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Evaluation of Residual Stresses in Human Face as a Function of Growth

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***Abstract*—**Growth and remodeling of biological structures have gained lots of attention over the past decades. Determining the response of living tissues to mechanical loads is necessary for a wide range of developing fields such as prosthetics design or computer-assisted surgical interventions. It is a well-known fact that biological structures are never stress-free, even when externally unloaded. The exact origin of these residual stresses is not clear, but theoretically, growth is one of the main sources. Extracting body organ's shapes from medical imaging does not produce any information regarding the existing residual stresses in that organ. The simplest cause of such stresses is gravity since an organ grows under its influence from birth. Ignoring such residual stresses might cause erroneous results in numerical simulations. Accounting for residual stresses due to tissue growth can improve the accuracy of mechanical analysis results. This paper presents an original computational framework based on gradual growth to determine the residual stresses due to growth. To illustrate the method, we apply it to a finite element model of a healthy human face reconstructed from medical images. The distribution of residual stress in facial tissues is computed, which can overcome the effect of gravity and maintain tissues firmness. Our assumption is that tissue wrinkles caused by aging could be a consequence of decreasing residual stress and thus not counteracting gravity. Taking into account these stresses seems therefore extremely important in maxillofacial surgery. It would indeed help surgeons to estimate tissues changes after surgery.

***Keywords*—**Finite element method, growth, residual stress, soft tissue.

I. Introduction

GROWTH and remodeling are one of the main features of living tissues. Within the growth mechanics, living system can be formulated by mechanical phenomena and continuum mechanics [1]. As shown by Fung's experiments, the biological tissues are not stress-free even when entirely unloaded [2]-[4]. It is a well-known fact that the residual stresses affect the distribution of stresses in tissues [5]. Chuong and Fung [3] showed that in the vessel walls, the circumferential stress gradient is reduced due to the presence of residual stresses. The exact origin of residual stresses in the living tissues is not clear, but one of the main causes of these stresses is tissue growth and remodeling.

In early past decades, modeling tissues growth and computing the resulting residual stresses in the biological structures has gained lots of attention. In the case of soft tissues (in contrast with hard tissues such as bony structures), because of some complexity such as nonlinear behavior, anisotropy and large deformations, fewer models have been proposed. As a pioneering work on soft tissue growth, Rodriguez et al. [1] proposed a continuum formulation of volumetric growth. The authors defined the concept of "fictitious configuration" with considering a virtual state between zero-stressed reference and the current configurations. Holzapfel and Ogden [6] introduced a multi-layer model of arterial tissue in which each layer is composed of an isotropic matrix and two families of fibers that induce the anisotropy. Instead of using fictitious configuration, they assumed an open sector of artery as a stress-free reference configuration which produces residual stresses when it is closed. Among all works, Taber, Epstein and Maugin, Lubarda and Hoger, Guillau and Ogden have contributed to the understanding of tissues growth and the resulting residual stresses [7]-[10]. Recently, experimental analyses showed that the *in-vivo* stiffness of thin biological membranes like mitral leaflet is different from its measured *ex-vivo* stiffness by up to three orders of magnitude [11]. Rausch and Kuhl [11], using the inverse finite element method, showed that the main reason of this disagreement is the existing prestrain which relates to the growth-induced residual stresses. Hence ignoring the residual stresses and its effects may lead to erroneous results [12].

In this paper, we have implemented the concept of fictitious configuration for growth mechanics in order to determine the constitutive formulation of growth in the tissues. However, growth itself causes or changes the stress state in the tissues, but growth under loading changes the equilibrium state of stress. Hence determining the stresses due to growth requires an iterative method. In this paper, we propose a gradual growth method together with a loading-growth-unloading procedure to estimate tissues residual stresses. The method is applied to a model of healthy human facial tissues.

