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Evolved Bat Algorithm Based Adaptive Fuzzy Sliding Mode Control with LMI Criterion

P.-W. Tsai, C.-Y. Chen, C.-W. Chen

***Abstract*—**In this paper, the stability analysis of a GA-Based adaptive fuzzy sliding model controller for a nonlinear system is discussed. First, a nonlinear plant is well-approximated and described with a reference model and a fuzzy model, both involving FLC rules. Then, FLC rules and the consequent parameter are decided on via an Evolved Bat Algorithm (EBA). After this, we guarantee a new tracking performance inequality for the control system. The tracking problem is characterized to solve an eigenvalue problem (EVP). Next, an adaptive fuzzy sliding model controller (AFSMC) is proposed to stabilize the system so as to achieve good control performance. Lyapunov's direct method can be used to ensure the stability of the nonlinear system. It is shown that the stability analysis can reduce nonlinear systems into a linear matrix inequality (LMI) problem. Finally, a numerical simulation is provided to demonstrate the control methodology.

***Keywords*—**Adaptive fuzzy sliding mode control, Lyapunov direct method, swarm intelligence, evolved bat algorithm.

I. Introduction

IN recent years, adaptive fuzzy control system designs have attracted a lot ofattentionand have shown promising as a way to approach nonlinear control problems [7], [11], [15], [32]. The fundamental idea of adaptive fuzzy controlsystems is that one first constructs fuzzy models to describe the input/output behavior of the controlled system, based on the universal approximation theorem [32], [33] after which, these fuzzy models are used to design a controller. Adaptive laws are devised afterwards to adjust the parameters of the fuzzy models. A lot of effort has already gone into improving the robustness of adaptive fuzzy system, including studies on the design of the adaptive fuzzy sliding mode controller (AFSMC) [17], [25], [26], [31], [37] and on the integration of the sliding mode controller to improve the fuzzy rule base [10], [20]-[22], [29], [34], [35].

In addition to the above, deciding on the fuzzy rules and the initial values of the parameter vector values for the AFSMC is very important. A genetic algorithm is usually used as an optimization technique in the self-learning or training strategy for decidingon the fuzzy control rules and the initial values of the parameter vector [2], [3], [9], [14], [16], [18]. Using this type of GA-based AFSMC improves the immediate response, stability androbustness of the control system.

The design of the H^∞ fuzzy controller has attracted a lot of interest [19], [28], [36]. Fuzzy controlschemes have already been developed to guarantee the H^∞ tracking performance—that is, the induced L_2 -norm. Both the lumped matching error and the external disturbances to the tracking error must be equal to or less than the prescribed value. Generally, the *Lyapunov* stability criterion for the H^∞ fuzzy controller can be characterized in terms of solving a linear matrix inequality (LMI) or an eigenvalue problem (EVP) [1], [5], [17].

Let us consider an n^{th}-order single-input/single-out (SISO) system. The order of the motion equation forits sliding motion is usually equal to $n-1$, which makes it difficult to use conventional AFSMC to derive the entire system states so as to achieve H^∞ tracking performance [2], [19], [28], [36]. In order to consider the H^∞ tracking performance throughout the entire system states, we need to develop a new *Lyapunov* stability criterion for the proposed control strategy. It will be shown that the proposed controller (based on the *Lyapunov* theory) [22], [30], can guarantee good H^∞ tracking performance throughout the entire system states.

This study focuses on designing a robust tracking control for a class of nonlinear uncertain system involving plant uncertainties and external disturbances. To achieve this task, H^∞ tracking control is incorporated into the AFSMC. The proposed design is called the H^∞ AFSMC. The nonlinear system is described via fuzzy models.A genetic algorithm is used to find thefuzzy rules and the initial values of the parameter vector which the model is based on.The AFSMC is integrated with the H^∞ tracking control technique to design the control law. Based on the *Lyapunov* theory [22], [30], it will be shown that the proposed controller can guarantee good H^∞ tracking performance throughout the entire system states. In order to solve the control problem more efficiently itis characterized in terms of an eigenvalue problem (EVP) [1], [8], [24], [31]. In other words, the *Lyapunov* stability condition is transformed into the form of a certain linear matrix inequality (LMI) problem.One can then efficiently obtain the parameters of the controller by using convex optimization techniques to solve either the EVP or the LMI problem.

