

# Experimental Investigation of Heat Transfer and Flow of Nano Fluids in Horizontal Circular Tube

Abdulhassan Abd. K, Sattar Al-Jabair, Khalid Sultan

**Abstract**—We have measured the pressure drop and convective heat transfer coefficient of water – based  $Al_2O_3(25nm)$ ,  $Al_2O_3(30nm)$  and  $CuO(50nm)$  Nanofluids flowing through a uniform heated circular tube in the fully developed laminar flow regime. The experimental results show that the data for Nanofluids friction factor show a good agreement with analytical prediction from the Darcy's equation for single-phase flow. After reducing the experimental results to the form of Reynolds, Rayleigh and Nusselt numbers. The results show the local Nusselt number and temperature have distribution with the non-dimensional axial distance from the tube entry. Study decided that then Nanofluid as Newtonian fluids through the design of the linear relationship between shear stress and the rate of stress has been the study of three chains of the Nanofluid with different concentrations and where the  $Al$ ,  $Al_2O_3$  and  $CuO$  – water ranging from (0.25 - 2.5 vol %). In addition to measuring the four properties of the Nanofluid in practice so as to ensure the validity of equations of properties developed by the researchers in this area and these properties is viscosity, specific heat, and density and found that the difference does not exceed 3.5% for the experimental equations between them and the practical. The study also demonstrated that the amount of the increase in heat transfer coefficient for three types of Nano fluid is  $Al$ ,  $Al_2O_3$ , and  $CuO$  – Water and these ratios are respectively (45%, 32%, 25%) with insulation and without insulation (36%, 23%, 19%), and the statement of any of the cases the best increase in heat transfer has been proven that using insulation is better than not using it. I have been using three types of Nano particles and one metallic Nanoparticle and two oxide Nanoparticle and a statement, whichever gives the best increase in heat transfer.

**Keywords**—Newtonian, NUR factor, Brownian motion

## I. INTRODUCTION

**N**ANOFUIDS fluids are engineering colloids made of a base fluid and nanoparticles. Nanoparticles are particles that are between 1 and 100 nm in diameter.

Nano fluids typically employ metal or metal oxide Nanoparticles, such as copper and alumina, and the base fluid is usually a conductive fluid such as water or ethylene glycol. Nano fluids are studied because of their heat transfer properties: they enhance the thermal conductivity and convective properties over the properties of the base fluid.

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Typical thermal conductivity enhancement is in the range of 15 – 40 % over the base fluid and heat transfer coefficient enhancement have been found up to 40% [1]. Increases in thermal conductivity of the added Nanoparticles, and there must be other mechanisms attributed to the increase in performance. Conventional fluids, such as water, engine oil and ethylene glycol are normally used as heat transfer fluids. Although various techniques are applied to enhance the heat transfer, the low heat transfer performance of these conventional fluids obstructs the performance enhancement and compactness of heat exchangers. The past decade has seen many research activities in the experimental heat transfer characteristics of various Nano fluids. For forced convective heat transfer [2]. Studied the heat transfer behavior in parallel channels using an unspecified Nano fluid and observed reduction in thermal resistance by a factor of 2. [3] Performed experiments on convective heat transfer of two types of Nano fluids i.e.  $\gamma Al_2O_3$  and  $TiO_2$  dispersed in water, under turbulent flow conditions. Even though the Nusselt number (Nu) was found to increase with the particle volume fraction and the Reynolds number, the heat transfer coefficient (h) actually decreased by 3 – 12 %. The experimental results of [4] illustrated that the convective heat transfer coefficient of Cu/water based Nano fluids varied significantly with the flow velocity and the volumetric loading of particles. For example, for 2 vol% of Cu Nanoparticles in water, the Nusselt number increased by about 60%. [5] Investigation convective heat transfer of  $CuO$  and  $Al_2O_3$ / water – based Nano fluids under laminar flow condition through an annular copper tube. In their study, the heat transfer coefficient was found to increase with increasing particle volume fraction as well as Peclet number.  $Al_2O_3$ / water – based Nano fluids showed higher enhancement of heat transfer coefficient compared to  $CuO$  / water – based Nano fluids. [6] Studied  $Al_2O_3(20nm)$ / DIW – based Nano fluids flowing through 1mm sized stainless steel test tube and subject to constant wall heat flux and low Reynolds number ( $Re < 270$ ). Compared to the base fluid, the Nusselt number of this Nano fluid had a maximum enhancement of 8 % for 1 vol% Nanoparticle at  $Re=270$ . [7] Conducted convective heat transfer experiments for  $Al_2O_3$ / water – based Nano fluid in a rectangular micro channel ( $50 \times 50 \text{ micro}^2$ ) under laminar flow conditions. The convective heat transfer coefficient increased by more than 32 % for 1.8 vol % of Nanoparticles in base fluids. The Nusselt number increases with increasing Reynolds number in the laminar flow regime ( $5 < Re < 300$ ). Based on the results, they

proposed a new convective heat transfer correlation for Nano fluids in micro channels.

