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Evaluation of the Effect of Rotor Solidity on the Performance of a H-Darrieus Turbine Adopting a Blade Element-Momentum Algorithm

G. Bedon, M. Raciti Castelli, E. Benini

***Abstract*—**The present study aims to evaluating the effect of rotor solidity - in terms of chord length for a given rotor diameter - on the performances of a small vertical axis Darrieus wind turbine. The proposed work focuses on both power production and rotor power coefficient, considering also the structural constraints deriving from the centrifugal forces due to rotor angular velocity. Also the smoothness of the resulting power curves have been investigated, in order to evaluate the controllability of the corresponding rotor architectures.

***Keywords*—**Vertical axis wind turbine, Darrieus, solidity, Blade Element-Momentum

I. Introduction

GREEN energy solutions to produce energy will enter in the day-life in a massive way in a near future, due to the increasingly awareness of possible fossil fuel shortages and its environmental un-sustainability. Wind energy experienced a consistent spread in the last decade, especially with the offshore and onshore horizontal axis wind turbines (HAWTs). These solutions represent a first approach for the exploitation of an important source of energy like the wind. Their diffusion is nevertheless limited, due to the inherent defects that characterize their architecture. In fact, the production of a consistent amount of energy requires the installation of a high tower and a big rotor: the investment cost and the social rejection limit the possible installation sites. In addition to this, wind turbine installations require a connection to the electrical grid that, due to their isolated locations, can be quite expensive. In this context, the vertical axis wind turbine (VAWT) represents a solution that could achieve a trade-off between production and integration. In fact, its architecture does not require a tower installation and, thanks to the lightweight of the components, can be installed on the building roofs, as well as inside urban areas. In addition, grid connection results to be easier, since the power is produced where it is consumed. Among the VAWTs, the Darrieus type represents the most promising design with respect to the general efficiency of the energy conversion system.

Gabriele Bedon is a M.Sc. Student in Mechanical Engineering at the University of Padua, Via Venezia 1, 35131 Padova, Italy (e-mail: gabriele.bedon@studenti.unipd.it).

Marco Raciti Castelli is a Research Associate at the Department of Industrial Engineering of the University of Padua, Via Venezia 1, 35131 Padova, Italy(e-mail: marco.raciticastelli@unipd.it).

Ernesto Benini is an Associate Professor at the Department of Industrial Engineering of the University of Padua, Via Venezia 1, 35131 Padova, Italy (e-mail: ernesto.benini@unipd.it).

The design strategy of this architecture is still object of research: different methods, based on both computational algorithms and physical models, are available in literature.

Raciti Castelli et al. [1]-[2] proposed the use of Computational Fluid Dynamic (CFD), in order to estimate the aerodynamic performance of a VAWT: this method allows a great flexibility in the investigation of several blade profiles and permits a deep understanding of the air flow inside the turbine. On the other hand, it requires consistent computational resources.

Bedon et al. [3] combined the BE-M code developed by Raciti Castelli et al. [4] and a genetic algorithm, obtaining an optimization tool, named W.O.M.B.A.T. (Weatherly Optimization Method for Blade of Air Turbine), that can be used in the design procedure of a VAWT. Several test cases were performed, considering different objectives as proof of concept. The increase of power coefficient, with respect to the baseline configuration, resulted up to 11%.

Numerical codes based on the Blade Element-Momentum (BE-M) Theory permit wind turbine designers to provide a good estimation of rotor performance, in an amount of time considerably lower than that required by modern CFD codes. This approach has been adopted by several authors to simulate and predict the aerodynamic performance of their turbines [5] [6] [7] [8] [9] [10].

Technical data required to perform the calculations are the operative conditions and geometrical details of the turbine, including the aerodynamic coefficients of the blade profiles, as highlighted by Bak et al. [11].

Strickland's Multiple-Streamtube model [7] is one of the most analyzed and improved BE-M algorithms. This model considers the same interaction between air and rotor, both in the upwind and in the downwind portions of rotor blade revolution, following an approach called *Single Disk*.

