)3\sum_i k_i\cdot\|\phi-x^*\|_\Delta^2 e^{-\lambda t}$$

The rest proofs are similar to the of Theorem 3.1, which complete the proof.

IV. Existence and stability of periodic solution

Consider the following DCNNs with periodic input vector function $u(t)$ of period ω

$$\frac{dx(t)}{dt}=-x(t)+Af(x(t))+Bf(x(t-\tau(t)))+u(t),(t\geq0) \quad (8)$$

In this section, we shall give the stability criteria for periodic solution of system (8).

Theorem 4.1: There exists a unique ω-periodic solution of system (8) and all other solutions converge exponentially to the ω-periodic solution as $t\to\infty$ if the coefficient matrices of system (8) satisfies

$$\rho(M'K'+N'K')<1,$$

where

$$M'=diag\{a_1',a_2',\ldots,a_n'\}, a_i'=3\sum_{j=1}^n|a_{ij}|^2,$$

$$N'=diag\{b_1',b_2',\ldots,b_n'\}, b_i'=3\sum_{j=1}^n|b_{ij}|^2,$$

$$K=(k_{ij})_{n\times n}, k_{ij}=1, i,j=1,2,\ldots,n$$

Proof For all $\phi(t), \psi(t)$, which are continuous functions on $[-\tau^*, 0]$, denote the solutions of system (8) through $(0, \phi), (0, \psi)$ by $x_\phi(t)$ and $x_\psi(t)$, respectively. Then

$$\begin{aligned} &\frac{d(x_\phi(t) - x_\psi(t))}{dt} \\ &= -(x_\phi(t) - x_\psi(t)) + A(f(x_\phi(t)) - f(x_\psi(t))) \\ &+B(f(x_\phi(t-\tau(t))) - f(x_\psi(t-\tau(t)))). \end{aligned} \tag{9}$$

Set $y(t) = x_\phi(t) - x_\psi(t)$, $h(y(t)) = f(x_\phi(t)) - f(x_\psi(t))$, $h(y(t-\tau(t))) = f(x_\phi(t-\tau(t))) - f(x_\psi(t-\tau(t)))$ then we can rewrite the above equation as

$$\frac{dy(t)}{dt} = -y(t) + Ah(y(t)) + Bh(y(t-\tau)).$$

Like previous proof, we can obtain

$$\begin{aligned} y(t) &= y(0)e^{-t} + \int_0^t e^{-(t-s)}[Ah(y(s)) + Bh(y(s-\tau(t)))]ds \\ &= I'_{1i} + I'_{2i} + I'_{3i}, (i = 1, 2, 3) \end{aligned} \tag{10}$$

Following from lemma2.3, when n=3, the following inequality holds

$$|y(t)|^2 \le 3(|I'_{1i}|^2 + |I'_{2i}|^2 + |I'_{3i}|^2),$$

for all $t \ge 0$, it yields,

$$e^{\lambda t}|y(t)|^2 \le 3e^{\lambda t}(|I'_{1i}|^2 + |I'_{2i}|^2 + |I'_{3i}|^2)$$

Denote $G_j(t) = \sup_{0 \le s \le t} |y_j(s)|^2 e^{\lambda s}, j = 1, 2, \ldots, n$ where $0 < \lambda < 1$. Similar to the proof of lemma3.1, lemma3.2, lemma3.3 and theorem3.1, we can obtain

$$\|y(t)\| \le M(\alpha)\|y(0)\|e^{-\frac{\lambda}{2}} \le M(\alpha)\|\phi - \psi\|e^{-\frac{\lambda}{2}}.$$