## II. EXPERIMENTAL SYSTEM

The experimental loop was designed for convective heat transfer in laminar flow domain. The horizontal test section was Pyrex tube with an inner diameter (ID) of 4mm and length of 2.5m. The tube surface is electrically heated by coil made from tungsten matter connecting to an AC power supply to generate heat flux. To measure the wall temperature of the Pyrex tube and the bulk mean temperature of the fluids at the inlet and out let of the tube ten thermocouples (T – types) are soldered at place along the test section and two thermocouple (T – types) are inserted at the inlet and out let of the test section. The pressure drop is measured by two gauge pressure. The range pressure operates over range of [0 – 400 mbar] at beginning of the test section and other end the range pressure operates over range [0 – 150 mbar]. To preserve a constant temperature at the inlet of the test section the heated fluid returns to reservoir tank passing spiral Pyrex heat exchanger to a cooler fluid. The flow meter was positioned just after the pump discharge. The Fig. 1,,2 represent picture of the test section for the two cases with and without insulation.

There are two cases will be taken in to experimental as follows:

- The first test section without insulation
- The second test section with insulation

The second test section was thermally isolated with fiber glass to minimize the heat loss from the tube to the ambient. The Fig.3 show experimental apparatus for investigation of heat transfer and flow of Nano fluids in horizontal circular tube in fully developed laminar flow regime with uniform heat flux.

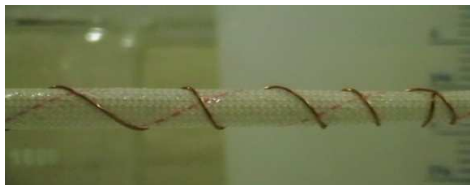


Fig. 1 Test section with insulation

TABLE I  
UNITS FOR NANOFUIDS PROPERTIES

Symbol	Quantity	units
$\bar{q}$	Heat flux	W/m <sup>2</sup>
$\Delta P$	Pressure drop	pa
f	Friction factor	
$d_p$	Particle diameter	nm
$v_{eT}$	Thermophoretic velocity	m <sup>2</sup> /s
$D_B$	Brownian diffusion coefficient	m <sup>2</sup> /s
$k_B$	Boltzman's constant	J/k
$v_{eg}$	Nanoparticle settling velocity	m/s
$k_n$	Thermal conductivity of Nano fluid	w/m <sup>2</sup> .k
PHi	Nanoparticle volume fraction	-
$\mu_n$	Dynamic viscosity Nano fluid	N.s/m <sup>2</sup>



Fig. 2 Test section without insulation



Fig. 3 Experimental apparatus

## III. PROPERTIES OF NANO FLUID

All physical properties of the water – based AL, AL<sub>2</sub>O<sub>3</sub>, CuO Nano fluids needed to calculate the pressure drop and the convective heat transfer are measured .the dynamic viscosity ( $\mu$ ) is measured using Brook Field Digital Viscometer Model Dv–E. In this study, measurement of viscosity in practice where he found that there is a difference very little between the practical and empirical relations used by researchers such as [8], [9], [10], [11]. [10] is the closest to practical difference does not exceed 1.72%. Figs.4 – 6 shows the dynamic viscosity of water – based AL, AL<sub>2</sub>O<sub>3</sub>, CuO are increased about 2.1% for 0.25vol% at 25C<sup>0</sup> compared with that of pure water. The thermal conductivity, specific heat and density are measured. In addition, measured values of the density agrees well with the values calculated From mixing theory [12] as shown in Fig.7 and Fig.8 shows the density of water – based AL, AL<sub>2</sub>O<sub>3</sub>, CuO are increased about 3.2% compared with that of pure water.