After a brief review of the growth continuum mechanics, the gradual growth method to determine the growth multiplier and the loading-growth-unloading procedure are presented. As a simple verification, the performance and efficiency of the proposed method are examined on a cantilever beam as an illustrative example. The proposed method is then applied to a finite element model of the human face. The residual stresses due to isotropic growth under gravity are represented on that face model.

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II. Continuum Modeling of Growth

A. Continuum Theory and Constitutive Formulation

Let X be the position vector of a material point in the reference configuration $\mathbf{B_0}$ at time t_0, and x its position vector in the current configuration $\mathbf{B_t}$ at time t. Deformation from $\mathbf{B_0}$ to $\mathbf{B_t}$ is denoted by $x = \Phi(X, t)$ and the deformation gradient is;

$$F = Grad\Phi = \frac{\partial x}{\partial X} \cdot \tag{1}$$

Using multiplicative decomposition of the deformation gradient, we introduce F_e and F_g which are the elastic and the growth deformation tensors respectively,

$$F = F_e F_g \tag{2}$$

To model the behavior of soft tissue, we use a 5-parameter Mooney-Rivlin function as the energy strain potential with an incompressible constraint which represents a nonlinear isotropic hyperelastic material. Since only the elastic tensor F_e generates stress, we use it instead of total deformation gradient F in expression of strain energy potential [13]

$$\psi(F, F_g) = \psi_e(F_e) \tag{3}$$

Using further decomposition, we introduce the volumetric and isochoric parts of elastic strain energy potential

$$\psi_e = \psi_e^{vol} + \bar{\psi}_e \tag{4}$$

This leads to Flory's decomposition of elastic tensor

$$F^e = F_e^{vol} . \bar{F}_e \tag{5}$$

with $F_e^{vol} = (J_e)^{1/3} I$ and $\bar{F}_e = (J_e)^{-1/3} F_e$, we have $J_e = \det(F_e)$ and thus $J_e^{vol} = \det(F_e^{vol}) = J_e$ hence $\bar{J}_e = \det(\bar{F}_e) = 1$. Therefore, the isochoric part of elastic right Cauchy-Green deformation tensor can be determined:

$$\bar{C}_e = \bar{F}_e^T . \bar{F}_e = (J_e)^{-2/3} C_e \tag{6}$$

Considering (3)-(6) for an isotropic material, we can express the elastic strain energy function and its related isochoric and volumetric parts as a function of the invariants of the isochoric part of the elastic right Cauchy-Green deformation tensor and J_e the determinant of total elastic tensor,

$$\bar{\psi}_e = c_1(\bar{I}_1 - 3) + c_2(\bar{I}_2 - 3) + c_3(\bar{I}_1 - 3)^2 + c_4(\bar{I}_2 - 3)(\bar{I}_1 - 3) + c_5(\bar{I}_2 - 3)^2 \tag{7}$$

$$\psi_e^{vol} = (J_e - 1)^2 \tag{8}$$

$$\psi_e = c_1(\bar{I}_1 - 3) + c_2(\bar{I}_2 - 3) + c_3(\bar{I}_1 - 3)^2 + c_4(\bar{I}_2 - 3)(\bar{I}_1 - 3) + c_5(\bar{I}_2 - 3)^2 + p(J_e - 1)^2 \tag{9}$$

where

$$\bar{I}_1 = tr(\bar{C}_e) \tag{10}$$

$$\bar{I}_2 = \frac{1}{2}((tr(\bar{C}_e))^2 - tr(\bar{C}_e^2)) \tag{11}$$

and p is the Lagrange multiplier to enforce the incompressibility constraint. Considered material constants are presented in Table I [14].

