# Estimation of the Bit Side Force by Using Artificial Neural Network 

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#### Abstract

Horizontal wells are proven to be better producers because they can be extended for a long distance in the pay zone. Engineers have the technical means to forecast the well productivity for a given horizontal length. However, experiences have shown that the actual production rate is often significantly less than that of forecasted. It is a difficult task, if not impossible to identify the real reason why a horizontal well is not producing what was forecasted. Often the source of problem lies in the drilling of horizontal section such as permeability reduction in the pay zone due to mud invasion or snaky well patterns created during drilling. Although drillers aim to drill a constant inclination hole in the pay zone, the more frequent outcome is a sinusoidal wellbore trajectory. The two factors, which play an important role in wellbore tortuosity, are the inclination and side force at bit. A constant inclination horizontal well can only be drilled if the bit face is maintained perpendicular to longitudinal axis of bottom hole assembly (BHA) while keeping the side force nil at the bit. This approach assumes that there exists no formation force at bit. Hence, an appropriate BHA can be designed if bit side force and bit tilt are determined accurately. The Artificial Neural Network (ANN) is superior to existing analytical techniques. In this study, the neural networks have been employed as a general approximation tool for estimation of the bit side forces. A number of samples are analyzed with ANN for parameters of bit side force and the results are compared with exact analysis. Back Propagation Neural network (BPN) is used to approximation of bit side forces. Resultant low relative error value of the test indicates the usability of the BPN in this area.


Keywords- Artificial Neural Network, BHA, Horizontal Well, Stabilizer.

## I. Introduction

BOTTOM hole assembly (BHA) is the part of drill string that affects the trajectory of borehole by bit side force and tilt. The bit side force is the controlling factor for hard formations (drilling rates 1 to $10 \mathrm{ft} / \mathrm{h}$ ). For formations that are soft to medium hard, the side force is not the only component that will influence the inclination and direction of the bit; the bit tilt becomes influential as well. Because of the curvature of BHA near the bit, the bit is canted or tilted in some resultant direction and inclination, some what like the bent housing and bent sub side force becomes a controlling factor once again for very soft formations (drilling rates excess of $100 \mathrm{ft} / \mathrm{h}$ ) [1]. For a given BHA, the bit side force and tilt depends on a number of parameters. These parameters are hole, drill collar and stabilizer sizes, BHA material properties, mud weight,

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stabilizer locations, borehole inclination, direction, contact length, location between pipe and hole, and finally weight on bit. Unfortunately, analytical techniques derived to estimate bit side force ignores some of these parameters for the sake of simplicity. For example the analytical method developed based on three moment equation ignores the effect of pipe to wall contact as well as hole curvature [2], [3]. Other techniques have been developed to handle the well bore curvature, variable gauge holds and combination BHA components, and situations in which pipe/ wall contact occurs between the bit and stabilizers, as well as the cases in which increases in weight on bit force the creation of additional points of contact [4], [5], [6]. Nowadays Artificial Neural Network (ANN) is used in many of engineering problem. The back propagation network (BPN) is trained to estimate the bit side force in three cases: slick BHA, single stabilizer BHA and two stabilizers BHA. The results show that, an ANN can be estimated the bit side force with high accuracy. By using this method the time of analyzing will be very short.

## II. Estimation of the bit side force based on existing analytical techniques

A set of equations has been derived and published [3] to determine the bit side force based on three moment equation. These equations are derived depending on how many stabilizers are attached such as slick BHA, single-stabilizer BHA and two-stabilizer BHA. In all three cases, the essence of this technique is to determine the point of contact between the pipe and wall of the hole called tangency point.