This assumption is not realistic, since the air passing through the upwind zone is slowed down by the interaction with the rotor blade: this model was thus improved, leading to the Double *Disk* approach, that takes into account the velocity variation between the upwind and downwind portions of rotor blade revolution [9] [12] [13] [14]. The resulting combined algorithm is called Double Multiple-Streamtube model.

The present work aims to investigating the effect of rotor solidity on the aerodynamic performance of a small Darrieus VAWT using a BE-M algorithm based on the Double Multiple-Streamtube model. Several works already analyzed this problem using both CFD and experimental measurements.

Raciti Castelli et al. [15] adopted a commercial CFD code to investigate the effect of blade number on a Darrieus VAWT, registering both a reduced aerodynamic efficiency and a reduced optimal angular velocity for an increased rotor solidity.

Shangmao and Yan [16] performed both static and dynamic investigations on the influence of rotor solidity adopting a CFD code. It was proved that, increasing the solidity, the maximum power production is achieved at a lower tip speed ratio. On the other hand, too large solidity decreased the absolute value of the power coefficient. By means of both experimental measurements and CFD simulations, Sabaeifard at al. [17] performed a preliminary analysis of the influence of the solidity - in terms of both chord and number of blades - for a H-Darrieus turbine equipped with NACA 0018 blades.

The obtained results allowed the determination of an optimal value for the solidity, which maximized the power coefficient of the rotor. In the present work, the Double Disk Multiple-Streamtube algorithm developed by Raciti Castell et al. [4] is adopted.

This code includes an algorithm for the simulation of hysteresis effects on rotor blade aerodynamic coefficients, based on the works of Gormont [18], Strickland et al. [19] and Berg [20]. The proposed algorithm was validated by Bedon et al. [21] with the use of the database of aerodynamic coefficients provided by Sheldahl and Klimas [22]. The corrections from Gormont-Strickland works were already proved to be reliable for the estimation of small rotor performances and were included in the W.O.M.B.A.T. optimization algorithm [3].

II. BE-M Theory

Several BE-M algorithms are available in literature. Some of them include specific corrections to keep both stremtube expansion and rotor blade dynamic stall into account [23]. The numerical code adopted in the present work was developed by Raciti Castelli et al. [4].

As shown in Fig. 1, the swept area of the analyzed rotor was discretized into several vertical and azimuthal subdivisions.

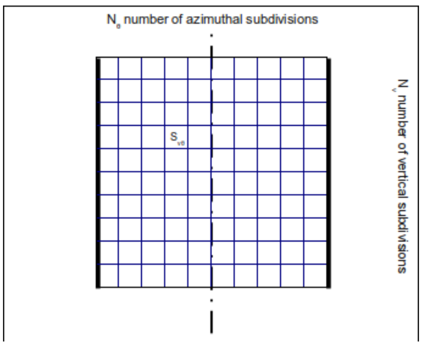

Fig. 1 Schematic view of the mesh generation for a straight-bladed rotor

The scheme of the adopted double disk BE-M model, reporting the six considered characteristic wind speeds and considering also the expansion of the streamtube due to air-blade interaction in the upwind zone, is reported in Fig. 2.

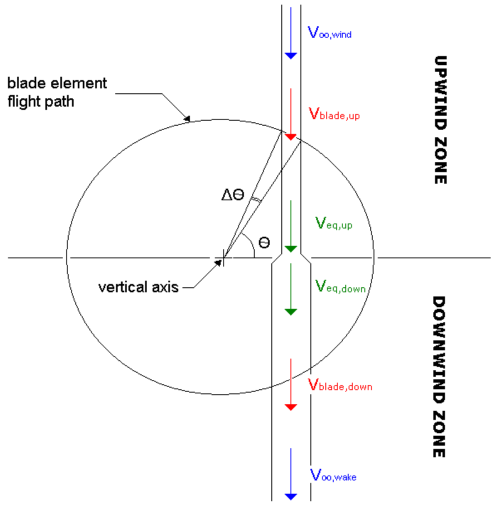

Fig. 2 Plan view of rotor cross-section and visualization of the six streamtube characteristic flow velocities for the adopted Double-Disk Multiple-Streamtube configuration, considering also the correction for streamtube expansion