II. Reference Modeling for a Nonlinear System

A. Problem Formulation

Consider an n^{th}-order nonlinear system given in (1):

P.-W. Tsai, C.-Y. Chen, and C.-W. Chen (Corresponding author) are with Department of Maritime Information and Technology, National Kaohsiung Marine University, Kaohsiung 80543, Taiwan (phone: +886-7-8100888#5310; fax: +886-7-571-7301; e-mail: peri.tsai@gmail.com, cychen613@gmail.com, chengwu@mail.nkmu.edu.tw).

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= f(x)+g(x)\cdot u+d \\ y &= x_1 \end{aligned} \tag{1}$$

where $x=[x_1,x_2,\cdots,x_{n-1},x_n]^T\in R^n$ is the state vector of the system; $u\in R$ is the control signal; f and g are smooth nonlinear functions; d denotes the external disturbance $d(t)$, which is unknown but is usually bounded. The states $x=[x_1,x_2,\cdots,x_{n-1},x_n]^T$ are assumed to be available.

B. GA-Based H^∞ AFSMC for Nonlinear Systems

Consider the square MIMO system (a system with the same number of inputs and outputs) with p inputs and p outputs.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{(n_1-1)} &= x_{n_1} \\ \dot{x}_{n_1} &= f_1(x)+g_{11}(x)\cdot u_1+\cdots+g_{1p}(x)\cdot u_p+d_1(t) \\ \dot{x}_{(n_1+1)} &= x_{(n_1+2)} \\ &\vdots \\ \dot{x}_{(n_1+n_2-1)} &= x_{(n_1+n_2)} \\ \dot{x}_{(n_1+n_2)} &= f_2(x)+g_{21}(x)\cdot u_1+\cdots+g_{2p}(x)\cdot u_p+d_2(t) \\ &\vdots \\ \dot{x}_m &= f_p(x)+g_{p1}(x)\cdot u_1+\cdots+g_{pp}(x)\cdot u_p+d_p(t) \end{aligned} \tag{2}$$

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= x_{(n_1+1)} \\ &\vdots \\ y_p &= x_{(m-n_p+1)} \end{aligned}$$

where $m=n_1+n_2+\cdots+n_p$, and $x\in R^m$ are assumed to be available.

Differentiating $y_1,y_2,\cdots,y_p$ with respect to time for $n_1,n_2,\cdots,n_p$ times, respectively, until the inputs appear, one obtains an input/output form of (2), such as:

$$y_i^{(n_i)}=f_i(x)+\sum_{j=1}^{P}g_{ij}(x)\cdot u_j+d_i(t),\quad \text{for } i=1,\cdots,p. \tag{3}$$

Define

$$G(x)=\begin{bmatrix} g_{11} & \cdots & g_{1p} \\ \vdots & \ddots & \vdots \\ g_{p1} & \cdots & g_{pp} \end{bmatrix} \tag{4}$$

(2) can then be represented as:

$$\begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix}=\begin{bmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{bmatrix}+G(x)\cdot\begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix}+\begin{bmatrix} d_1(t) \\ \vdots \\ d_p(t) \end{bmatrix} \tag{5}$$

Assumption 1: *G(x) is bounded away from singularity, i.e., $G^{-1}(x)$ exists and has a bounded norm over a compact set $\zeta\in R^m$. Specifically, $\sigma_p(G(x))\ge b_1>0,\forall x\in\zeta$, where $\sigma_p(G(x))$ represents the smallest singular value of matrix G(x).*