## A. Slick BHA

Slick BHA has no stabilizers attached to it. The tangency point in this case is the first point where pipe departs from the borehole wall above the bit. $\mathrm{F}_{\mathrm{B}}$ is the bit side force. Equation (1) is determined the bit side force.
$\mathrm{F}_{\mathrm{B}}=-0.5 \mathrm{~W}_{\mathrm{C}} \mathrm{L}_{\mathrm{T}} \mathrm{B}_{\mathrm{C}} \sin \phi+\frac{\left(\mathrm{WOB}-0.5 \mathrm{~W}_{\mathrm{C}} \mathrm{B}_{\mathrm{C}} \mathrm{L}_{\mathrm{T}} \cos \phi\right) 1_{3}}{\mathrm{~L}_{\mathrm{T}}}(1)$
Where $L_{T}$ is the first point tangency and is measured from the bit. It has to be estimated before substituting in equation (1). $L_{T}$ is determined by trial and error as follows:

Assume an initial guess for $\mathrm{L}_{\mathrm{T}}$ and calculate new $\mathrm{L}_{\mathrm{T}}^{\prime}$ from equation (2). If $\mathrm{L}_{\mathrm{T}}$ is not equal to $\mathrm{L}_{\mathrm{T}}^{\prime}$, then set new $\mathrm{L}_{\mathrm{T}}=\mathrm{L}_{\mathrm{T}}^{\prime}$ and repeat the procedure until $\mathrm{L}_{\mathrm{T}}$ agrees with $\mathrm{L}_{\mathrm{T}}^{\prime}$.
$\mathrm{L}_{\mathrm{T}}^{\prime}=\left[\frac{24 \mathrm{EI} .1}{\mathrm{~W}_{\mathrm{C}} \mathrm{B}_{\mathrm{C}} \mathrm{X} \sin \phi}\right]^{\frac{1}{4}}$
$\mathrm{X}=\frac{3(\tan \mathrm{u}-\mathrm{u})}{\mathrm{u}^{3}}$
$\mathrm{u}=\frac{\mathrm{L}_{\mathrm{T}}}{2} \sqrt{\frac{\mathrm{WOB}-0.5 \mathrm{~W}_{\mathrm{C}} \mathrm{B}_{\mathrm{C}} \mathrm{L}_{\mathrm{T}} \cos \phi}{\mathrm{EI}}}$
in these equations, $\mathrm{B}_{\mathrm{C}}$ is buoyancy factor, E is young's modulus, $1_{3}$ is hole clearance around drill collars, $\mathrm{W}_{\mathrm{C}}$ is unit weight of drill collars, WOB weight on bit, I is axial moment of inertia and $\phi$ is hole inclination at bit.

## B. Single - stabilizer BHA

Equation (5), as in the previous case, side force can be determined as a function of tangency point:

$$
\begin{align*}
\mathrm{F}_{\mathrm{B}}= & -\frac{\mathrm{B}_{\mathrm{C}} \mathrm{~W}_{\mathrm{C} 1} \mathrm{~L}_{\mathrm{T}} \sin \phi}{2} \\
& +\frac{\left(\mathrm{WOB}-0.5 \mathrm{~W}_{\mathrm{C} 1} \mathrm{~B}_{\mathrm{C}} \mathrm{~L}_{\mathrm{T}} \cos \phi\right) 1_{1}-\mathrm{m}}{\mathrm{~L}_{\mathrm{T}}} \tag{5}
\end{align*}
$$