The non-dimensional streamwise force is defined as:

$$F_x^* = \frac{Nc}{8\pi r}\left(\frac{W}{U_\infty}\right)^2\left(C_n - C_t\frac{\cos(\theta)}{\sin(\theta)\sin(\beta)}\right) \quad (1)$$

where N is the number of blades, c the rotor chord, r the rotor radius relative to the considered blade element, W the relative velocity in blade element cross-sectional plane, U_∞ the unhindered wind speed, θ the azimuthal coordinate of the blade, β the inclination of the blade element with respect to the horizontal plane and C_t and C_n respectively the blade element tangential and normal aerodynamic coefficients, respectively defined as:

$$C_t = C_L\sin(\alpha) - C_D\cos(\alpha) \quad (2)$$
$$C_n = C_L\cos(\alpha) + C_D\sin(\alpha) \quad (3)$$

being C_L and C_D respectively the lift and drag aerodynamic coefficients and α the blade angle of attack between chord and relative air velocity.

Defining the axial induction factor as:

$$a = 1 - \frac{V_{blade,i}}{U_\infty} \quad (4)$$

where $V_{blade,i}$ is the flow velocity at each blade section (*i* can be up or down, as shown in Fig. 2), the streamwise momentum equation can be written as:

$$a = F_x^* + a^2 \quad (5)$$

forming the basis for an iterative solution.

Once the streamwise momentum equation is solved, the torque for each streamtube can be calculated by means of the following equation:

$$T_s = \frac{1}{2}\rho r C_t \frac{c\,\Delta h}{\sin(\beta)} W^2 \tag{6}$$

where ρ is air density (assumed 1.225 kg/m^3) and Δh is the height of each single streamtube.

Finally, the average power produced by the rotor can be calculated as:

$$P_{out} = \omega \frac{N}{N_\theta} \sum_1^{N_\theta} \sum_1^{N_V} T_S \tag{7}$$

being ω rotor angular speed, N_θ and N_V respectively the number of horizontal and vertical mesh subdivisions.

The rotor power coefficient can be estimated by the formula:

$$C_p = \frac{P_{out}}{0.5\,\rho\,U_\infty{}^3\,S} \tag{8}$$

where S is the swept-area of the turbine.

Two different dynamic stall models, based on Gormont works [18] were implemented: Gormont-Strickland [19] and Gormont-Berg [20] models. Furthermore, the finite aspect-ratio correction from Viterna and Corrigan studies [24] was implemented.

III. The Case Study

The present work investigates the influence of blade chord length in a three-bladed Darrieus VAWT characterized by a uniform blade section along its span and a thickness to chord ratio equal to 0.18. Table 1 summarizes the main geometrical features of the rotor.

TABLE I
Main Geometrical Characteristics of the Tested Rotor

Name	Value
Height	3.236 m
Diameter	2 m
Number of blades	3
Blade profile	NACA 0018
Blade shape	linear

Rotor solidity, defined as:

$$\sigma = \frac{N\,c}{2\,\pi\,R} \tag{9}$$

was varied by considering different chord lengths, comprised in the range between 0.15 m and 0.30 m, determining a solidity ranging from 7.2% to 14.3%.

The wind speed considered in the simulations ranged from 4 up to 13 m/s, with a step of 1 m/s. The wind turbine was supposed to be coupled with a variable speed generator, capable of varying the rotational speed in a range comprised between 75 and 450 rpm.

The main performance parameter considered was the power produced P and the power coefficient C_p, as defined in (8).

IV. Results and Discussion

The proposed simulations were performed considering a constant wind speed and a variable rotor angular velocity, in order to find the optimal rotational value to maximize the power production. The resulting power curves for a chord length equal to 0.15 m, 0.20 m and 0.30 m are reported in Figs. from 3 to 5.