If the control goal is for the plant output y to track reference trajectories $y_{r_1},\cdots,y_{r_p}$, then the reference control inputs $r_1,\cdots,r_p$ can be defined as the following reference model:

$$\begin{aligned} r_1 &= y_{r_1}^{(n)}+\alpha_{1(n_1-1)}y_{r_1}^{(n-1)}+\alpha_{1(n_1-2)}y_{r_1}^{(n-2)}+\ldots+\alpha_{11}\dot{y}_{r_1}+\alpha_{10}y_{r_1} \\ &\vdots \\ r_p &= y_{r_p}^{(n)}+\alpha_{p(n_p-1)}y_{r_p}^{(n-1)}+\alpha_{p(n_p-2)}y_{r_p}^{(n-2)}+\ldots+\alpha_{p1}\dot{y}_{r_p}+\alpha_{p0}y_{r_p} \end{aligned} \tag{6}$$

where $\begin{bmatrix} \alpha_{1(n_1-1)} & \alpha_{1(n_1-2)} & \cdots & \alpha_{11} & \alpha_{10} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_{p(n_p-1)} & \alpha_{p(n_p-2)} & \cdots & \alpha_{p1} & \alpha_{p0} \end{bmatrix}$

are chosen such that all the polynomials

$$\begin{aligned} &\ell^{n_1}+\alpha_{1(n_1-1)}\ell^{n_1-1}+\alpha_{1(n_1-2)}\ell^{n_1-2}+\ldots+\alpha_{11}\ell+\alpha_{10} \\ &\vdots \\ &\ell^{n_p}+\alpha_{p(n_p-1)}\ell^{n_p-1}+\alpha_{p(n_p-2)}\ell^{n_p-2}+\ldots+\alpha_{p1}\ell+\alpha_{p0} \end{aligned} \tag{7}$$

are Hurwitz, and where ℓ here denotes the complex *Laplace* variable.

If $f_i(x), g_{ij}(x)$ are known and ***Assumption1*** is satisfied, then the control law (for all $x\in\zeta$) can be defined as

$$\begin{bmatrix} u_1 \\ \vdots \\ u_p \end{bmatrix}=G^{-1}(x)\cdot \left[-\begin{bmatrix} f_1(x) \\ \vdots \\ f_p(x) \end{bmatrix}-\begin{bmatrix} d_1(t) \\ \vdots \\ d_p(t) \end{bmatrix}-\begin{bmatrix} \alpha_{1(n_1-1)}y_1^{(n_1-1)}+\cdots+\alpha_{11}\dot{y}_1+\alpha_{10}y_1 \\ \vdots \\ \alpha_{p(n_p-1)}y_p^{(n_p-1)}+\cdots+\alpha_{p1}\dot{y}_p+\alpha_{p0}y_p \end{bmatrix}+\begin{bmatrix} r_1 \\ \vdots \\ r_p \end{bmatrix}\right] \tag{8}$$

The linearized systems then become

$$\begin{aligned} &(y_{r_1}^{(n_1)}-y_1^{(n_1)})+\alpha_{1(n_1-1)}(y_{r_1}^{(n_1-1)}-y_1^{(n_1-1)})+\cdots+\alpha_{11}(\dot{y}_{r_1}-\dot{y}_1)+\alpha_{10}(y_{r_1}-y_1)=0 \\ &\vdots \\ &(y_{r_p}^{(n_p)}-y_p^{(n_p)})+\alpha_{p(n_p-1)}(y_{r_p}^{(n_p-1)}-y_p^{(n_p-1)})+\cdots+\alpha_{p1}(\dot{y}_{r_p}-\dot{y}_p)+\alpha_{p0}(y_{r_p}-y_p)=0 \end{aligned} \tag{9}$$

Define $e_1 = y_{r_1} - y_1, \cdots, e_p = y_{r_p} - y_p$, then we can obtain the following error equations:

$$\begin{aligned} &e_1^{(n_1)} + \alpha_{1(n_1-1)} e_1^{(n_1-1)} + \cdots + \alpha_{11}\dot{e}_1 + \alpha_{10} e_1 = 0 \\ &\qquad\vdots \\ &e_p^{(n_p)} + \alpha_{p(n_p-1)} e_p^{(n_p-1)} + \cdots + \alpha_{p1}\dot{e}_p + \alpha_{p0} e_p = 0 \end{aligned} \tag{10}$$

