International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:7, No:10, 2013

# Experimental Testing of Statistical Size Effect in Civil Engineering Structures

Jana Kaděrová, Miroslav Vořechovský

*Abstract*—The presented paper copes with an experimental evaluation of a model based on modified Weibull size effect theory. Classical statistical Weibull theory was modified by introducing a new parameter (correlation length $l_\rho$) representing the spatial autocorrelation of a random mechanical properties of material. This size effect modification was observed on two different materials used in civil engineering: unreinforced (plain) concrete and multi-filament yarns made of alkali-resistant (AR) glass which are used for textile-reinforced concrete. The behavior under flexural, resp. tensile loading was investigated by laboratory experiments. A high number of specimens of different sizes was tested to obtain statistically significant data which were subsequently corrected and statistically processed. Due to a distortion of the measured displacements caused by the unstiff experiment device, only the maximal load values were statistically evaluated. Results of the experiments showed a decreasing strength with an increasing sample length. Size effect curves were obtained and the correlation length was fitted according to measured data. Results did not exclude the existence of the proposed new parameter $l_\rho$.

*Keywords*—Statistical size effect, concrete, multi-filaments yarns, experiment, autocorrelation length.

## I. Introduction

THE behavior of concrete structures in tension is usually described by simplified models reducing, or even neglecting the tensile strength of concrete. The tendency to exploit the material effectively calls for more precise models describing the tensile behavior, which is covered by fracture mechanics. Introducing the models of fracture mechanics quasibrittle materials (as concrete) was initiated a long time after these models occurred in steel structures (mainly in aircraft, naval and nuclear engineering). One of the reasons to consider the fracture mechanics of concrete is the effect of size on a nominal strength of a structure.

The scope of this paper is a description of experimental testing of two types of materials used in concrete structures when their size increases. The first material was plain unreinforced concrete, the second type were multifilament yarns from alcali resistant (AR) glass used for production of textile reinforcement for concrete. In the first case, series of beams of constant cross–section with an increasing length was designed (5 different lengths) and tested in flexure. In the letter case, yarn specimens of 6 different lengths were prepared to be tested in tension. Obtained data were statistically processed and compared with prediction according to the classical and modified Weibull size effect theory.

The experimental testing was done in the laboratory of Institute of Structural mechanic, Brno University of Technology on the testing machine Z100 Zwick/Roell Gruppe equipped by two load cells measuring the force (20 kN and 2.5 kN). For the bending tests, special equipment for 3PB and 4PB was used, while for the tensile tests of yarns, mechanical tensile clamps of combined type (self-locking with pre-stressing screws) were installed [1], [2].

J. Kaděrová and M. Vořechovský are with the Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Czech Republic, (e-mail: kaderova.j@fce.vutbr.cz).

In Section II, the classical statistical Weibull's size effect theory is shortly described. The spatial autocorrelation influence is considered and some modification of the size effect law is introduced. Section III describes the theoretical presumptions and the designed experiment investigating the size effect on samples of plain concrete. In Sec. IV a bundle model together with tensile experiments on AR-glass yarns are presented. The main obtained results are summarized in last Sec. V.

## II. Statistical size effect theory

### A. Classical Weibull Theory

The definition of classical Weibull integral for strength of structures can be derived from illustrative example of structural segments coupled in series (chain model). Each segment of the chain is independent of others and its strength is a random variable with a given probability distribution function. If the cumulative density function (CDF) is identical for all segments of the chain, then we call segments as independent and identically distributed (IID). All the segments share the same loading $\sigma$ (due to a common force $F$).

The probability of failure of any segment $P_1(\sigma)$ is equal to the strength CDF. The probability of survival of one segment is the complement $1 - P_1(\sigma)$. The probability of survival of the whole chain is $1 - P_\mathrm{f}$ and is given by condition that all the segments must survive (the collapse of one segment means the collapse of the whole chain). For independent segments, the survival probability is the product of survival probabilities of individual segments linked in series:

$$1 - P_\mathrm{f} = \underbrace{(1 - P_1)(1 - P_1)...(1 - P_1)}_{N-\mathrm{times}} = (1 - P_1)^N \quad (1)$$

By taking the logarithm of the equation, we obtain:

$$\ln(1 - P_\mathrm{f}) = N \ln(1 - P_1) \quad (2)$$

As the probability of chain failure $P_\mathrm{f}$ is a very low number in practical situations, the expression can be simplified by substitution $\ln(1 - P_1) \approx -P_1$, which leads to approximation:

$$P_\mathrm{f}(\sigma) = 1 - e^{-NP_1(\sigma)} \quad (3)$$

$$P_\mathrm{f}(\sigma) = 1 - \exp\left[-\frac{l}{l_\mathrm{r}} P_1(\sigma)\right] \quad (4)$$

where $P_1(\sigma)$ is the probability distribution of failure of a reference length $l_r$ for a given stress level $\sigma$. The reference length can be understood as a part of the total length $l$ of structure (chain) and each segment of such a length is considered independent of other parts. The number of independent chain segments is then $N = l/l_r$.