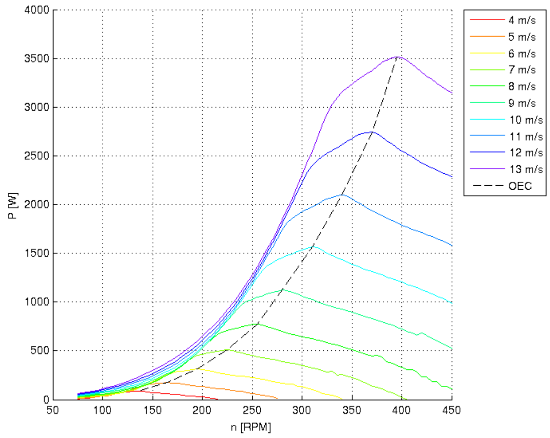

Fig. 3 Power production as a function of rotor angular velocity with respect to the wind speed; chord length of 0.15 m (σ = 7.2%)

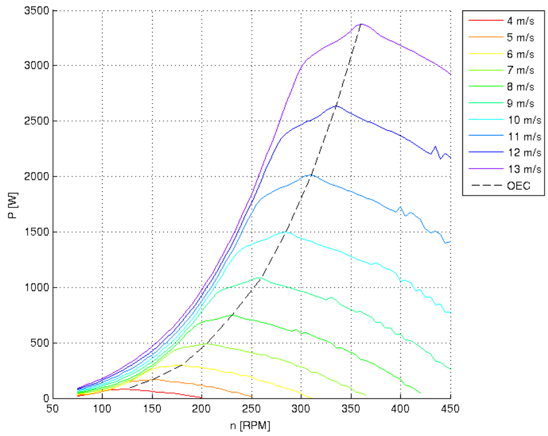

Fig. 4 Power production as a function of rotor angular velocity with respect to the wind speed; chord length of 0.20 m (σ = 9.5%)

As can be clearly seen, higher rotor solidities require a lower angular velocity to obtain the maximum amount of power produced for a certain wind speed. This phenomenon is also evidenced from Fig. 6, showing the evolution of the optimal rotor angular velocity as a function of both wind speed and rotor solidity. Moreover, a slight reduction in rotor efficiency with the increase of rotor solidity can be observed, as evidenced also from Fig. 7, showing the evolution of the maximum power production as a function of both wind speed and rotor solidity.

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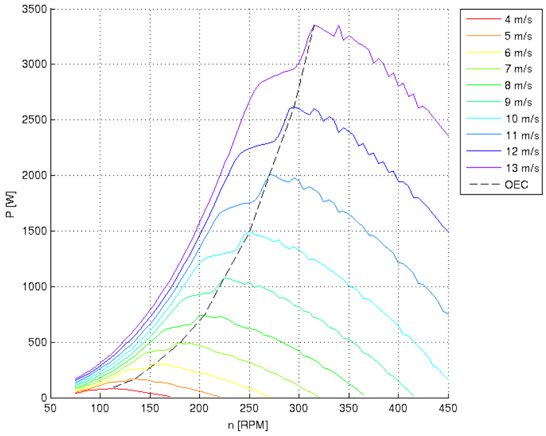

Fig. 5 Power production as a function of rotor angular velocity with respect to the wind speed; chord length of 0.30 m (σ = 14.3%)

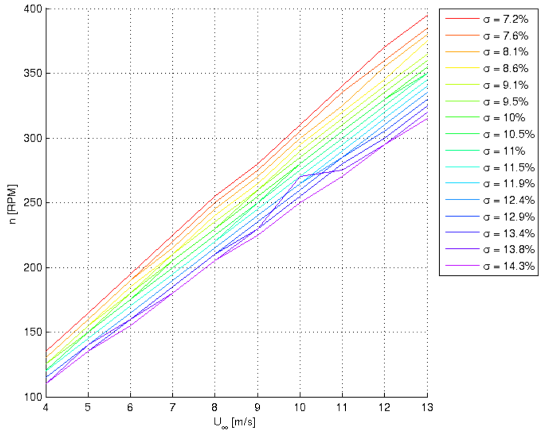

Fig. 6 Optimal rotational speed as a function of both wind speed and rotor solidity