The tangency point $\mathrm{L}_{\mathrm{T}}$ is the first point where pipe departs from the borehole wall above the stabilizer.
However, $\mathrm{L}_{\mathrm{T}}$ stands for the length from stabilizer up to the tangency point. The same analysis technique can be applied to determine $L_{T}$ by assuming an initial guess for $L_{2}$ and calculating $L_{T}$ from equation (6). If $L_{T}$ is not equal to $L_{2}$, then set new $\mathrm{L}_{2}=\mathrm{L}_{\mathrm{T}}$ and repeat the procedure until $\mathrm{L}_{\mathrm{T}}$ agrees with $\mathrm{L}_{2}$.
$\mathrm{L}_{\mathrm{T}}=\left[\frac{24 \mathrm{EI}\left(1_{3}-1_{1}\right)-4 \mathrm{~mL}_{\mathrm{T}}^{2} \mathrm{~W}_{2}}{\mathrm{q}_{2} \mathrm{x}_{2}}\right]^{\frac{1}{4}}$
$\mathrm{m}=\frac{-\frac{\mathrm{q}_{1} \mathrm{~L}_{1}^{2}}{4} \mathrm{X}_{1}-\frac{\mathrm{q}_{2} \mathrm{~L}_{2}^{3} \mathrm{I}_{1}}{4 \mathrm{~L}_{1} \mathrm{~L}_{2}} \mathrm{X}_{2}+\frac{6 \mathrm{EI}_{1} \mathrm{l}_{1}}{\mathrm{~L}_{1}^{2}}+\frac{6 \mathrm{EI}_{1}\left(\mathrm{l}_{1}-1_{3}\right)}{\mathrm{L}_{1} \mathrm{~L}_{2}}}{2\left(\mathrm{~V}_{1}+\frac{\mathrm{L}_{2} \mathrm{I}_{1}}{\mathrm{~L}_{1} \mathrm{I}_{2}} \mathrm{~V}_{2}\right)}$
$\mathrm{q}_{1}=\mathrm{W}_{\mathrm{C} 1} \mathrm{~B}_{\mathrm{C}} \sin \phi$
$\mathrm{q}_{2}=\mathrm{W}_{\mathrm{C} 2} \mathrm{~B}_{\mathrm{C}} \sin \phi$
$\mathrm{W}_{2}=\frac{3}{\mathrm{u}_{2}}\left[\frac{1}{\sin \left(2 \mathrm{u}_{2}\right)}-\frac{1}{2 \mathrm{u}_{2}}\right]$
$V_{i}=\frac{3}{2 u_{i}}\left[\frac{1}{2 u_{i}}-\frac{1}{\tan \left(2 u_{i}\right)}\right]$
Where $i=1$ and 2
$X_{i}=\frac{3\left(\tan u_{i}-u_{i}\right)}{u_{i}^{3}}$
$\mathrm{u}_{1}=\frac{\mathrm{L}_{1}}{2} \sqrt{\frac{\mathrm{WOB}-0.5 \mathrm{~W}_{\mathrm{Cl}} \mathrm{B}_{\mathrm{C}} \mathrm{L}_{1} \cos \phi}{\mathrm{EI}_{1}}}$
$\mathrm{u}_{2}=\frac{\mathrm{L}_{2}}{2} \sqrt{\frac{\mathrm{WOB}-0.5 \mathrm{~W}_{\mathrm{C} 2} \mathrm{~B}_{\mathrm{C}} \mathrm{L}_{2} \cos \phi-\mathrm{W}_{\mathrm{C} 1} \mathrm{~B}_{\mathrm{C}} \mathrm{L}_{1}}{\mathrm{EI}_{2}}}$
$\mathrm{l}_{1}=0.5\left(\mathrm{~d}_{\mathrm{b}}-\mathrm{d}_{\mathrm{SI}}\right)$
$1_{3}=0.5\left(\mathrm{~d}_{\mathrm{b}}-\mathrm{d}_{2}\right)$
in these equations, $m$ is bending moment, $d_{b}$ is bit diameter, $\mathrm{d}_{2}$ is drill collar diameter, $\mathrm{d}_{\mathrm{S} 1}$ is 1 st stabilizer diameter, $1_{1}$ and $1_{3}$ are respectively hole clearance around 1 st stabilizer and hole clearance around drill collars.