The behavior of Weibull probability distribution is demonstrated for increasing number of chain segments in Fig. 1. The random strength of each segment is given by Weibull PDF and CDF as:

$$\begin{aligned} F_1(\sigma; s, m) &= 1 - \exp\left[-(\sigma/s)^m\right] \\ f_1(\sigma; s, m) &= \begin{cases} (m/s)(\sigma/s)^{m-1} \exp\left[-(\sigma/s)^m\right] & \\ \qquad\qquad \sigma \geq 0; s, m > 0 & \\ 0 \qquad \sigma < 0 & \end{cases} \end{aligned} \tag{5}$$

Using (1) we can express the CDF and PDF of Weibull distribution for $N$ number of elements:

$$\begin{aligned} F_N &= 1 - [1 - F_1(\sigma; s, m)]^N \\ f_N &= \frac{\partial F_N}{\partial \sigma} = N \cdot f_1(\sigma; s, m)\,[1 - F_1(\sigma; s, m)]^{N-1} \end{aligned} \tag{6}$$

Graphs of the probability densities (full lines) and the cumulative distribution functions (dash lines) are plotted in Fig. 1 for different $N$. The trend of decreasing mean value and the standard deviation with increasing number of elements can be observed.

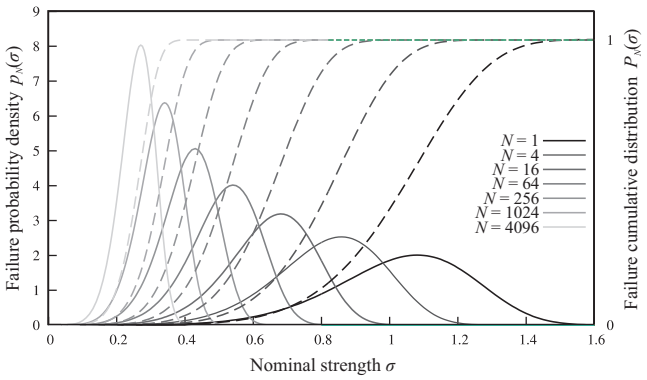


Fig. 1. Weibull strength distribution PDF (full line) and CDF (dashed line).

This reduction of strength can be even more clearly shown in the double logarithmic plot of strength as a function of number of segments. For chosen level of failure probability $P_f = 0.5$ (median strength) the size effect curve is presented in Fig. 2. In logarithmic coordinates, the curve appears as a straight line with a slope given by the shape parameter $(-1/m)$.

The above described derivation for a chain strength can be generalized for continuous bodies. Consider a body (structure) under uniform stress containing randomly distributed flaws, see Fig. 3a. The size of the body is characterized by its length $l$ (e.g. the length of a fiber). The structure fails once the stress at the weakest point (cross section) reaches its

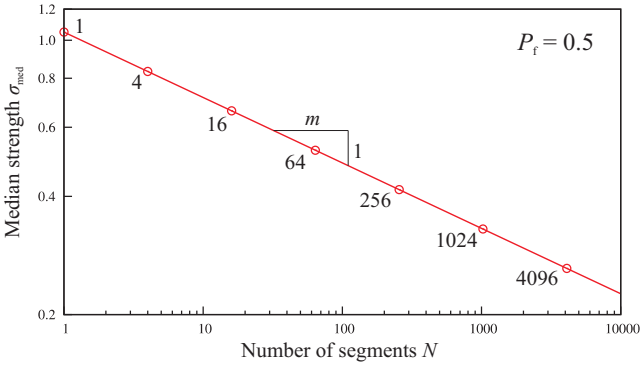


Fig. 2. Weibull median strength $\sigma_{med}$ vs. number of segments $N$ in double-logarithmic scale.

local strength. Assume that the local strength is random and characterized by the Weibull distribution (two parametric). Using the weakest-link model together with the Weibull-type function for concentration of defects, the probability of failure $P_f$ at a given level of stress $\sigma$ is expressed as the so-called Weibull integral [3]:

$$P_f(\sigma) = 1 - \exp\left[-\int_l \left\langle \frac{\sigma}{s_0} \right\rangle^m \frac{\mathrm{d}l}{l_0}\right] \tag{7}$$

where the Malacuya brackets stand for positive part $\langle \bullet \rangle = \max(\bullet, 0)$. The argument in the Malacuya brackets with its power $m$ represents a particular choice of concentration function. It represents a contribution to the failure probability of the whole structure. For a given Weibull modulus (shape parameter) $m$, we have a reference length $l_0$ with corresponding scale parameter $s_0$ of the Weibull strength. The uniform stress level is independent of the position over the length and therefore we can rewrite (7) as $-\ln(1 - P_f) = (\sigma/s_0)^m l/l_0$. Now, the stress level for a chosen probability of failure $P_f$ can be expressed as a function of the structural size (length $l$):

$$\begin{aligned} \sigma(l) &= s_0 (l_0/l)^{1/m} [-\ln(1 - P_f)]^{1/m} \\ &= s_0 f_W(l) \quad [-\ln(1 - P_f)]^{1/m} \end{aligned} \tag{8}$$

This function is a power law and therefore its graph in a double logarithmic plot for arbitrary level of probability $P_f$ (quantile) is a straight line with decreasing slope of $-1/m$. For example, the mean strength of the structure depends on its length as $\overline{\sigma}(l) = s_0 \Gamma(1 + 1/m)(l_0/l)^{1/m} = s(l)\Gamma(1 + 1/m)$, where $\Gamma$ is the Gamma function. The effect of length in this equation and also in (8) has been inserted into the scale parameter which then reads

$$s(l) = s_0 (l_0/l)^{1/m} = s_0 f_W(l) \tag{9}$$

From here on, we call $f_W(l)$ the Weibull length dependent function. The strength distribution of such a structure is Weibull for arbitrary length:

$$F(x) = 1 - \exp\left\{-\left[\frac{x}{s(l)}\right]^m\right\} \tag{10}$$

and its shape (parameter $m$) is independent of the structure size and the corresponding coefficient of variation (CoV) of fiber strength distribution is a constant depending solely on the Weibull modulus $m$:

$$\mathrm{CoV} = \sqrt{\Gamma\left(1+2/m\right)/\Gamma^2\left(1+1/m\right) - 1} \qquad (11)$$

There is a strong relation between the theory of *extreme values* and the *weakest link model*.