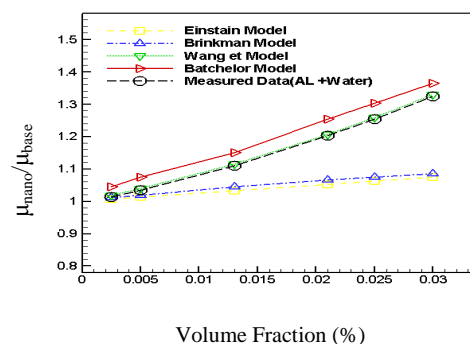


Fig. 4 Viscosity ratio water based - AL

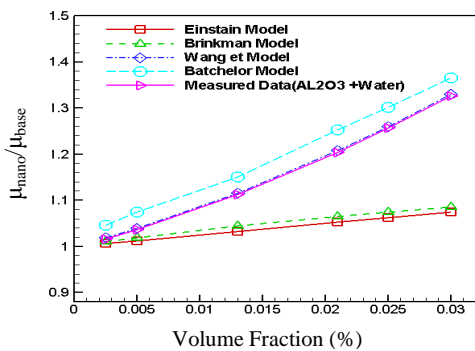


Fig. 5 Viscosity ratio water based – AL<sub>2</sub>O<sub>3</sub>

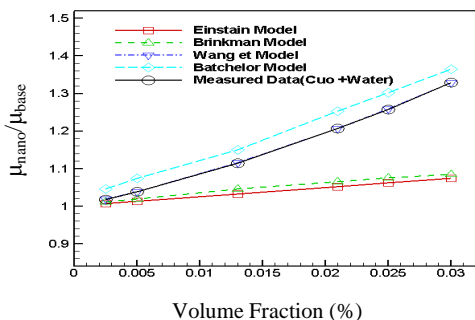


Fig. 6 Viscosity ratio water based– CuO

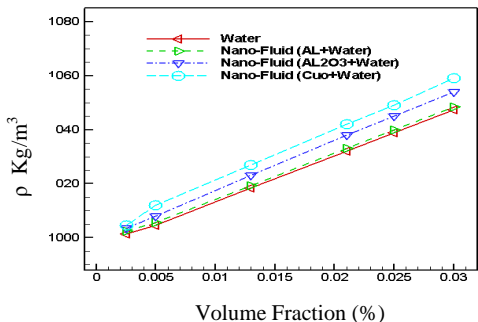


Fig. 7 Density of water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO

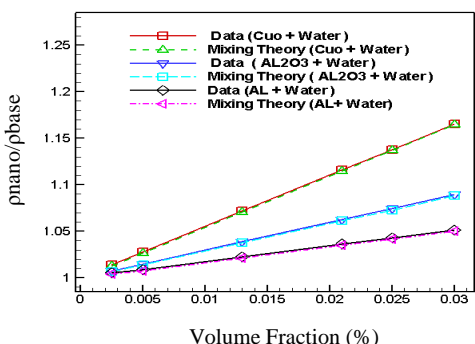


Fig. 8 Density ratio of water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO

The experimental result for thermal conductivity indicated at comparison with the equations or models of thermal conductivity developed by researchers such as [13], [14], [15], [16].

The Hamilton and Crosser is the closest to practical difference does not exceed 3.27%. Figs. 9, 10, 11 shows the thermal conductivity of water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO are enhanced about 1.03% for 0.25vol% at 25C<sup>0</sup> compared with that of pure water and Fig. 12 shown thermal conductivity ratio for three types of the Nano fluids AL, AL<sub>2</sub>O<sub>3</sub> and CuO + water. The experimental result for specific heat was compared with two models of specific heat are user in all research in this area where he was the second model is the closest to a practical and a difference of 2.25% and Fig. 13 shown comparison with two models [17,18].