TABLE I
MOONEY-RIVLIN MATERIAL CONSTANTS USED IN THIS PAPER

C_1	C_2	C_3	C_4	C_5
2.5E+3	0	1.175E+3	0	0

To complete our continuum formulation, we introduce the second Piola-Kirchhoff stress, the Cauchy stress, and the fourth order elasticity tensor in (12)-(14) respectively,

$$S_e = 2\frac{\partial \psi_e}{\partial C_e} = S_e^{vol} + \bar{S}_e \tag{12}$$

$$\sigma_e = J_e^{-1} F_e . S_e . F_e^T \tag{13}$$

$$\mathbb{C}_e = 4\frac{\partial^2 \psi_e}{\partial C_e \otimes \partial C_e} = 2\frac{\partial S_e}{\partial C_e} = \mathbb{C}_e^{vol} + \bar{\mathbb{C}}_e \tag{14}$$

Because of limitation on space, the detailed derivations of equations are not given here. For more detail, see [11] and [13].

B. Growth Model

Although, the exact origin of residual stresses in biological tissues is still not clear, it is a well-known fact that such residual stresses are changed by growth. Fig. 1 shows the various configurations based on the concept of fictitious configuration and transformations, which are consequents of multiplicative decomposition. It should be noted that configuration $\mathbf{B_g}$ is fictitious grown incompatible configuration.

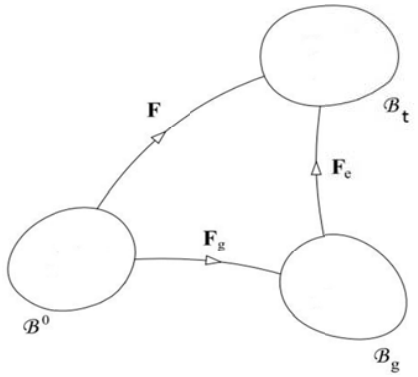


Fig. 1 Different configurations and transformation based on multiplicative decomposition of the deformation gradient

We use the isotropic growth law, as one of the most widely used models in the literature. It is indeed the simplest form of growth that can generate residual stress [13]. We thus parameterize the growth tensor, F_g as a multiple of a growth multiplier Θ_g by second order identity tensor I ,

$$F_g = \Theta_g I \tag{15}$$

Growth is related to time by following the evolution law,

$$\dot{\Theta}_g = \frac{1}{\tau} tr(E_e) \tag{16}$$

where τ is the time constant and E_e is the elastic Lagrangian strain tensor, $E_e = \frac{1}{2}(C_e - I)$.

Based on (15) and (16), the elastic part of the deformation is required to determine the growth and its related tensor F_g; however, F_e itself is computed following the growth as $F_e = FF_g^{-1}$. Hence, an iterative method is necessary to compute the growth factor Θ_g .

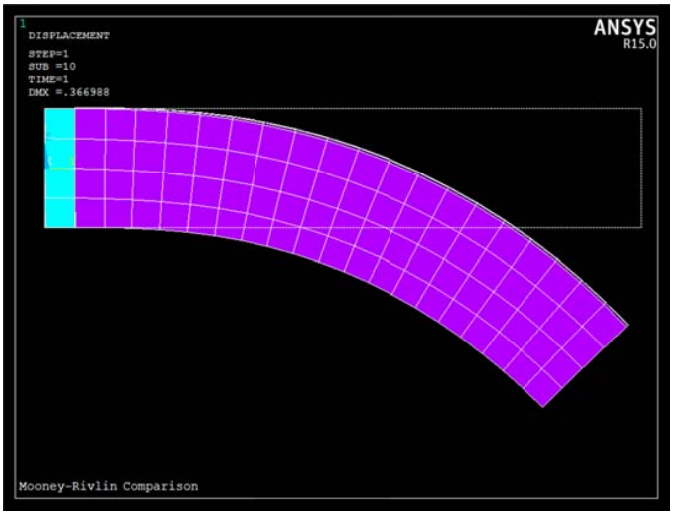
III. Computational Modeling of Growth

The constitutive formulations presented in the previous section are implemented in Ansys Mechanical APDL (Ansys®, 2015) by using the ability of user defined material (UMAT). To determine the residual stresses due to growth, we propose a gradual growth method together with a loading-growth-unloading protocol. Based on this, we first solve the finite element model under external mechanical loading and boundary condition. Then, under loading condition, growth is applied by using the growth multiplier Θ_g . In each step, this growth multiplier is increased to a maximum value with a specified velocity. Finally, in the unloading step, the external loads on the grown model are removed. The remained stresses are thus supposed to represent the residual stresses due to growth.