The important property readily seen from the above equations is that the scale parameter of the Weibull distribution can be adjusted by for any length $l_1$ to deliver the same $P_\mathrm{f}$ as for the original reference length $l_0$: $s_1/s_0 = f_W(l_1) = (l_0/l_1)^{1/m}$. This demonstrates the inherent feature of the Weibull distribution in the context of the weakest-link model already revealed in (8): it is arbitrarily scalable with respect to the reference length $l_0$; there is no length scale inside. Realizing that the reference length of one segment $l_1$ is arbitrarily scalable, we may perform this randomization with arbitrary segment length, including very small $l_1 \to 0$ with the scaling parameter $s_1 \to \infty$ and still obtain the same size effect. The extreme value theory gives us an analytical solution, which was recently proposed to simplify computations of large structures with stochastic finite element method [4], [5].

However, it has been argued [6] that the independence assumption of neighboring strengths is not correct for a real spatial distribution of strength in a material and must be abandoned *at a certain length scale*. Also, the *strength must remain bounded* for short segments. The origin of the strength bound is not discussed here, but surely, it is not possible to measure arbitrarily high strength with very short specimens. This discrepancy calls for solution.

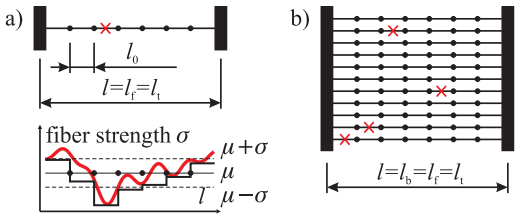


Fig. 3. Unidirectional fibrous structures with breaks at peak load: a) one fiber discretized into segments of length $l_0$ with a sketch of a strength random field and its minima; b) Daniels's bundle of (discretized) fibers.

### B. Modified Theory with Length Parameter

In order to impose an upper bound on the strength of small structures in the Weibull theory, the independence assumption of any pair of local substructures must be abandoned [7]. A plausible and physically acceptable assumption is that neighboring segments of a structure are statistically dependent, while two remote segments are independent. This can be easily modeled by an autocorrelated random field. In other words, it is assumed here that the local strengths are dependent via autocorrelation function. The autocorrelation can be just a function of Eucledian norm of two points,

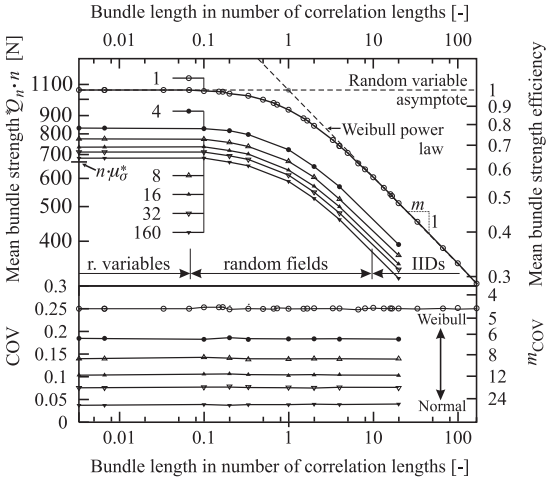


Fig. 4. Top: mean size effect curves for an increasing number of fibers $n$ in a bundle for fiber strength described by Weibull random process, $m = 4.52$. The curves nearly overlap for $l > 160$. Bottom: effective Weibull modulus $m$ (11).

moreover, it can be isotropic, i.e. the autocorrelation can be independent of direction. An example of such a function can be the squared exponential function (power $p = 2$): $R(\Delta d) = \exp\left[-\left(|\Delta d|/l_\rho\right)^p\right]$. In the model, the strength random field is homogeneous and isotropic, meaning simply that the local distribution is identical in all points of the structure. To remain consistent with the previous text, the strength is assumed to be Weibull distributed from here on. In addition, the relation between the pair of reference shape and scale parameter of the distribution and the autocorrelation length must be formulated explicitly. The reason is that the simple scaling relation $s_1/s_0 = f_W(l_1) = (l_0/l_1)^{1/m}$ does not hold anymore. Why? Because a statistical length scale in a form of the autocorrelation length $l_\rho$ have been incorporated. As a consequence, the strength dependence upon the size (length) is not a power law anymore. The autocorrelation has the effect of imposing an upper bound on strength for infinitely small (short) structures. When the structural size converges to zero, the weakest link mechanism disappears and the strength is identical to the elemental distribution (the highest attainable strength of the model at the currently modeled scale such as micro, meso, macro, etc.). By adding more material (increasing length), the weakest link effect gradually overtakes and causes the decrease of structural strength (both, the mean of strength and also its standard deviation reduces). In limit, one can show that the large size asymptotic behavior is the classical Weibull size effect. In other words, for very large structures the effect of relatively small autocorrelation length becomes insignificant and the model can again be treated as the weakest link model of independent identically distributed random strength elements. The crossover length is the autocorrelation length. To conclude, the fiber strength has the same form as in (8), but with a different length dependent function. In particular, a smooth interpolation function proposed recently in [7], [8] has correct asymptotes: the left asymptote at the small size limit is horizontal and the right asymptote is the classical