It is clear that $e_1, \cdots, e_p$ will approach zero if α_{ij} is chosen such that all polynomials like (7) are Hurwitz.The control objective can be achieved by a control law designed as follows:

$$u_j = u_{j,eq} + u_{j,d} \tag{11}$$

where $u_{j,eq}$ has equivalent control and defines $u_{j,d}$ as

$$\begin{bmatrix} u_{1,d} \\ \vdots \\ u_{p,d} \end{bmatrix} = G^{-1}(x) \cdot \begin{bmatrix} S_1/\rho_1 \\ \vdots \\ S_p/\rho_p \end{bmatrix} \tag{12}$$

If $f_i(x), g_{ij}(x)$ are known, we can then design the FLC (13) to approximate $u_{j,eq}$

$$\bar{u}_{j,eq}(\bar{\theta}_j) = \sum_{k=1}^{m} R_k(S) \cdot \bar{\theta}_{kj} \tag{13}$$

where m is the sum of the fuzzy rules; $\bar{\theta}_{kj}$, *i.e.* $|\bar{\theta}_{kj}| \le \theta_{\max}$, indicates the adjustable consequent parameters of the FLC; and $R(S) = [R_1(S), R_2(S), \cdots, R_m(S)]^T$ is the vector of fuzzy basis function [33],which is defined as

$$R_k(S) = R_k(\|S_i - C_{ki}\|) = \frac{\prod_{i=1}^{n} \mu_k(\|S_i - C_{ki}\|)}{\sum_{k=1}^{m}\left[\prod_{i=1}^{n} \mu_k(\|S_i - C_{ki}\|)\right]} \tag{14}$$

where $k = 1, \cdots, m$ and $i = 1, \cdots, n$ with μ_k represent the degree of membership. The S_i in μ_k can be chosen by

$$\mu_k(\|S_i - C_{ki}\|) = \exp\left(-\left(\frac{\|S_i - C_{ki}\|}{\beta}\right)^2\right) \tag{15}$$

Given the approximation property of the fuzzy system, an uncertain and nonlinear plant can be well-approximated and described via the fuzzy model involving FLC rules to achieve the control object.

Assumption 3*: For $x \in \zeta \subset R^p$, there exists adjustable parameter vectors $\bar{\theta}_j = [\bar{\theta}_{1j}, \bar{\theta}_{2j}, \cdots, \bar{\theta}_{mj}]^T$, for $j = 1,2,\cdots, p$, such that the fuzzy system $\bar{u}_{j,eq}(S, \bar{\theta}_j) = \bar{\theta}_j{}^T R(S)$ can approximate the continuous function u_j with accuracy $\varepsilon_{\max}$ over the set $\zeta \subset R^p$, that is, $\exists\, \bar{\theta}_j$ such that*

$$\sup\left| \bar{u}_{j,eq}(S, \bar{\theta}_j) - u_{j,eq}(S) \right| \le \varepsilon_{\max}, \ \forall S \in \zeta \tag{16}$$

Let $\hat{\theta}_j$ denote the estimate of $\bar{\theta}_j$ at time *t*. Because of this, we cannow define the estimated control output $\hat{u}_{j,eq}(S, \hat{\theta}_j)$ with

$$\hat{u}_{j,eq}(S, \hat{\theta}_j) = \sum_{k=1}^{m} R_k(S) \cdot \hat{\theta}_{kj} = \hat{\theta}_j{}^T R(S) \tag{17}$$

and decide on the initial values of the consequent parameter vector $\hat{\theta}_j$ based on the genetic algorithm.

First, if we define the parameter error vector at time *t* by $\tilde{\theta}_j = \bar{\theta}_j - \hat{\theta}_j$, then

$$\tilde{\theta}_j{}^T R(S) = \bar{u}_{j,eq}(S, \bar{\theta}_j) - \hat{u}_{j,eq}(S, \hat{\theta}_j). \tag{18}$$

According to ***Assumption 3***, we can define the modeling error ε_j as

$$\varepsilon_j = u_{j,eq} - \bar{u}_{j,eq}(S, \bar{\theta}_j), \tag{19}$$

where $|\varepsilon_j| \le \varepsilon_{\max}$.