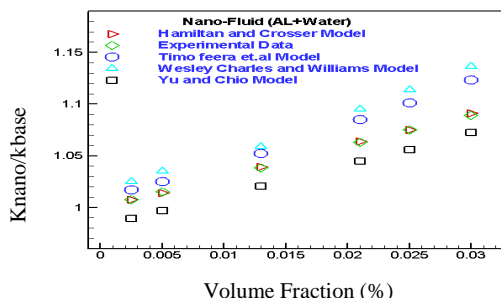


Fig. 9 Thermal conductivity ratio of water based– AL

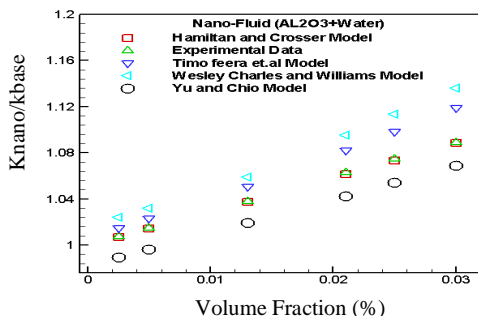


Fig. 10 Thermal conductivity ratio of water based– AL<sub>2</sub>O<sub>3</sub>

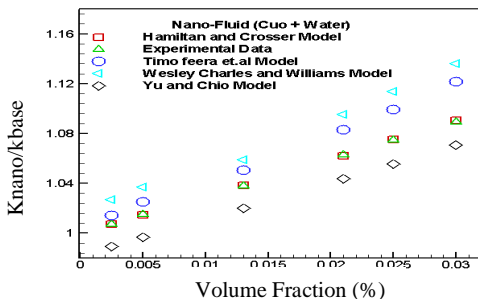


Fig. 11 Thermal conductivity of ratio of water based– CuO

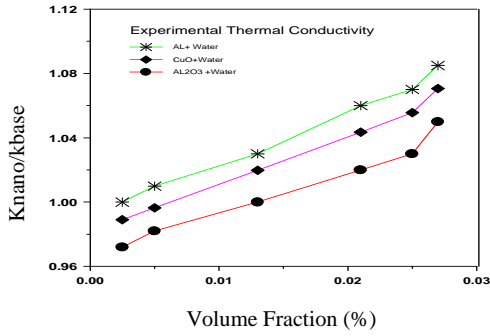


Fig. 12 Thermal conductivity ratio of water based – AL, AL<sub>2</sub>O<sub>3</sub> and CuO

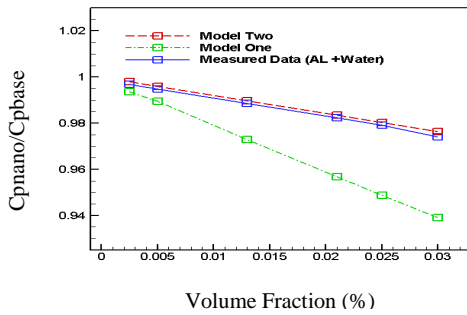


Fig. 13 Comparison of specific heat correlation With two models

#### IV. DATA ANALYSIS AND VALIDATION

It is necessary to know the heat transfer coefficient used in this study clearly before making any calculations. The local heat transfer coefficient of the equation as follows.

$$h_z = \frac{\bar{q}}{(\Delta T)_z} \quad (1)$$

Where  $(\Delta T)_z$  have been taken as the difference between the temperature of the surface localized  $(T_s)_z$  and the temperature of the fluid  $(T_b)_z$  at length  $Z$  from the entrance of the tube. Also, with the energy balance in a tube the mean temperature of fluid can be given by:

$$(T_b)_z = T_i + \frac{\bar{q} P}{m C_p} z \quad (2)$$

Where:  $T_i$ ,  $p$ ,  $\dot{m}$  and  $C_p$  are the fluid temperature at the inlet of test section, the surface perimeter, the mass flow rate and the heat capacity, respectively.

Thus the local heat transfer coefficient becomes

$$h_z = \frac{\bar{q}_w}{(T_s)_z - (T_b)_z} \quad (3)$$

Therefore, the local heat transfer coefficient can be obtained with Eqs. (3) and (2). In order to verify the accuracy and the reliability of this experimental system, the pressure drop and the local heat transfer coefficient are experimentally measured using water before obtaining those of water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluids. The experiments on the pressure drop are conducted within the Reynolds number of 600, and entry length is within 2% of total tube length. Therefore, we

can assume the fully developed laminar flow regime. Fig.14 shows that pressure drop data for water and analytical predictions using Eq.(4) have a good agreement with less than 1.5% error.

$$\Delta p = 32 \frac{\mu m \mu L}{d_{tube}^2} \quad (4)$$

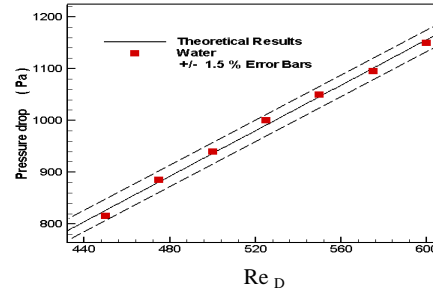


Fig. 14 Comparison between theoretical and experimental pressure drops of water

The heat transfer coefficients calculated by Eqs. (3) and (2) are compared with the following Shah equation [19,20] of the local Nusselt number in circular tube for a constant heat flux condition according to the Reynolds number of 400 in Fig.15.