Weibull function $f_W(l)$ from (8):

$$f_V(l) = \left(\frac{l_\rho}{l + l_\rho}\right)^{1/m} \tag{12}$$

At large sizes, self-similar behavior is recovered (the double logarithmic plot is a straight decreasing line with a slope $-1/m$). At small sizes, the weakest link mechanism is suppressed by the fact that all substructures share an identical strength due to their perfect positive dependence. Note that this relation is supported by numerical simulations of extremes (minima) of random fields and is an alternative to currently available analytical results [9]. Note also that the shape ($m$ or CoV) of the distribution remains independent of size $l$ in (8). An illustration of the mean fiber size effect exploiting (12) is provided in Fig. 4 with comparison to the classical Weibull dependence.

## III. Flexural Strength of Plain Concrete

### A. Theoretical Strength

According to the theory of elasticity, the nominal strength of a beam under four point bending (4PB) is attained when in bottom fibers of the cross section within the bending span $s_\mathrm{b}$ reach tensile strength. In the bending span, the bending moment has a constant value $M = F \cdot s_\mathrm{s}$ and the beam is loaded by pure flexure as the shear force is equal to zero.

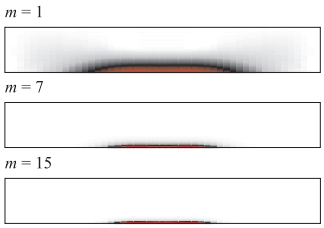


Fig. 5. Elastic stress field: reduction of the region contributing to Weibull integral (7) depending on parameter $m$.

The distribution of the longitudinal tensile stress is linear over the beam's depth $D$. The tensile stress enters the Weibull integral (7) and contributes to it. Fig. 5 shows the region contributing with its stress field into the Weibull integral. The region size reduces with the increasing shape parameter $m$. Nevertheless, it can be observed, that only a very narrow area with a tensile stress is relevant for the calculation of the Weibull integral. This area can be then easily modeled as a chain, where the local strength is represented by elements with identically distributed random strengths sharing the same load, Fig. 6 right. Failure due to the crack initiation from a smooth surface can appear anywhere inside the bending span in a place with the weakest local strength (weakest link of the chain).

As discussed in Sec. II-B, the size effect law plotted in a double-logarithmic scale as a straight line was modified by introducing a new left asymptote expressing the strength of a single chain element, which is never infinitely high, and a parameter $l_\rho$ describing the correlation length of a random field of a material strength.

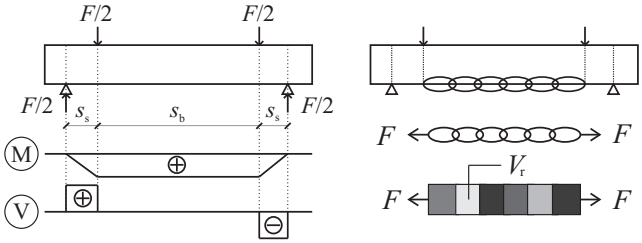


Fig. 6. Left: Geometrical scheme of bent beam. Right: Random strength of chain segments.

### B. Experimental Testing

The experiment was focused on the observation of effect of size (resp. length $s_\mathrm{b}$) on the specimens' strength. Consequently, a wide range of specimen lengths was desired with emphasis on production of the longest possible bending span, so that the behavior in this region could be mapped. The length groups were suggested with equal distribution of their logarithms, as the size effect curve is usually visualized in the double-logarithmic scale.

The bending experiment was inspired by results mentioned in [10]. The material of plain concrete was designed with respect to the loading capacity of the experimental equipment, the size of concrete specimens was designed according to Table I keeping the cross–section and the length of edges $o$ and shear span $s_\mathrm{s}$ constant, Fig. 6 left. The bending span $s_\mathrm{b}$ varied in range from 0 (for 3PB) to 300 mm; five different length groups of concrete beams were produced, Fig. 7.

TABLE I
Size of Concrete Specimens

| Group nr. | Shear span $s_\mathrm{s}$ [mm] | Bending span $s_\mathrm{b}$ [mm] | Edge $o$ [mm] | Total length $L$ [mm] | Number of specimens [-] |
|---|---|---|---|---|---|
| 1 | 100 | 0 | 30 | 260 | 26 |
| 2 | 100 | 50 | 30 | 310 | 29 |
| 3 | 100 | 100 | 30 | 360 | 29 |
| 4 | 100 | 200 | 30 | 460 | 27 |
| 5 | 100 | 300 | 30 | 560 | 28 |
| Cross–section: $60 \times 60$ [mm$^2$] | | | | | $\sum$ 139 |

Concrete beams were produced in 4 days, each day a new batch of fresh concrete was mixed. Despite the effort to follow the given recipe, the series from different batches embodied different strength characteristics. The total number of produced specimens was 139. The testing was performed always 35 days after casting. The testing schedule was created to respect some elementary principles of laboratory testing and design of experiments. Samples of different size were present in each of the testing series. The loading rate was chosen to ensure the static response of the samples: concrete samples were loaded by rate 5 mm/min.