We can say that

$$u_{j,eq} = \hat{u}_{j,eq}(S, \hat{\theta}_j) + \tilde{\theta}_j{}^T R + \varepsilon_j. \tag{20}$$

Substituting (12) and (20) into (11), we can then obtain the error dynamic equation in (21):

$$e_i^{(n_i)} + \alpha_{i(n_i-1)} e_i^{(n_i-1)} + \ldots + \alpha_{i1}\dot{e}_i + \alpha_{i0} e_i = \sum_{j=1}^{p} g_{ij}(x) \cdot \left(\tilde{\theta}_j{}^T R(S) + \varepsilon_j\right) - \frac{1}{\rho_i} S_i \tag{21}$$

and so, we can now define the augmented error as

$$S_i = \beta_{i(n_i-1)} e_i^{(n_i-1)} + \ldots + \beta_{i1}\dot{e}_i + \beta_{i0} e_i, \quad \text{for} \quad i = 1, \cdots, p \tag{22}$$

where $\beta_{i(n-1)}, \cdots, \beta_{i1}, \beta_{i0}$ in (26) and $\alpha_{i(n-1)}, \cdots, \alpha_{i1}, \alpha_{i0}$ in (10) are chosen such that

$$\hat{M}_i(\ell) = \frac{\beta_{i(n-1)}\ell^{n-1} + \ldots + \beta_{i1}\ell + \beta_{i0}}{\ell^n + \alpha_{i(n-1)}\ell^{n-1} + \ldots + \alpha_{i1}\ell + \alpha_{i0}} = \frac{N_i(\ell)}{D_i(\ell)} \tag{23}$$

is a SPR (strictly positive real) transfer function, and $N_i(\ell)$ and $D_i(\ell)$ are coprime.

If we define $e_{im} = [\, e_i, \cdots, \overset{(n_i-1)}{e_i}\,]^T$ as the states of (21), then (21) can be realized as

$$\dot{e}_{im}(t) = \Lambda_i \cdot e_{im}(t) + b_i \cdot [\sum_{j=1}^{p} g_{ij} \cdot (\tilde{\theta}_j^{\,T} R(S) + \varepsilon_j) - \frac{1}{\rho_i} S_i] \quad (24)$$

$$S_i(t) = c_i^{\,T} e_{im}(t) \quad (25)$$

where $\Lambda_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\alpha_{i0} & -\alpha_{i1} & -\alpha_{i2} & \cdots & -\alpha_{i(n-2)} & -\alpha_{i(n-1)} \end{bmatrix}_{n\times n}$, $b_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 1 \end{bmatrix}_{n\times 1}$,

$c_i = \begin{bmatrix} \beta_{i0} & \beta_{i1} & \cdots & \beta_{i(n-1)} \end{bmatrix}^T$ and let $\beta_{i(n-1)} = 1$, *for* $i = 1, \cdots, p$.

It is clear that the lumped uncertainty $\omega_i = \sum_{j=1}^{p} g_{ij}(x) \cdot \varepsilon_j$ would directly affect the tracking error. In order to achieve the control objective, the following H^∞ tracking performance related to the tracking error vector e_{im} should be requested as (26) [2], [19], [28], [36]:

$$\int_0^T e_{im}^T Q_i e_{im}\, dt \le e_{im}^T(0) P_i e_{im}(0) + \frac{1}{\gamma_i} \tilde{\theta}_i^{\,T}(0) H_i \tilde{\theta}_i(0) + \rho_i^{\,2} \int_0^T \omega_i^{\,2} dt \quad (26)$$

for all $\omega_i \in L_2[0,T]$, $\forall T \in [0,\infty)$, where Q_i and P_i are symmetrically positive definite weighting matrices, and where $0 < \rho_i < 1$ is a prescribed attenuation level.