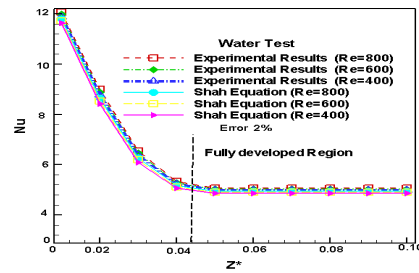


Fig. 15 Comparison between Shah Equation and experimental data

$$Nu_z = \begin{cases} 1.302z^{-\frac{1}{3}} - 1 & Z_* \leq 0.00005 \\ 1.302z^{-\frac{1}{3}} - 0.5 & 0.00005 \leq Z_* \leq 0.0015 \\ 4.364 + 8.68(10^3 Z_*)^{-0.506} \exp(-41Z_*) & Z_* \geq 0.001 \end{cases} \quad (5)$$

Where:  $Nu_z = \frac{h_z d}{k}$ ,  $Z_* = \frac{2(z/d)}{Re Pr}$

For fully;

Developed laminar flow regimes, the experimental data compared with Shah Equation [19, 20] have good agreement within 2% error and are constant. To show the effect of the Nano fluids on heat transfer Rate we introduce a variable called Nusselt number Ratio (NUR) with its definition given as:

$$NUR = \frac{Nu(\Phi)|with\ nanofluid}{Nu(\Phi = 0)|pure\ fluid}$$

If the value of NUR greater than 1 indicated that the heat transfer rate is enhanced on that fluid, whereas reduction of heat transfer is indicated when NUR is less than 1. The Nusselt number is used as an indicator of heat transfer enhancement

where an increase in Nusselt number corresponds to enhancement in heat transfer.

V. RESULT AND DISCUSSION

The pressure drop of water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluids flowing through a circular tube is experimentally measured to investigate flow characteristics of the Nano fluids. Based on the pressure drop of water-based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluids ,we can express the Darcy friction factor, which is dimensionless parameter defined as:

$$f = \frac{\Delta p}{\rho U m^2 / 2} \tag{6}$$

For fully developed laminar flow, it follows that

$$f = \frac{64}{Re_{dtube}} \tag{7}$$

Substituting the pressure drop into Eq.(6), the Darcy friction factor can be calculated . Figs 16, 17, 18 shows that the Darcy friction factor of water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluids in fully developed laminar flow regime has a good agreement with Eq.(7) with less than 2% error.

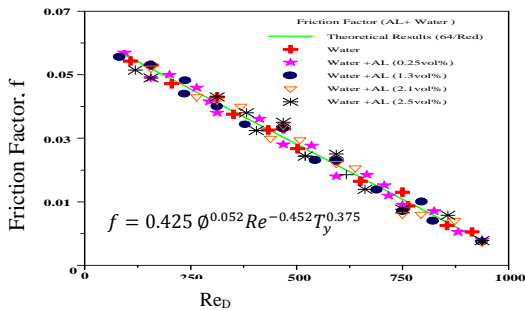


Fig. 16 The friction factor of water + based AL Nano Fluid in fully developed laminar flow

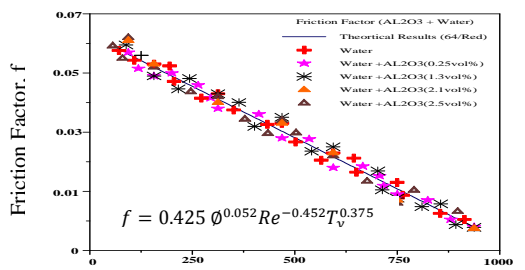


Fig. 17 The friction factor of water based + AL<sub>2</sub>O<sub>3</sub> Nano Fluid in fully developed laminar flow

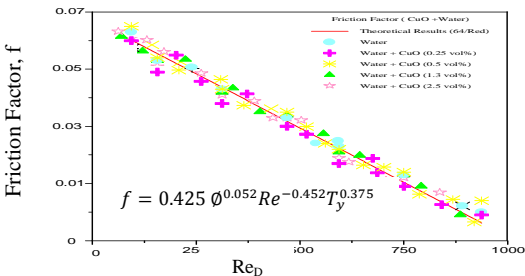


Fig. 18 The friction factor of water– based CuO

This implies that the friction factor correlation for single – phase flow can be extended to water – based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluid. The Nusselt number of water-based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluids flowing through a circular tube for a constant heat flux is experimentally measured to understand convective heat transfer characteristics of water-based AL, AL<sub>2</sub>O<sub>3</sub> and CuO Nano fluids. Figs. 19-24 with insulation and Figs. 25–30 without insulation shows the Nusselt number for three types of Nano fluids as function of non-dimensional axial distance at different Reynolds number and Rayleigh number for volume fraction 0.25vol% and 2.5vol%.