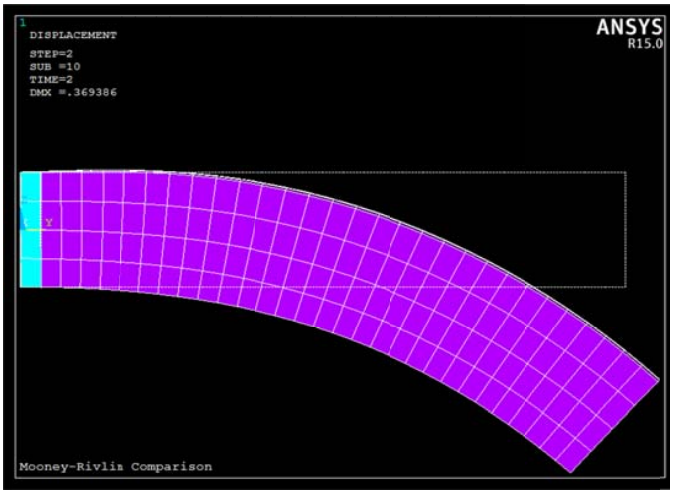
IV. Verification of Method

In order to verify the presented method, we implemented it on a cantilever beam with one fixed end. A 5-parameter Mooney-Rivlin model was used for that beam. As concerns boundary conditions, one end of the beam has been fixed. The other end of the beam is subjected to a transverse force applied perpendicularly to the longitudinal axis of the beam. After solving and determining bending stresses, a gradual growth is applied to the beam model. Finally, by removing the external loads in an unloading step, only the residual stresses due to growth remain in the beam. Fig. 3 shows the deformation of the beam at the end each loading, growth, and unloading steps, respectively.

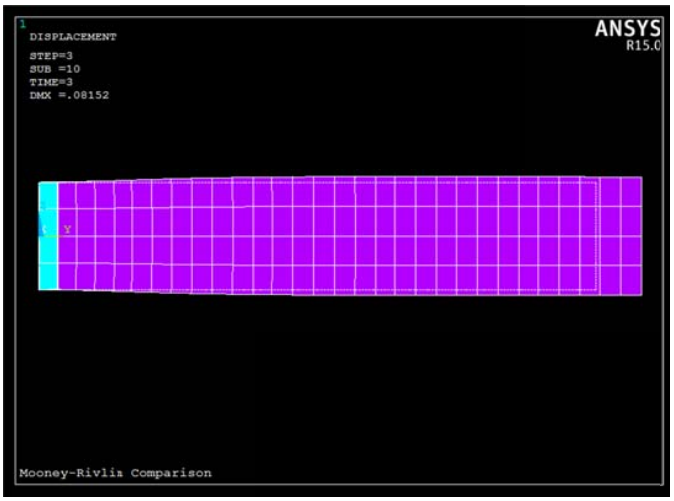
As shown in Fig. 3 (c), there is an indelible deformation in the beam due to growth, even when the model is entirely unloaded.