The maximal measured load (force) was in case of bending test transformed to the value of nominal strength $\sigma_\mathrm{N}$ according to (13), where the moment contribution due to self–weight

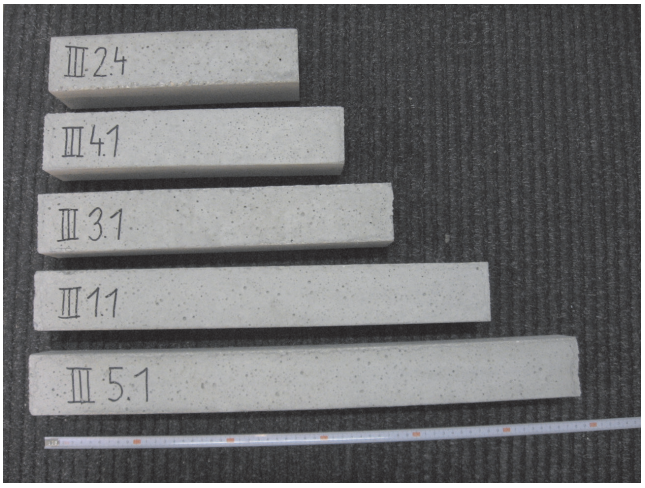


Fig. 7. Five different lengths of concrete samples.

$M_{\mathrm{sw}}$ in the spot of beam failure was added.

$$\sigma_{\mathrm{N}} = \frac{M_{\max}}{W} = \frac{M_{\mathrm{sw}}(x) + M_{\mathrm{F}}}{W} = \frac{M_{\mathrm{sw}}(x) + F/2 \cdot s_{\mathrm{s}}}{W} \tag{13}$$

TABLE II
RESULTS OF BENDING TESTS: AVERAGE, STD AND COV OF YARN STRENGTH.

| *Length group nr.* | $s_{\mathrm{b}}$ [cm] | $\sigma_{\max}$ avr [MPa] | $\sigma_{\max}$ std [MPa] | $\sigma_{\max}$ CoV [%] | $n_{\mathrm{sam}}$ [-] |
|---|---|---|---|---|---|
| 1 | 0 | **6.367** | 0.525 | 8.24 | 26 |
| 2 | 5 | **5.771** | 0.640 | 11.10 | 29 |
| 3 | 10 | **5.848** | 0.484 | 8.28 | 29 |
| 4 | 20 | **5.683** | 0.592 | 10.24 | 27 |
| 5 | 30 | **5.508** | 0.586 | 10.64 | 28 |

The nominal strength was plotted in a double–logarithmic scale as a function of the bending span $s_{\mathrm{b}}$. The value of bending span for the 3PB samples was set 0.1 cm (instead of 0), so that the strength values are visible in a double–logarithmic plot. The individual test results are shown in plots in Fig. 8 as circles. Each plot represents one production series, full line connects the average values.

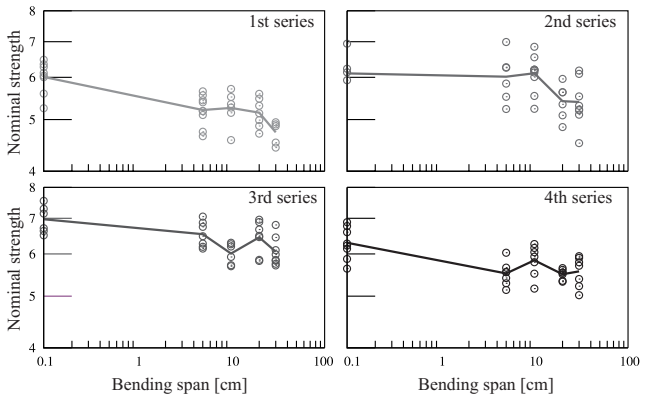


Fig. 8. Bending test: realizations of individual nominal strengths

As can be observed from measured data, some strength reduction with increasing bending span is visible. Nevertheless, production series embody high strength variance among them. This fact was caused by the heterogeneity of concrete batches. Due to this fact, it was not correct to consider the material of the whole data set (137 specimens) as homogeneous. Although, the statistical evaluation of the non-homogeneous data was done and results are presented in Tab. II. The graphical representation of results of the whole data set in form of double–logarithmic plot is in Fig. 9.

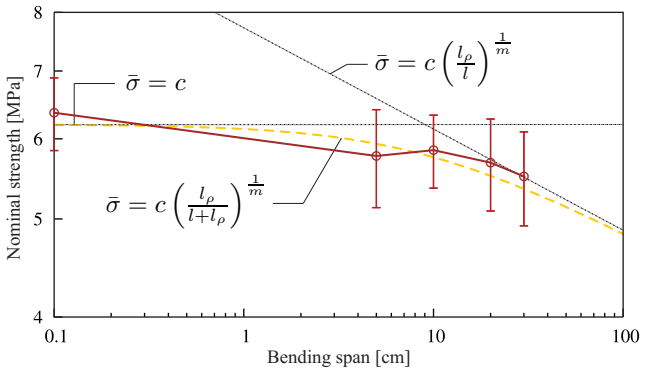


Fig. 9. Bending test: Mean values and std of nominal strength vs. bending span

Due to technological limitations, it was not possible to produce specimens with longer bending span, that would show the further development of the size effect curve. Nevertheless, it can be expected, that the sample strength would follow the decreasing trend according to the classical Weibull theory.