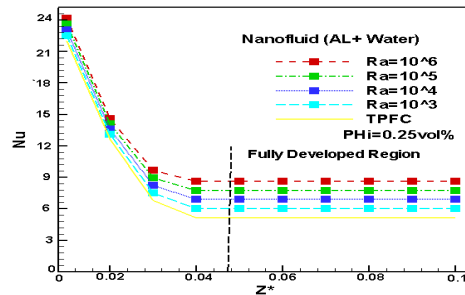


Fig. 19 The local Nusselt number of water based+ AL Nano fluid with insulation, different Rayleigh number and 0.25vol%

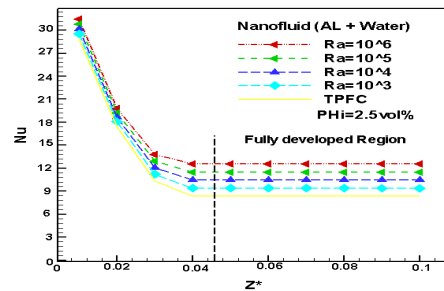


Fig. 20 The local Nusselt number of water based + AL Nano fluid with insulation, different Rayleigh number and 2.5vol%

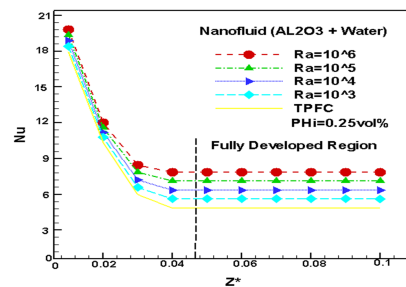


Fig. 21 The local Nusselt number of water based+ AL<sub>2</sub>O<sub>3</sub> Nano fluid with insulation, different Rayleigh Number and 0.25vol%

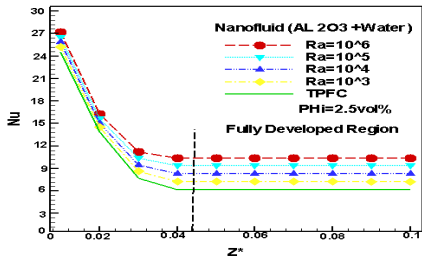


Fig. 22 The local Nusselt number of water based+  $AL_2O_3$  Nano fluid with insulation, different Rayleigh Number and 2.5 vol%

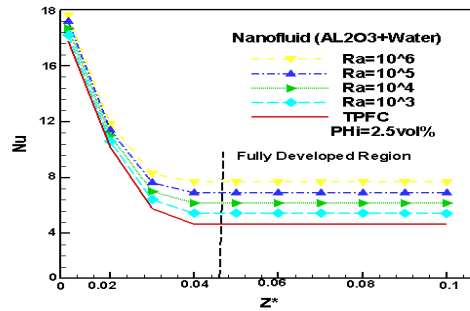


Fig. 26 The local Nusselt number of water based+  $AL_2O_3$  Nano fluid without insulation, different Rayleigh Number and 2.5vol%

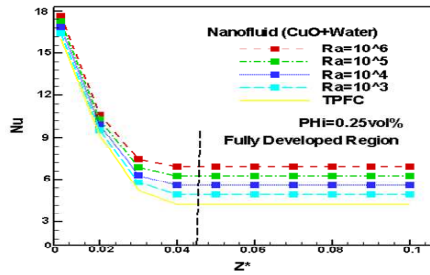


Fig. 23 The local Nusselt number of water based+CuO Nano fluid with insulation, different Rayleigh Number and 0.25 vol%

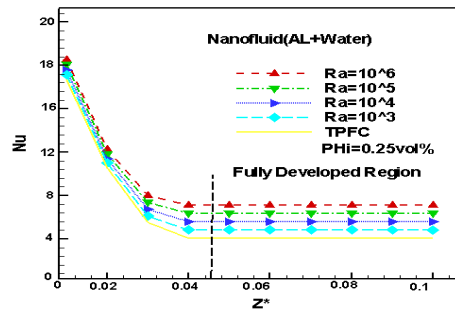


Fig. 27 The local Nusselt number of water based+ AL Nano fluid without insulation, different Rayleigh number and 0.25v%

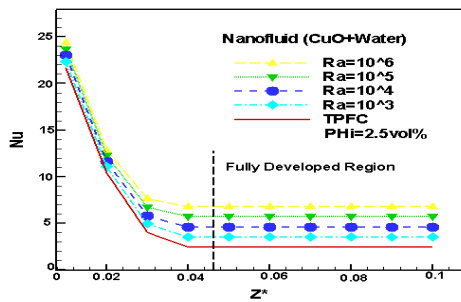


Fig. 24 The local Nusselt number of water based+CuO Nano fluid with insulation, different Rayleigh Number and 2.5vol%

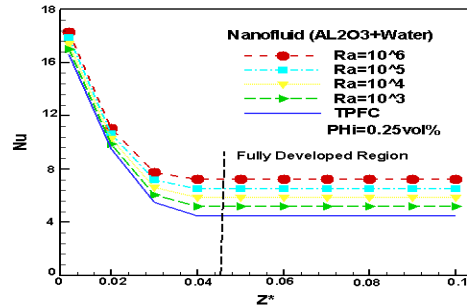


Fig. 28 The local Nusselt number of water based+  $AL_2O_3$  Nano fluid without insulation, different Rayleigh Number and 0.25vol%