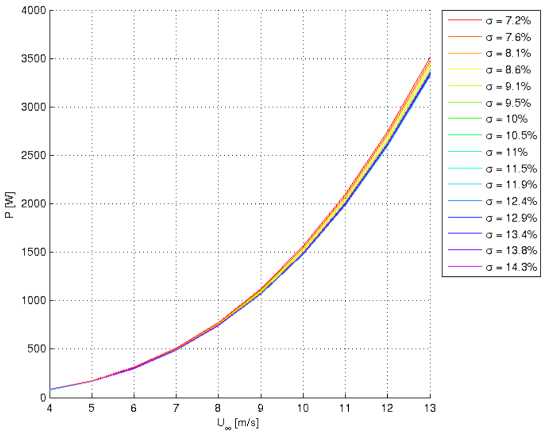

Fig. 7 Maximum power production as a function of both wind speed and rotor solidity

The decrease of aerodynamic performance due to the increment of rotor solidity can be better evaluated from Fig. 8, showing the evolution of the maximum power coefficient as a function of both wind speed and rotor solidity.

Considering the extreme curves (corresponding to a rotor solidity of respectively 7.2% and 14.3%), rotor overall aerodynamic efficiency registered an average decrease of 2 percentage points.

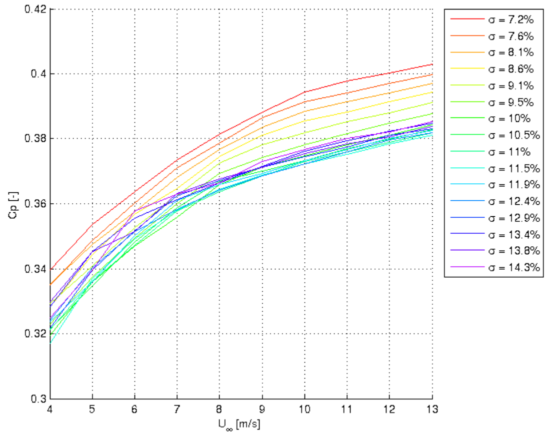

Fig. 8 Maximum rotor power coefficient as a function of both rotor angular velocity and wind speed

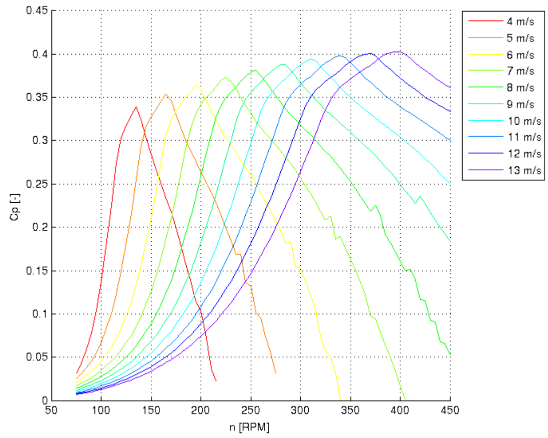

Fig. 9 Rotor power coefficient as a function of both rotor angular velocity and wind speed; chord length of 0.15 m (σ = 7.2%)

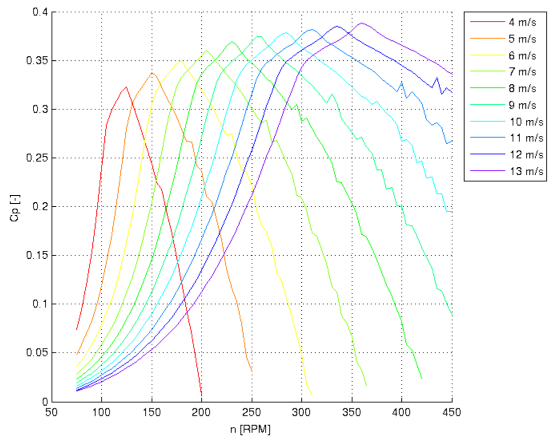

Fig. 10 Rotor power coefficient as a function of both rotor angular velocity and wind speed; chord length of 0.20 m (σ = 9.5%)

From the comparison between Figs. 3 and 5, it can be observed that, for lower values of rotor solidity, the power curve appears to be quite flat close to its maximum. This allows an easier control policy, since the rotational speed can be easily adjusted by the control system.