Besides,

$$H_i = \begin{bmatrix} g_{i1}(x) \cdot \mathrm{I}_{m\times m} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{ip}(x) \cdot \mathrm{I}_{m\times m} \end{bmatrix}_{pm\times pm} \quad (27)$$

It is seen that if the system starts with the initial conditions $e_{im}(0) = 0$, $\tilde{\theta}_i(0) = 0$, then the H^∞ tracking performance of (26) becomes (28).

$$\int_0^T e_{im}^T Q_i e_{im}\, dt \le \rho_i^{\,2} \int_0^T \omega_i^{\,2} dt \quad (28)$$

or, $\sup_{\omega \in L_2[0,T]} \frac{\|e_{im}\|_{Q_i}}{\|\omega_i\|} \le \rho_i$, where $\|e_{im}\|_{Q_i} = (\int_0^T e_{im}^T Q_i e_{im}\, dt)^{1/2}$.

That is, the L_2-gain form ω_i to e_{im} must be equal to or less than the prescribed value of ρ_i [19], [28], [36]. Thus, the following result can be obtained:

Theorem 1: *Consider the nonlinear uncertain system* $\overset{(n)}{y_i} = f_i(x) + \sum_{j=1}^{p} g_{ij}(x) \cdot u_j + d_i$ *that satisfies assumptions* $(\bar{\theta}_j, \hat{\theta}_j)$ *.Suppose that the unknown control input* $u_{j,eq}$ *which is approximated by* $\hat{u}_{j,eq}(S, \hat{\theta}_j)$ *as* (17), S_i *is given by* (29), Q_i *isa symmetric positive definite weighting matrix, and* $0 < \rho_i < 1$ *is the design constant that serves as an attenuation level.*

Let $K_i = K_i^{\,T} > 0$ be the solution of the following LMI:

$$\Lambda_i^{\,T} K_i + K_i \Lambda_i + \frac{1}{\rho_i} K_i b_i b_i^{\,T} K_i - \frac{1}{\rho_i} c_i c_i^{\,T} + \frac{1}{\rho_i} Q_i \le 0 \quad (29)$$

III. Evolved Bat Algorithm for Finding Stable System Parameters

The Evolved Bat Algorithm (EBA) [27] is a newly developed swarm intelligence algorithm inspired by bat echolocation in the natural world. The computational speed of the EBA is fast because its structure is designed with simple and light computations. Unlike other swarm intelligence algorithms (e.g., Particle Swarm Optimization (PSO) [4], Artificial Bee Colony (ABC) optimization [12], [13], or Cat Swarm Optimization [5] [6], there is only one major variable which should be determined before using EBA, i.e., the medium for spreading sound waves. The chosen medium determines the step size of the movement of the artificial agent in the solution space. In general, the step size has a direct influence on the search result. In this paper, the chosen medium is air, based on the natural environment where bats live.The distance between the sound wave source and the target, which bounds the wave back, is defined by (30):`

$$D = \frac{340}{2}\left(m/\mathrm{sec.}\right) \cdot \Delta T(\mathrm{sec.}) = \frac{0.34}{2}\left(km/\mathrm{sec.}\right) \cdot \Delta T(\mathrm{sec.}) = 0.17 \Delta T \quad (30)$$

where D denotes the distance andit is known that the sound wave travels *340* meter-per-second in the air.

In our experiments, we use a random number in the range of [-1, 1] to denote ΔT. The negative part of ΔT comes from the moving direction in the coordinate. ΔT is given with a negative value when the transmission direction of the sound wave is opposite to the axis of the coordinate. The movement of the bat in EBA is defined by (31):

$$x_i^t = x_i^{t-1} + D \quad (31)$$

where x_i indicates the coordinate of the i^{th} artificial agent; and is the iteration number.