Choose a positive integer m such that $M(\alpha)e^{-\frac{m\alpha\omega}{2}} \le \frac{1}{2}$. Define a Poincare mapping:

$$P : C([-\tau^*, 0], R^n) \to C([-\tau^*, 0], R^n)$$

by $P\phi = x_\phi(\omega)$. Then we derive that

$$\|P\phi - P\psi\| = \|x_\phi(\omega) - x_\psi(\omega)\| \le M(\alpha)\|\phi - \psi\|_\Delta e^{-\frac{\alpha}{2}\omega}$$

$$\begin{aligned} \|P^2\phi - P^2\psi\| &= \|Px_\phi(\omega) - Px_\psi(\omega)\| \\ &= \|x_{x_\phi(\omega)}(\omega) - x_{x_\psi(\omega)}(\omega)\| \\ &= \|x_\phi(2\omega) - x_\psi(2\omega)\| \\ &\le M(\alpha)\|\phi - \psi\|_\Delta e^{-\frac{\alpha}{2}2\omega} \end{aligned} \tag{11}$$

By induction and $M(\alpha)e^{-\frac{m\alpha\omega}{2}} \le \frac{1}{2}$, we have

$$\|P^m\phi - P^m\psi\| \le M(\alpha)\|\phi - \psi\|_\Delta e^{-\frac{\alpha}{2}m\omega} \le \frac{1}{2}\|\phi - \psi\|_\Delta$$

This implies that P^m is a contraction mapping, hence there exists a unique fixed point $\varphi \in C$such that $P^m\varphi = \varphi$. Thereby we have $P^m(P\varphi) = P(P^m\varphi) = P\varphi$. This shows that $P\varphi \in C$ is also a fixed point of P^m, so $P\varphi = \varphi$, i.e.$x_\varphi(\omega) = \varphi$ Let $x_\varphi(t)$ be a solution of system (8) through $(0, \varphi)$, then $x_\varphi(t + \omega)$ is also a solution of system (8) and

$$x_\varphi(t + \omega) = x_{x_\varphi(\omega)}(t) = x_\varphi(t), (t \ge 0),$$

which implies that $x_\varphi(t)$ is a $\omega - periodic$ solution of system (8) and we know that all other solutions of system (8) converge exponentially to this $\omega - periodic$ solution as $t \to \infty$ and hence this $x_\varphi(t)$ is a unique $\omega - periodic$ solution of system (8). Similar to the proof of Theorem3.2 and Theorem4.1 we can easily get the following Theorem.

Theorem 4.2: There exists a unique ω-periodic solution of system (8) and all other solutions converge exponentially to the ω-periodic solution as $t \to \infty$ if the coefficient matrices of system (8) satisfy $\|A\|_2^2 + \|B\|_2^2 < \frac{1}{3}$.

Notice that $\rho(A) \le \|A\|$ for any $A \in R^{n\times n}$, in which $\|\cdot\|$ is an arbitrary matrix norm. Moreover, for any matrix norm and any nonsingular matrix S, a matrix norm $\|A\|_S$ can be given by $\|A\|_S = \|S^{-1}AS\|$. For the convenience of calculation, in general, taking $S = diag\{s_1, \ldots, s_n\} > 0$. Therefore, corresponding to the matrix norm widely appliedł the row norm, column norm and Frobenius norm, we can obtain the following corollary.

Corollary 4.1: The equilibrium of system (7) is exponential stability provided one of the following conditions hold

$$(1) \quad \sum_{j=1}^m [\frac{s_i}{s_j}(a_i + b_i)] < 1, (i = 1, 2, \ldots, m),$$

$$(2) \quad \sum_{j=1}^m [\frac{s_j}{s_i}(a_j + b_j)] < 1, (i = 1, 2, \ldots, m),$$

$$(3) \quad \sum_{i=1}^m \sum_{j=1}^m [\frac{s_i}{s_j}(a_i + b_i)] < 1.$$

where $s_1, s_2, \ldots, s_m$ are positive real numbers.