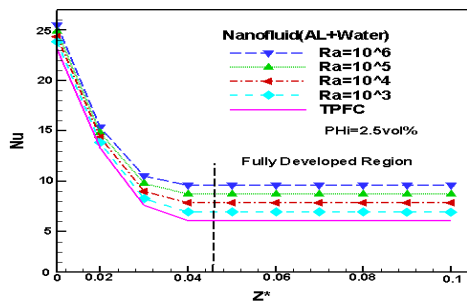


Fig. 25 The local Nusselt number of water based+ AL Nano fluid without insulation, different Rayleigh number and 2.5v%

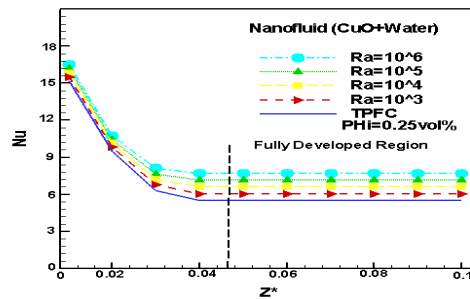


Fig. 29 The local Nusselt number of water based+CuO Nano fluid without insulation, different Rayleigh Number and 0.25vol%

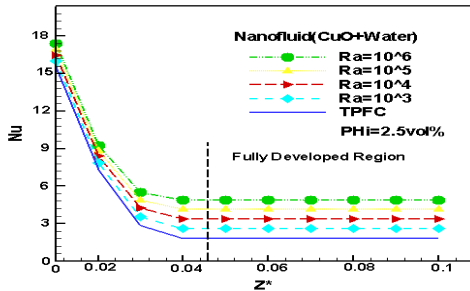


Fig. 30 The local Nusselt number of water based+CuO Nano fluid without insulation, different Rayleigh Number and 2.5vol%

The experimental results clearly show that Nanoparticles suspended in water enhance the heat transfer and the local Nusselt number of water-based AL,  $AL_2O_3$  and CuO Nano fluids increases with volume fraction of AL,  $AL_2O_3$  and CuO Nanoparticles as shown in Figs.( 19 – 24 )with insulation and Figs.(25 – 30 )without insulation. The study also demonstrated that the amount of the increase in Nusselt number for three types of Nano fluid is AL,  $AL_2O_3$  and CuO+ Water and these ratios are respectively (45%, 32%, 25%) with insulation and without insulation (36%, 23%, 19%) compared with that pure water and the statement of any of the cases the best increase in heat transfer has been proven that using insulation is better than not using it. I have been using three types of Nano particles and one metallic Nanoparticle and two oxide Nanoparticle and a statement, whichever gives the best increase in heat transfer and thermal losses in the absence of insulation does not exceed 15% has been proven that these losses increased with the increasing concentration and size of the particles. It should be noted the enhancement of heat transfer greatly depend on Nano particle type, Nano particles size, base fluid, flow regime and specially B.C. Nano fluids that contain metal Nanoparticles AL(25nm) + water show more enhancement compared to oxide Nano fluids  $AL_2O_3$ (30nm) + water and CuO(50nm) + Water . CuO+ water Nano fluid presents fewer enhancements in local Nusselt number at same Nanoparticle concentration. It is related to large particle size and high viscosity of CuO+ water and high thermal conductivity of AL+ water. The thermal losses in the absence of insulation does not exceed 15% has been proven that these losses increased with the increasing concentration and size of the particles. Moreover to NUR factor are used to present the corresponding flow and heat transfer inside the tube. Figs.(31 – 33)represents shear stress versus shear rate for three series of Nano fluid in various concentrations . it can be found that there is almost linear relation between shear stress and shear rate for all concentrations of Nano fluids which confirm a Newtonian behavior for AL,  $AL_2O_3$  and CuO in volume fraction 0.25 – 2.5%.

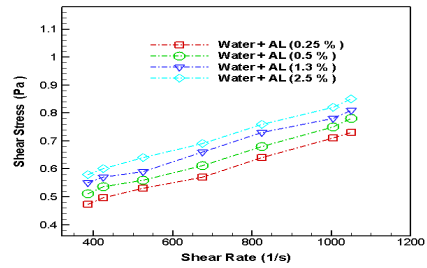


Fig. 31 Shear stress versus shear rate for based water + AL Nano fluid at different concentration

Fig. 34 and Fig. 35 show wall temperature distribution on the internal surface of the tube and the relation between temperatures and dimension less axial distance is linear relation for Nano fluid (AL+ water)with and without insulation.