In addition, if a bat moves into the random walk process, its location will be updated by(32):

$$x_i^{t_R} = \beta \cdot \left(x_{best} - x_i^t\right),\ \beta \in [0, 1] \quad (32)$$

where β is a random number; x_{best} indicates the coordinate of the near best solution found so far overall artificial agents; and $x_i^{t_R}$ represents the new coordinates of the artificial agent after the operation of the random walk process.The operation of EBA contains the following 4 steps:

Step 1. Initialization: the artificial agents are spread throughout the solution space by randomly assigning the coordinates to them.
Step 2. Movement: the artificial agents are moved by (30)-(31). A random number is generated and then it is checked whether it is greater than the fixed pulse emission rate. If the result is positive, the artificial agent is moved using the random walk process, as defined by (32).
Step 3. Evaluation: the fitness of the artificial agents is calculated by the user defined fitness function and updated to the stored near best solution.
Step 4. Termination: the termination conditions are checked to decide whether to go back to step 2 or terminate the program and output the near best solution.

IV. Our Proposed Idea

Theorem 1 states the stability of a T-S fuzzy controller system and the stabilization can be achieved by finding a common symmetric positive definite matrix *P* for *r* subsystems. Hence, the stability analysis is converted to the problem of solving eigen values using the interior-point method associated with LMI techniques. The stability condition can be reduced to that of linear system when $r = 1$. In this paper, we propose the concept of utilizing swarm intelligence method to find the common symmetric positive definite matrix *P*, which satisfies the LMI stability conditions, instead of using the conventional methods.

To employ EBA solving problems of optimization, a fitness function should be defined at the first beginning. The fitness function is the mathematic representation of the evaluation condition for the target problem. In our design, we're going to use EBA to find a common *P* matrix, which satisfies the condition listed in (34):

$$\left(A_i - B_i K_l\right)^T P + P\left(A_i - B_i K_l\right) < 0 \tag{34}$$

where $P = P^T > 0$ and $i, l = 1,2,\ldots,r$.According to Hsiao et al.'s report [10], The equilibrium point of a closed-loop fuzzy system is asymptotically stable in the large, if there exists a common positive definite matrix *P*, which satisfies (34).The objective of our proposed method is to choose the proper common matrix *P* for the T-S fuzzy controller system which satisfies the stability condition listed in (34). The Duffing equation can describe a mechanical system with a hardening spring and can display rich nonlinear phenomena such as chaos and bifurcation. As a result, in recent years, the Duffing equation has become a test-bed for various advanced nonlinear and/or adaptive control techniques [23].

For the purpose of fulfilling the stability conditions of the theorem, EBA is employed to find the feasible *P* matrix. Each particle contains a symmetric positive definite matrix. The fitness function we design for this application is listed in (35):

$$F = \alpha \times \beta \tag{35}$$

where F denotes the fitness value, and × stands for the AND operation in Boolean logic; α and β come from (36) and (37).

$$\alpha = \begin{cases} 1, & \text{if}(A_i - B_i K_l)^T P + P(A_i - B_i K_l) < 0 \\ 0, & \text{otherwise} \end{cases} \tag{36}$$

$$\beta = \begin{cases} 1, P = P^T > 0 \\ 0, \text{ otherwise} \end{cases} \tag{37}$$

V. Conclusion

The stability analysis of a GA-Based H^∞ adaptive fuzzy sliding model controller for a nonlinear system is discussed in this study. We first tracked the reference trajectory for an uncertain and nonlinear plant, and made sure that it waswell-approximated and described via the fuzzy model adopting FLC rules. Then, we decided on the initial values of the consequent parameter vector $\hat{\theta}_j$ via a genetic algorithm. After this, we guaranteed a new H^∞ tracking performance inequality for the control system and the H^∞ tracking problem was characterized in terms of a linear matrix inequality (LMI) or an eigenvalue problem (EVP). It could then be efficiently solved by using convex optimization techniques. Next, an adaptive fuzzy sliding model controller was proposed to stabilize the system; good H^∞ control performance was achieved at the same time. A stability criterion was also derived from *Lyapunov*'s direct method to ensure the stability of the nonlinear system. It wasalso shown that the stability analysis of nonlinear systems could be reduced into LMI problems. Finally,concept of using EBA to find the common *P* matrix, which satisfies the stability criteria of the nonlinear system, is presented in this paper. Based on this criterion, the fuzzy controller design, with the LMI technique, can be used to stabilize the proposed fuzzy systems.

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