The registered flattening of rotor power coefficient close to its maximum for higher values of rotor solidity can also be evaluated from Figs. 9 to 11, where the beneficial influence of an increased blade Reynolds number – determining an increment of the performances of the blade sections as a consequence of the increase of the relative velocity in each blade element cross-sectional plane – due to an increased wind velocity and rotor angular speed is also visible.

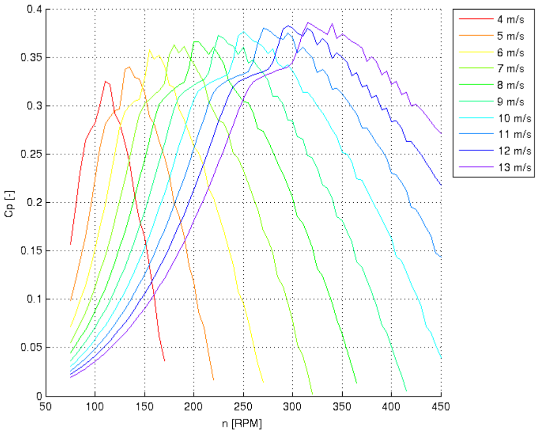

Fig. 11 Rotor power coefficient as a function of both rotor angular velocity and wind speed; chord length of 0.30 m (σ = 14.3%)

V. Conclusions and Future Work

The effect of rotor solidity on the performance of a three-bladed VAWT of 2 m diameter has been investigated by varying the chord length in the range 0.15 – 0.30 m. From the analysis of the resulting rotor power curves, it can be derived that a low value of rotor solidity leads to a higher production and a more controllable turbine. On the other hand, the optimal angular velocity is higher, leading to more consistent structural constraints. Considering the two opposite tendencies (increased rotational speed vs increased power production), a trade-off configuration should be preferably adopted, so as to gain high aerodynamic efficiency and ease of rotor control, as well as reduced structural loads on rotor blades.

Further work should be performed, in order to investigate also the effect of blade number on the performances of the analyzed rotor geometry.

Nomenclature

a [-]	axial induction factor
c [m]	airfoil chord
C_D [-]	airfoil drag coefficient
C_L [-]	airfoil lift coefficient
C_n [-]	blade element normal coefficient
C_P [-]	rotor power coefficient
C_t [-]	blade element tangential coefficient
F_x^*[N]	streamwise force exerted by the blade element as it passes through the streamtube
Δh [m]	streamtube height
n [rpm]	rotor angular velocity
N [-]	number of rotor blades
N_θ [-]	number of horizontal mesh subdivisions
N_V [-]	number of vertical mesh subdivisions
P_{out} [W]	aerodynamic power produced by the turbine
r [m]	rotor radius relative to the considered blade element (coincident with rotor radius for a H-shaped geometry)
R [m]	wind turbine radius
T_s [Nm]	blade element torque for each streamtube
S [m^2]	rotor swept area
U_∞ [m/s]	unperturbed wind speed
$V_{blade,i}$ [m/s]	flow velocity at blade section, *i* can be up or down
W [m/s]	relative velocity in blade element cross-sectional plane
α [rad]	blade angle of attack between chord line and relative velocity
β [rad]	blade element inclination with respect to the horizontal plane (corresponding to 90° for the proposed calculations)
ρ [kg/m^3]	air density (assumed 1.225 kg/m^3)
σ [-]	rotor solidity
θ [rad]	blade azimuthal coordinate
ω [rad/s]	rotor angular velocity

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