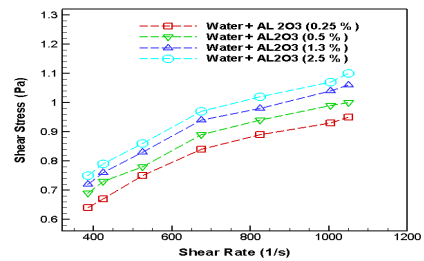


Fig. 32 Shear stress versus shear rate for based water +  $AL_2O_3$  Nano fluid at different concentration

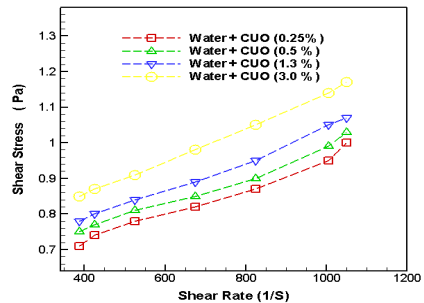


Fig. 33 Shear stress versus shear rate for based water +CuO Nano fluid at different concentration

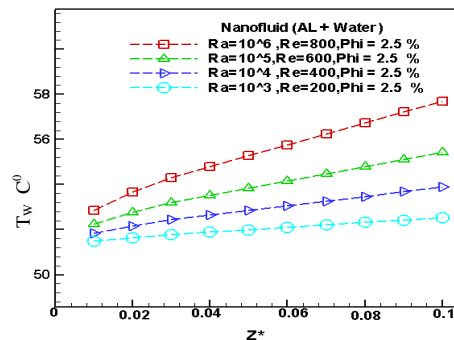


Fig. 34 Wall temperature for AL+water at different Ra,Re and Phi=2.5vol% with insulation

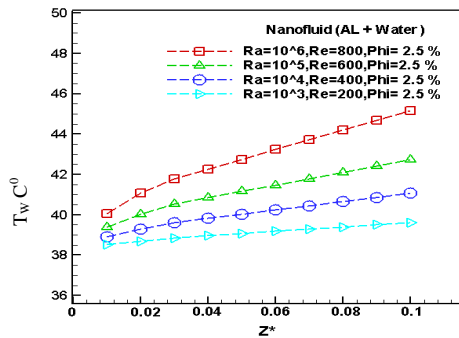


Fig. 35 Wall temperature for AL+water at different Ra,Re and  $\Phi=2.5\text{vol}\%$  without insulation

## VI. CONCLUSION

The present study was carried out to investigate about the convective heat transfer characteristics of three series of Nano fluids containing metallic and metal oxide nanoparticles in water through a circular tube under a laminar flow regime with a uniform heat flux boundary condition. Considering the experimental results, the following conclusions can be drawn:

- The Nano fluid has larger heat transfer (Nusselt number) than pure water under the same Reynolds number
- The type of Nano fluid is a key factor for heat transfer enhancement. The high values are obtained when using AL,  $\text{AL}_2\text{O}_3$  and CuO Nanoparticles respectively.
- NUR factor increase generally with Ra at constant PHI and increases with PHI at constant Ra.
- The statement of any of the cases the best increase in heat transfer has been proven that using insulation is better than not using it for Nano fluids.
- Heat transfer enhances with increasing Nanoparticles concentration.
- Nano fluids that contain metal Nanoparticles (AL) show more enhancements compared to oxide Nano fluids  $\text{AL}_2\text{O}_3$  – water and CuO –water. Also, CuO – water Nano fluids less enhancement in heat transfer at the same Nanoparticle concentration. it related to
- The increase in Nusselt number due to presence of Nanoparticles is much higher than prediction of single phase heat transfer correlation used with Nano fluid properties.
- There is an optimum concentration for each Nano fluid in which more enhancement are available. It is related to viscosity increasing by particle concentration.
- It is found that the Nano fluids Nusselt number enhancement is not only due to the increase of thermal conductivity but also other factors such as Nano fluid viscosity, dispersion, chaotic movement of Nanoparticles, Brownian motion, particle migration, fluctuation and interaction between particles.
- Increasing Nano fluid conductivity with increasing volume fraction but according to the model thermal conductivity when the particle size is decrease the random motion is larger and the convection like effect become dominant and

thermal conductivity for AL-water greater than  $\text{CuO}, \text{AL}_2\text{O}_3$ -water .

- The Nano fluid will not cause a penalty drop in pressure and there is no need for pump power. This may be because the small additional Nano particles in the base liquid do not cause the change in the flow behavior of the fluid.

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