(a)

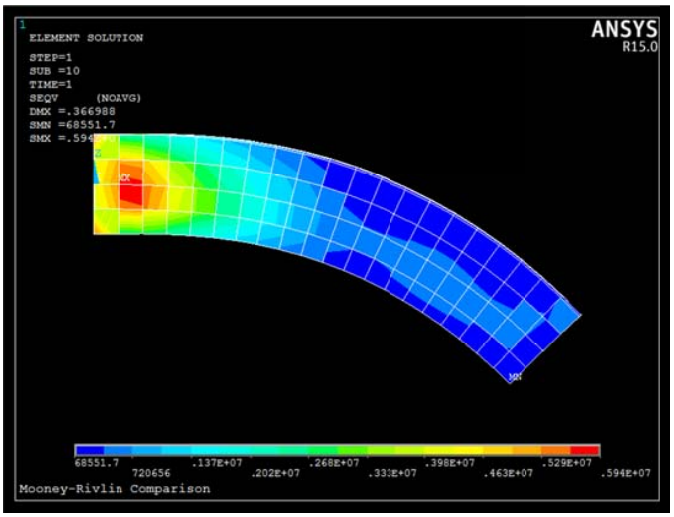


(b)

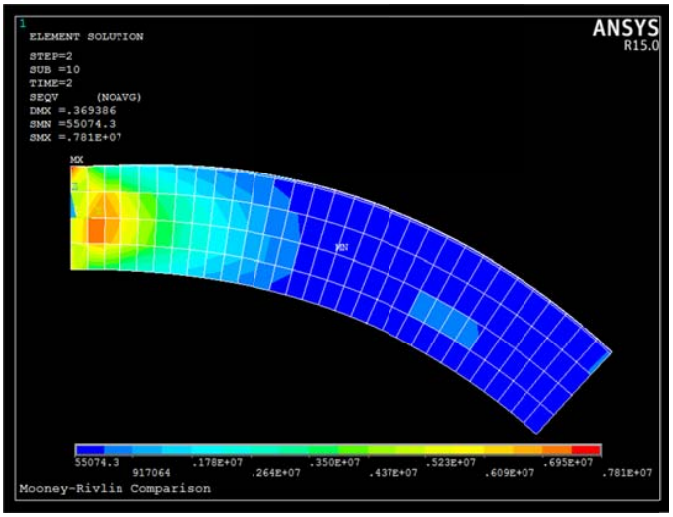


(c)

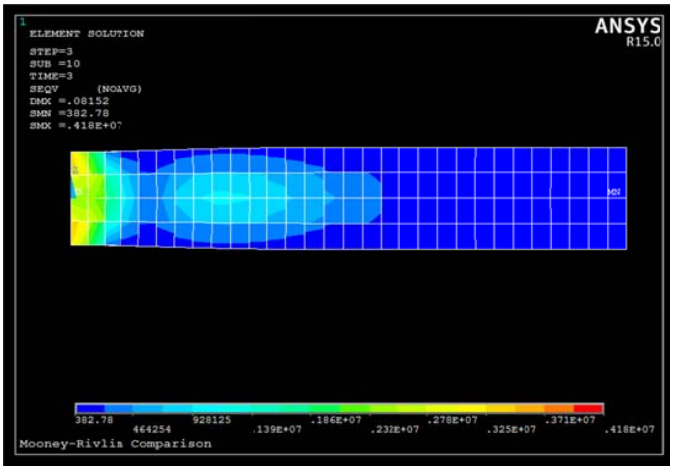
Fig. 3 Deformation of a beam at the end of loading (a), growth under loading (b) and unloading phases (c)



(a)



(b)



(c)

Fig. 4 Von-Mises stress distribution

The Von-Mises stresses after the loading, growth and unloading steps are shown in Fig. 4. By comparing Fig. 4 (a) and (b), it can be inferred that the distribution of stress becomes more uniform after the growth step with a reduction of stress concentration; such a result is consistent with the literature [5]. The computed residual stresses due to growth in unloaded configuration are shown in Fig. 4 (c).

V. Residual Stresses in Human Face

A. Human Face

To model growth and to determine its related residual stresses in the human face, the finite element model plotted in Fig. 5 is used. This model is based on an 8 nodes hex-dominant mesh representing the three layers of dermis and sub-dermis tissues generated with 10162 elements and 10068 nodes [14].