## IV. TENSILE STRENGTH OF MULTI-FILAMENT YARNS

### A. Strength of Fiber Bundles

The above described extension of Weibull theory can be readily incorporated into theory of strength of bundles with elastic-brittle fibers and with global load sharing [7]. The classical model formulated by Daniels [11] describes a situation of $n$ parallel fibers (or microbonds) with IID random strengths, equal lengths and elastic moduli, stretched between two clamps under global load sharing. The maximum tensile force of the bundle $Q_n^*$ is measured in terms of load per fiber. Daniels [11] derived a recursive formula for computing the cumulative density function (CDF) $G_n(x)$ of $Q_n^*$ depending on the fiber CDF $F(x)$ and number of fibers $n$:

$$\begin{aligned} G_n(x) &= \mathrm{P}(Q_n^* \le x) = \\ &= \sum_{k=1}^{n} (-1)^{k+1} \binom{n}{k} [F(x)]^k \, G_{n-k}\left(\tfrac{n\,x}{n-k}\right), \\ \text{where} \quad & G_1(x) \equiv F(x), \quad G_0(x) \equiv 1 \quad \text{and} \\ & \binom{n}{k} = \tfrac{n!}{(n-k)!\,k!} \end{aligned} \tag{14}$$

This formula is usable only for small number of parallel fibers (a few tens) as the computational demands and also the round-off errors explode with increasing number of fibers.

Moreover, Daniels proved that, under broad assumptions on $F(x)$, the asymptotic distribution of the maximum bundle load $Q_n^*$ is Gaussian, i.e. with $n \to \infty$, it tends to

$$\lim_{n\to\infty} G_n(x) = \Phi\left(\frac{x-\mu^*}{\gamma^*}\sqrt{n}\right) = \Phi\left(\frac{x-\mu^*}{\sigma^*}\right) \qquad (15)$$

where $\Phi(\bullet)$ is the standard Gaussian cumulative density function. The mean value $\mu^*$ depends on fiber $F(x)$ and not on the number of fibers $n$. The standard deviation $\sigma^*$ of bundle strength is proportional to the inverse of the square root of $n$, see e.g. [7] for details. Formulas for the mean value $\mu^*$ and standard deviation $\sigma^*$ in the case of Weibull strength distribution $F(x)$ of fibers will be given in (16).

The effect of parallel coupling seems to be captured well. The question remains what the *effect of length* of such a bundle on its strength is. It has been shown in [7] that, for the situation studied, the effect of length and parallel coupling can be treated separately and they are independent and do not interact. Simply, a change in the length of the fibers in the bundle results only in the change of the scaling parameter $s$ of the distribution $F(x)$ of fiber strength. This distribution then enters formulas for the bundle strength distribution. If, for example, we consider Weibull fibers, and, in analogy with (8), use the association of the length dependence with the scale parameter $s(l) = s_0 f(l)$, the resulting Weibull strength distribution can be plugged in Daniels's formulas for bundle strength. After a few simple manipulations [7] the resulting mean bundle strength reads $\mu^*(n,l) = \mu^*(n) f(l)$ thus manifesting the decomposed effects of length and parallel coupling. The bundle strength being a function of the amount of material (fiber length and number of parallel fibers) is plotted in Fig. 4. The figure compares the proposed incorporation of the statistical length scale $l_\rho$ using $f_V(l)$ with the classical Weibull theory that uses $f_W(l)$. It is shown that with increasing number of fibers (or microbonds) the crossover length $l_\rho$ propagates in the size effect plots unchanged.

The Gaussian variables have the mean value $\mu^*$ and standard deviation $\sigma^*$ given in [11] and refined in [12], see [7]. The formulas are rearranged here to be explicitly dependent on the length function $f(l_\mathrm{b})$ (see Fig. 3) for which there are two alternatives (the Weibull form $f_W$ and the proposed formula $f_V$ based on extremes of random processes):

$$\begin{aligned}
&\mu* = f(l_\mathrm{b})\ \mu_\rho \quad \text{where}\\
&\mu_\rho = \underbrace{s_\rho\left[m^{\frac{-1}{m}} c_m + 0.996 n^{\frac{-2}{3}} m^{\left(\frac{-1}{m}-\frac{1}{3}\right)} \exp\left(-\frac{1}{3m}\right)\right]}_{\mu_\rho(m,n,s)\ldots\text{for length}\,l_\rho},\\
&\sigma* = f(l_\mathrm{b})\ \sigma_\rho \quad \text{where}\\
&\sigma_\rho = \underbrace{s_\rho\left[m^{-1/m}\sqrt{c_m(1-c_m)}/\sqrt{n}\right]}_{\sigma_\rho(m,n,s)\ldots\text{for length}\,l_\rho}, \quad c_m = \exp\left(\tfrac{-1}{m}\right)
\end{aligned} \qquad (16)$$

### B. Experimental Testing

One yarn composes of several hundreds up to thousands of single fibers with diameter measured in tens of micrometers. The fineness of the yarn is defined by the "tex" unit (gram per 1000 meters) and depends on the average fiber diameter, the fibre material density and the number of fibers.