Corollary 4.2: There exists a unique ω-periodic solution of system (8) and all other solutions converge exponentially to the ω-periodic solution as $t \to \infty$ provided one of the following conditions hold

$$(1) \quad \sum_{j=1}^n [\frac{s_i}{s_j}(a'_i + b'_i)] < 1, (i = 1, 2, \ldots, n)$$

$$(2) \quad \sum_{j=1}^n [\frac{s_j}{s_i}(a'_j + b'_j)] < 1, (i = 1, 2, \ldots, n)$$

$$(3) \quad \sum_{i=1}^n \sum_{j=1}^n [\frac{s_i}{s_j}(a'_i + b'_i)] < 1$$

where $s_1, s_2, \ldots, s_n$ are positive real numbers.

V. Numerical Examples

Example 1. Consider the following system

$$\begin{cases} \frac{dx_1(t)}{dt} = -x_1(t) + 2f(x_1(t)) + \frac{1}{2}f(x_2(t)) \\ \qquad - 2f(x_1(t-\tau_1(t))) + \frac{1}{2}f(x_2(t-\tau_2(t))) + 2 \\ \frac{dx_2(t)}{dt} = -x_2(t) + \frac{2}{3}f(x_1(t)) + \frac{1}{4}f(x_2(t)) \\ \qquad + \frac{1}{3}f(x_1(t-\tau_1(t))) - \frac{1}{4}f(x_2(t-\tau_2(t))) - 1 \end{cases} \tag{12}$$

where $\tau_1(t) = \frac{1}{2}\sin t + \frac{1}{2}, \tau_2(t) = \frac{1}{2}\cos t + \frac{1}{2}$. It is easy to see that $0 \le \tau_i(t) \le 1, i = 1, 2$

If $(x_1, x_2) = (x_1^*, x_2^*)$ is an equilibrium point, then we have

$$\begin{cases} x_1^* = f(x_2^*) + 2 \\ x_2^* = f(x_1^*) - 1. \end{cases} \tag{13}$$

If $x_1^* \in (-\infty, -1)$, then $f(x_1^*) = -1$. By the second equation of (12), we have $x_2^* = -2$. By the first equation of (12), we have $x_1^* = 1 \notin (-\infty, 1)$. There is no solution of (12).

If $x_1^* \in [-1, 1]$, then $f(x_1^*) = x_1^*$. By the second equation of (12), we have $x_2^* = x_1^* - 1$. By the first equation of (12), we have $x_2^* = f(x_2^*) + 1$. We can easily get $x_2^* = 2, x_1^* = 1 + x_2^* = 3 \notin [-1, 1]$. There is no solution of (12).

If $x_1^* \in (1, \infty)$, then $f(x_1^*) = 1$. By the second equation of (12), we have $x_2^* = 0$. By the first equation of (12), we have $x_1^* = 2 \in (1, \infty)$. So (2,0) is a unique equilibrium of (12).

Let $y = x - x^*$, then the system(12) can be written as the following equivalent system

$$\begin{cases} \dfrac{dy_1(t)}{dt} = -y_1(t) + \dfrac{1}{2}f(y_2(t)) + \dfrac{1}{2}f(y_2(t - \tau_2(t))) \\ \dfrac{dy_2(t)}{dt} = -y_2(t) + \dfrac{1}{4}f(y_2(t)) - \dfrac{1}{4}f(y_2(t - \tau_2(t))). \end{cases} \tag{14}$$

Since $A_{22} = (1/4)_{1\times1}, B_{22} = (-1/4)_{1\times1}$ are 1-dimension matrices, then $K = (1)_{1\times1}$ is a 1-dimension matrix, and

$$\rho(MK+NK) = \rho(3\times(1/4)^2\times1+3\times(-1/4)^2\times1) = 3/8 < 1.$$

According to theorem 3.1, the equilibrium of system (12) is exponentially stable.

Remark 1. When $\tau_i(t) = \tau_i$, then system (1) becomes a cellular neural networks with discrete time delays. At this case, since $|a11| + |a12| + |b11| + |b12| = 5 > 1$, $|a21| + |a22| + |b21| + |b22| = \frac{3}{2} > 1$, the condition of Corollary 3 in [11] does not hold. Since $-(A + A^T) = \begin{pmatrix} -4 & -7/6 \\ -7/6 & 1/2 \end{pmatrix}$ is not positive definite, thus the condition (i) of Theorem 1 in [9] does not hold. Additional, since $-(A + B) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is not diagonally row dominant, thus the conditions of Theorem 3.2 in [12] are not applicable, from which one can see that the criteria obtained in this paper are less conservative.