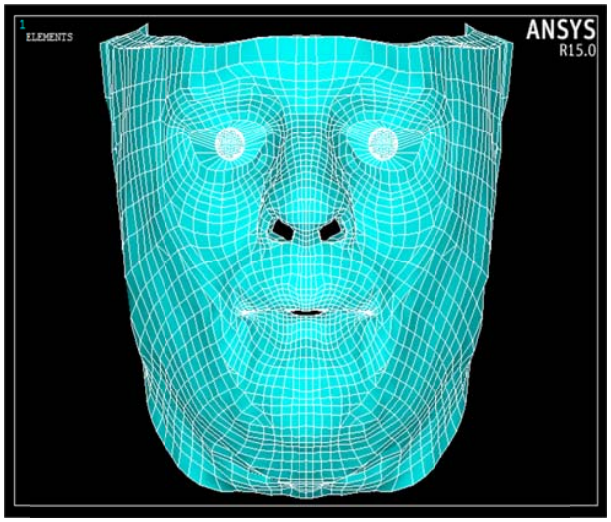


Fig. 5 Meshed human face model

As concerns boundary conditions, we assume that the inner layer of facial tissue is attached to the skull and therefore fixed in all directions.

B.Growth in Facial Tissue

There are different evolution laws in the literature. Researchers have proposed various driving forces for growth. Fiber stretches, Kirchhoff and Mandel stresses are some of widely used forces [15], [16]. In order to focus on residual stresses due to growth and not on the growth itself, we have selected isotropic growth. In this regard, we have used the trace of elastic strain tensor which is a reasonable assumption in isotropic growth [13].

The loading-growth-unloading sequence was implemented on the face model with a loading representing gravity. After this loading step, growth was applied gradually to the maximum value of $\Theta_g = 2$ (Fig. 7) which seems reasonable according to the literature [17], [18]. After removing the effect of gravity, the residual stresses due to growth in the facial tissue is then computed (Fig. 8).

Fig. 8 illustrates residual stress distribution while Figs. 6 and 7 show the stress distribution in non-grown and grown mechanically loaded configuration respectively. As shown in Fig. 7, the stress field has been made more uniform and homogenous as a consequence of growth. Moreover, Fig. 6 illustrates that residual stresses are greater in near the eyes, lips, and forehead.

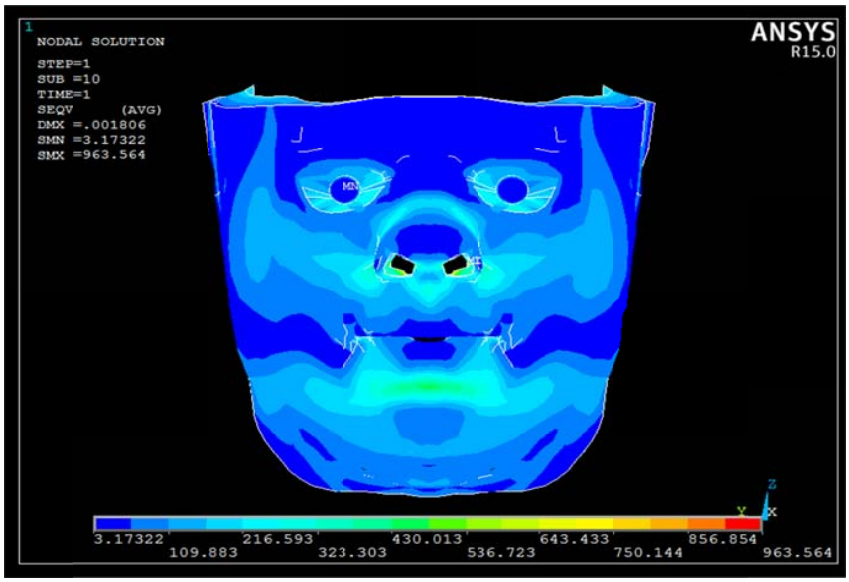


Fig. 6 Von-Mises stress distribution in non-grown mechanically loaded configuration