The shape of samples and the production technology was inspired by the experiments run previously at RWTH Aachen University [13] and other. The material selected for the tensile tests was the AR-glass yarn produced by Saint Gobain Vetrotex with brand name Cem-FIL Direktroving LTR 5325, 2400 tex. Six length groups were suggested with equal distribution of their logarithms – Table III, Fig. 10 (the longest possible specimen length was designed with regards to the testing equipment). The most problematic part of tensile testing was to deal with the anchoring of glass yarns into the machine. Basically, there are two possible ways how to create bundle supports: endings can be either directly coiled up on a cylindrical member or poured into anchoring blocks and then clamped. (Direct clamping of yarns is not possible, as the yarn is made of brittle material that would crush at the support point due to lateral compression in clamps.) Nevertheless, both techniques shows certain deficiencies. As the used testing machine was equipped with self–locking holders, yarn endings were poured into 75 mm long anchoring blocks made of epoxy resin. The total number of tested samples was 317 pieces.

TABLE III
LENGTH OF GLASS YARNS SPECIMENS

| *Length group nr.* | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Gauge length $L$ [mm] | 10 | 25 | 60 | 130 | 310 | 740 |
| Number of specimens $n_\mathrm{sam}$ | 53 | 48 | 48 | 55 | 53 | 60 |
| | | | | | | $\sum 317$ |

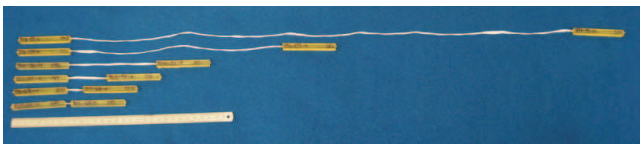

Fig. 10. Six different lengths of yarns.

Samples marked as outliers (with the relative error of maximal force with respect to the corresponding length group's average value greater then 0.35) were eliminated from the data set. These outliers (38 samples) belonged mostly to the first two production series, that were influenced by the still unsettled production procedure. The number of yarn samples used for the statistical evaluation was 279 (in each length group was 42–48 samples).

The obtained values of strength together with the average sample's free length $L$ and the number of samples used for the statistics $n_\mathrm{sam}$ are overviewed in Tab. IV. The effect of decreasing average and std of the strength with the increasing sample length can be observed. The value of CoV can be considered as stagnating in the range close to 15 %. The number of samples after the elimination of outliers exceeds required 30 pieces in each length group so that the obtained data set can be considered as a statistically representative with a high significance.

The graph with the samples' peak loads in a double-logarithmic scale is in Fig. 11. The plotted points represent

TABLE IV
RESULTS OF TENSILE TESTS: AVERAGE, STD AND COV OF YARN STRENGTH.

| *Length group nr.* | $L$ avr [mm] | $F_{\max}$ avr [N] | $F_{\max}$ std [N] | $F_{\max}$ CoV [%] | $n_{\text{sam}}$ [-] |
|---|---|---|---|---|---|
| 1 | 9.2 | **824.8** | 126.3 | 15.32 | 42 |
| 2 | 23.9 | **795.7** | 121.3 | 15.24 | 45 |
| 3 | 58.8 | **737.9** | 122.9 | 16.66 | 48 |
| 4 | 128.5 | **693.2** | 101.2 | 14.60 | 48 |
| 5 | 308.4 | **625.4** | 81.0 | 12.94 | 48 |
| 6 | 738.5 | **498.6** | 78.6 | 15.77 | 48 |

individual experiments, their color is assigned to the production series. Samples with relative error of strength exceeding $\pm 0.35$ as well as the whole series P01 and P02 are marked with a cross (outliers), the border lines separating the outliers from the accepted values are marked with dash line. The average of each length group strength (marked with a circle $\pm$ std) defines the size effect curve. The red color represents the modified (reduced) data set while the light grey shows the trend of the original complete set of samples (containing outliers). The fact that these two curves do not notably differ from each other confirms the claim of statistically sufficient number of samples.

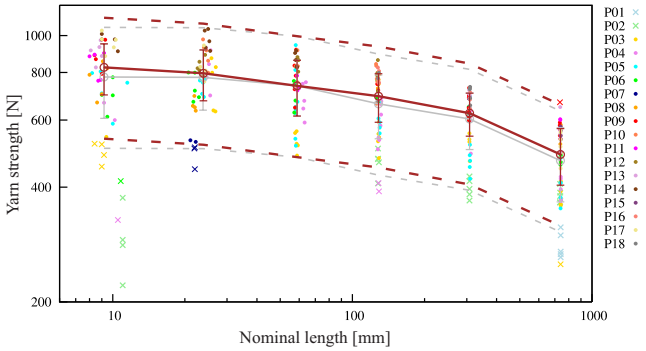


Fig. 11. Tension test: Yarn strengths vs. yarn lengths of tested sample groups and the size-effect curve as an average$\pm$std of modified (red) and original (grey) data set. Dash line separates the outliers.

Now, the curve can be fitted with the modified Weibull size-effect function with the included autocorrelation length (12). The curve–fit can be seen in Fig. 12. The parameters $l_\rho$ (autocorrelation length) defining the point of asymptotes' intersection, the strength value $c$ of the left asymptote and $m$ (the shape parameter of Weibullian distribution) governing the slope of the right asymptote in a double-logarithmic scale were fitted to the measured data.