Example 2. We consider the following system

$$\begin{cases} \dfrac{dx_1(t)}{dt} = -x_1(t) + a_{11}f(x_1(t)) + a_{12}f(x_2(t)) \\ \quad + a_{13}f(x_3(t)) + b_{11}f(x_1(t - \tau_1(t))) \\ \quad + b_{12}f(x_2(t - \tau_2(t))) + b_{13}f(x_3(t - \tau_3(t))) \\ \quad - a_{11} - b_{11} + a_{13} + b_{13} + 2 \\ \dfrac{dx_2(t)}{dt} = -x_2(t) + a_{21}f(x_1(t)) + a_{22}f(x_2(t)) \\ \quad + a_{23}f(x_3(t)) + b_{21}f(x_1(t - \tau_1(t))) \\ \quad + b_{22}f(x_2(t - \tau_2(t))) + b_{23}f(x_3(t - \tau_3(t))) \\ \quad - a_{21} - b_{21} + a_{23} + b_{23} \\ \dfrac{dx_3(t)}{dt} = -x_3(t) + a_{31}f(x_1(t)) + a_{32}f(x_2(t)) \\ \quad + a_{33}f(x_3(t)) + b_{31}f(x_1(t - \tau_1(t))) \\ \quad + b_{32}f(x_2(t - \tau_2(t))) + b_{33}f(x_3(t - \tau_3(t))) \\ \quad - a_{31} - b_{31} + a_{33} + b_{33} - 2, \end{cases} \tag{15}$$

where

$$\tau_1(t) = \frac{1}{2}\sin t + \frac{1}{2}, \tau_2(t) = \tau_3(t) = \frac{1}{2}\cos t + \frac{1}{2}.$$

It is easy to see that $0 \le \tau_i \le 1, i = 1, 2, 3$. Direct computation shows that $x^* = (2, 0, -2)$ is an equilibrium solution of system(15). Let $y = x - x^*$, and $|y_i(t)| \le 1$, then system(15) can be rewritten as the following equivalent system small

$$\begin{cases} \dfrac{dy_1(t)}{dt} = -y_1(t) + a_{12}g(x_2(t)) + b_{12}g(x_2(t - \tau_1(t))) \\ \dfrac{dy_2(t)}{dt} = -y_2(t) + a_{22}g(x_2(t)) + b_{22}g(x_2(t - \tau_2(t))) \\ \dfrac{dy_3(t)}{dt} = -y_3(t) + a_{32}g(x_2(t)) + b_{32}g(x_2(t - \tau_3(t))). \end{cases} \tag{16}$$

If $3(|a_{22}|^2 + |b_{22}|^2) < 1$, the existence interval of the solution of system (16) is $[0, \infty)$ and the equilibrium $x^* = (2, 0, -2)$ of system(15) is exponentially stable, moreover the result is independent of the parameters $a_{i1}, a_{i3}, b_{i1}, b_{i3} \in R, i = 1, 2, 3$ and $a_{12}, a_{32}, b_{12}, b_{32}$. When these coefficients are sufficiently large, the method on paper [5] can not decide the stability of the system in this example.

VI. Conclusions

In this paper, we have derived some sufficient conditions for exponential stability for the equilibrium point and the existence and global exponential stability of periodic solutions for DCNNs by dividing the state variables of the system. Compared with the previous methods, our method does not resort to any Lyapunov function, and the results derived in this paper improve and generalize some earlier works reported in the literature. The new conditions, which are associated with some initial values, are represented by some blocks of the feedback matrix. So the conditions are related to some elements of the feedback matrix, and do not depend on other parameters, and thus these parameters can be chosen arbitrarily.

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