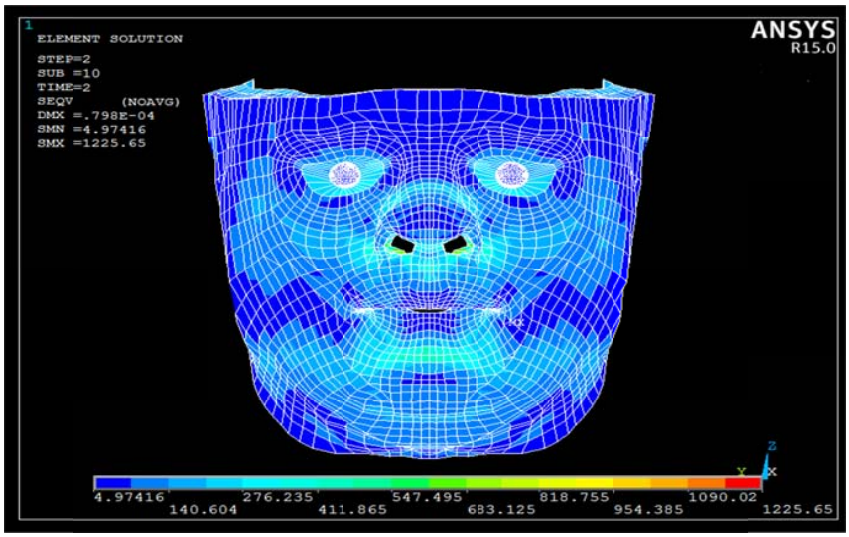


Fig. 7 Von-Mises stress distribution in grown mechanically loaded configuration

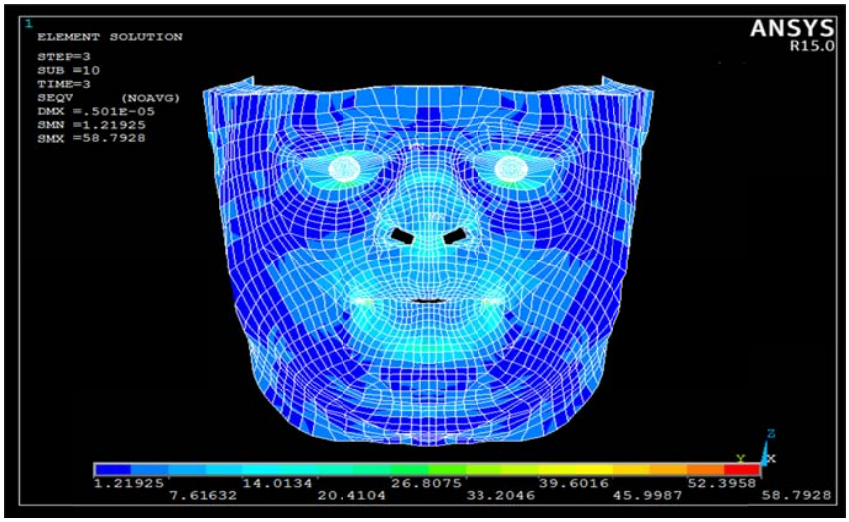


Fig. 8 Von-Mises stress distribution in grown mechanically unloaded configuration

VI. Conclusion

In this research, we proposed an approach to introduce and to estimate the soft tissues residual stresses due to isotropic growth. The results are promising since they seem to have a good agreement with the literature.

For future works, a more complex growth law should probably be used. In particular, to describe the material behavior, transversely isotropic and anisotropic material model will be considered. Validating the proposed method using experimental data is another perspective of this work.

References

[1] E. K. Rodriguez, A. Hoger, A. D. McCulloch, 'Stress-dependent finite growth in soft elastic tissue' *J. Biomech.* (1994)

[2] Yuan-Cheng Fung, *Biomechanics*, *Shock*, 1998, IX.