## V. CONCLUSION

Both of the two types of introduced experiments (bending tests on concrete beams and tensile tests on yarns) were focused on the observation of effect of size (resp. length) on the specimens' strength. Consequently, a wide range of specimen lengths was desired with emphasis on production of the longest possible bending span, resp. gauge length, so that the behavior in this region could be mapped. The length groups were suggested with equal distribution of their logarithms,

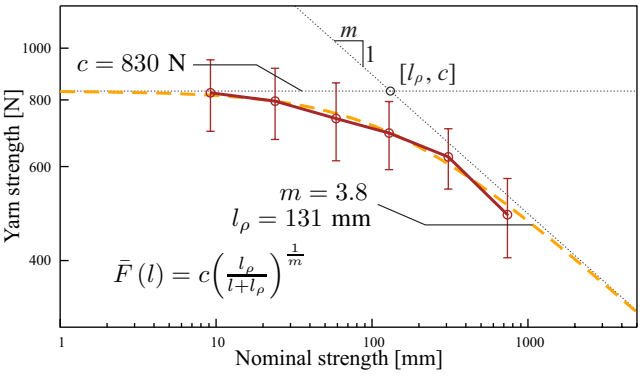


Fig. 12. Tension test: Estimation of size effect curve parameters.

as the size effect curve is usually visualized in the double-logarithmic scale.

A high number of samples were tested so that the obtained results are statistically significant. The most significant effect of the obtained data set was the strength reduction with the length extension. For each of the length groups, an average value of strength $F_{\max}$, its standard deviation and a coefficient of variation were calculated.

The experimentally obtained/measured dependencies of strength on size support the assumption of existence of correlation length of a strength random field.

## ACKNOWLEDGMENT

This outcome has been achieved with the financial support of the Czech Science Foundation under project No. 13-19416J and a Specific University Research project MŠMT No. FAST-J-12-21. The support is gratefully acknowledged.

## REFERENCES

[1] J. Kaděrová, "Testování statistického vlivu velikosti pomocí čtyřbodového ohybu (Experiments on statistical size effect in four point bent specimens)," Bachelor's thesis, Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Brno, Czech Republic, 2009.

[2] ——, "Multi-filament yarns testing for textile-reinforced concrete," Master's thesis, Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Brno, Czech Republic, 2012.

[3] W. Weibull, "The phenomenon of rupture in solids," *Royal Swedish Institute of Engineering Research (Ingenioersvetenskaps Akad. Handl.), Stockholm*, vol. 153, pp. 1–55, 1939.

[4] Z. P. Bažant, M. Vořechovský, and D. Novák, "Asymptotic prediction of energetic-statistical size effect from deterministic finite element solutions," *Journal of Engineering Mechanics (ASCE)*, vol. 133, no. 2, pp. 153–162, 2007.

[5] Z. P. Bažant, S. D. Pang, M. Vořechovský, and D. Novák, "Energetic-statistical size effect simulated by SFEM with stratified sampling and crack band model," *International Journal for Numerical Methods in Engineering (Wiley)*, vol. 71, no. 11, pp. 1297–1320, 2007.

[6] M. Vořechovský, "Incorporation of statistical length scale into Weibull strength theory for composites," *Composite Structures*, vol. 92, no. 9, pp. 2027–2034, 2010.

[7] M. Vořechovský and R. Chudoba, "Stochastic modeling of multi-filament yarns: II. Random properties over the length and size effect," *International Journal of Solids and Structures (Elsevier)*, vol. 43, no. 3-4, pp. 435–458, 2006.

[8] M. Vořechovský, "Statistical alternatives of combined size effect on nominal strength for structures failing at crack initiation," in *Problémy lomové mechaniky IV (Problems of Fracture Mechanics IV)*, M. Stibor, Ed. Academy of Sciences - Institute of physics of materials of the ASCR: Brno University of Technology, 2004, pp. 99–106, invited lecture.
[9] R. Leadbetter, G. Lindgren, and H. Rootzen, *Extremes and Related Properties of Random Sequences and Processes*, ser. Springer Series in Statistics. Springer London, Limited, 2011. [Online]. Available: http://books.google.cz/books?id=AGB9MQEACAAJ
[10] H. Koide, H. Akita, and M. Tomon, "Probability model of flexural resistance on different lengths of concrete beams," *Applications of Statistics and Probability, Melchers & Stewards (eds)*, pp. 1053–1057, 2000.
[11] H. Daniels, "The maximum of gaussian process whose mean path has a maximum, with an application to the strength of bundles of fibres," *Advances in Applied Probability*, vol. 21, pp. 315–333, 1945.
[12] R. Smith, "The asymptotic distribution of the strength of the series-parallel system with equal load-sharing," *Annals of Probability*, vol. 10, no. 1, pp. 137–171, 1982.
[13] R. Chudoba, M. Vořechovský, V. Eckers, and T. Gries, "Effect of twist, fineness, loading rate and length on tensile behavior of multifilament yarns (a multivariate study)," *Textile Research Journal (Sage)*, vol. 77, no. 11, pp. 880–891, 2007.

**Jana Kaděrová** is a Ph.D. student at Brno University of Technology, Institute of Structural Mechanics.

**Miroslav Vořechovský** is an associate professor at Brno University of Technology, Institute of Structural Mechanics.