[3] C J Chuong and Y C Fung, 'Residual Stress in Arteries', in *Frontiers in Biomechanics SE - 9*, ed. by G W Schmid-Schönbein, S.L-Y. Woo, and B W Zweifach (Springer New York, 1986), pp. 117–129.

[4] Y C Fung, 'Biomechanical Aspects of Growth and Tissue Engineering', in *Biomechanics SE - 13* (Springer New York, 1990), pp. 499–546.

[5] D. Ambrosi, G. A. Ateshian, E. M. Arruda, S. C. Cowin, J. Dumais, A. Goriely, G. A. Holzapfel, J. D. Humphrey, R. Kemkemer, E. Kuhl, J. E. Olberding, L. A. Taber, K. Garikipati, 'Perspectives on Biological Growth and Remodeling', *Journal of the Mechanics and Physics of Solids*, 59 (2011).

[6] Gerhard a. Holzapfel, Thomas C. Gasser and Ray W. Ogden, 'A New Constitutive Framework for Arterial Wall Mechanics and a Comparative Study of Material Models', *Journal of Elasticity*, 61 (2000), 1–48.

[7] Larry a. Taber, 'Biomechanics of Growth, Remodeling, and Morphogenesis', *Applied Mechanics Reviews*, 1995, 487.

[8] Marcelo Epstein and Gérard A. Maugin, 'Thermomechanics of Volumetric Growth in Uniform Bodies', *International Journal of Plasticity*, 16 (2000), 951–978.

[9] V. A. Lubarda and A. Hoger, *On the Mechanics of Solids with a Growing Mass*, *International Journal of Solids and Structures*, 2002, XXXI.

[10] A. Guillou and R W Ogden, 'Growth in Soft Biological Tissue and Residual Stress Development', *Growth (Lakeland)*.

[11] Manuel K. Rausch and Ellen Kuhl, 'On the Effect of Prestrain and Residual Stress in Thin Biological Membranes', *Journal of the Mechanics and Physics of Solids*, 61 (2013), 1955–1969.

[12] F Morin, H. Courtecuisse, M. Chabanas, Y. Payan 'Rest Shape Computation for Highly Deformable Model of Brain', *Computer Methods in Biomechanics and Biomedical Engineering*, 18 (2015), 2006–2007.

[13] M Genet, K. Rausch, L. C. Lee, S. Choy, X. Zhao, G. S. Kassab, S. Kozerke, J. M. Guccione, E. Kuhl, 'Heterogeneous Growth-Induced Prestrain in the Heart', *Journal of Biomechanics*, 48 (2015), 2080–2089.

[14] Mohammad Ali Nazari, P. Perrier, M. Chabanas, Y. Payan 'Simulation of Dynamic Orofacial Movements Using a Constitutive Law Varying with Muscle Activation', *Computer Methods in Biomechanics and Biomedical Engineering*, 13 (2010), 469–482L.

[15] C Lee, M. Genet, G. Acevedo-Bolton, K. Ordovas, J. M. Guccione, E. Kuhl, 'A Computational Model That Predicts Reverse Growth in Response to Mechanical Unloading', *Biomechanics and Modeling in Mechanobiology*, 14 (2015), 217–229.

[16] H F Choi, J. D'hooge, F. E. Rademakers, P. Claus, 'Influence of Left-Ventricular Shape on Passive Filling Properties and End-Diastolic Fiber Stress and Strain.', *Journal of biomechanics*, 43 (2010), 1745–53.

[17] Alexander M. Zöllner, Adrian Buganza Tepole and Ellen Kuhl, 'On the Biomechanics and Mechanobiology of Growing Skin', *Journal of Theoretical Biology*, 297 (2012), 166–175.

[18] Alexander M. Zöllner, A. B. Tepole, A. K. Gosain, E. Kuhl 'Growing Skin: Tissue Expansion in Pediatric Forehead Reconstruction', *Biomechanics and Modeling in Mechanobiology*, 11 (2012), 855–867.