## C. Two-stabilizer BHA

The two stabilizer BHA can also be solved with the same technique. The distance between the second stabilizer and the point of tangency ( $\mathrm{L}_{3}$ ) is unknown, and, as in the previous two cases, $\mathrm{L}_{3}$ must be guesses initially. This solution technique accommodates three different, the two moments, $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ can be determined and the bit side force can be calculated from the following equations:
$\mathrm{F}_{\mathrm{B}}=-\frac{\mathrm{W}_{\mathrm{C}} \mathrm{B}_{\mathrm{C}} \mathrm{L}_{\mathrm{T}}}{2} \sin \phi$

$$
\begin{equation*}
+\frac{\left(\mathrm{WOB}-\frac{\mathrm{W}_{\mathrm{Cl}} \mathrm{~B}_{\mathrm{C}} \mathrm{~L}_{\mathrm{T}}}{2} \cos \phi\right) \mathrm{l}_{1}-\mathrm{m}_{1}}{\mathrm{~L}_{\mathrm{T}}} \tag{17}
\end{equation*}
$$

$1_{1}=0.5\left(\frac{\mathrm{~d}_{\mathrm{b}}-\mathrm{d}_{\mathrm{Sl}}}{12}\right)$
$1_{2}=0.5\left(\frac{\mathrm{~d}_{\mathrm{b}}-\mathrm{d}_{\mathrm{S} 2}}{12}\right)$
$1_{3}=0.5\left(\frac{\mathrm{~d}_{\mathrm{b}}-\mathrm{d}_{2}}{12}\right)$
$\mathrm{L}_{\mathrm{T}}=\left[\frac{24 \mathrm{EI}_{3}\left(\mathrm{l}_{3}-\mathrm{l}_{2}\right)-4 \mathrm{~m}_{2} \mathrm{~L}_{3}^{2} \mathrm{~W}_{3}}{\mathrm{q}_{3} \mathrm{X}_{3}}\right]^{\frac{1}{4}}$
$\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ are solved simultaneously from the following two equations:
$2 m_{1}\left(V_{1}+\frac{L_{2} I_{1}}{L_{1} I_{2}} V_{2}\right)+m_{2} \frac{L_{2} I_{1}}{L_{1} I_{2}} W_{2}=$
$-\frac{\mathrm{q}_{1} \mathrm{~L}_{1}^{2}}{4} \mathrm{X}_{1}-\frac{\mathrm{q}_{2} \mathrm{~L}_{2}^{3} \mathrm{I}_{1}}{4 \mathrm{~L}_{1} \mathrm{I}_{2}} \mathrm{X}_{2}+\frac{6 E \mathrm{I}_{1} 1_{1}}{\mathrm{~L}_{1}^{2}}+\frac{6 E \mathrm{I}_{1}\left(\mathrm{l}_{1}-\mathrm{l}_{2}\right)}{\mathrm{L}_{1}^{2}}$
$\mathrm{m}_{1} \mathrm{~W}_{2}+2 \mathrm{~m}_{2}\left(\mathrm{~V}_{2}+\frac{\mathrm{L}_{3} \mathrm{I}_{2}}{\mathrm{~L}_{2} \mathrm{I}_{3}} \mathrm{~V}_{3}\right)=$
$-\frac{\mathrm{q}_{2} \mathrm{~L}_{2}^{2}}{4} \mathrm{X}_{2}-\frac{\mathrm{q}_{3} \mathrm{~L}_{3}^{2} \mathrm{I}_{2}}{4 \mathrm{~L}_{2} \mathrm{I}_{3}}+\frac{6 \mathrm{EI}_{2}\left(\mathrm{l}_{1}-1_{2}\right)}{\mathrm{L}_{2}^{2}}-\frac{6 \mathrm{EI}_{2}\left(\mathrm{l}_{3}-1_{2}\right)}{\mathrm{L}_{3}^{2}}$
Where
$\mathrm{q}_{\mathrm{i}}=\mathrm{W}_{\mathrm{C}} \mathrm{B}_{\mathrm{C}} \sin \phi$
$X_{i}=\frac{3\left[\tan \left(\frac{L_{i}}{2} \sqrt{\frac{p_{c i}}{E I_{i}}}\right)-\frac{L_{i}}{2} \frac{p_{c i}}{E I_{i}}\right]}{\left(\frac{\mathrm{L}}{2} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)^{3}}$

$$
\begin{align*}
& \mathrm{W}_{\mathrm{i}}=\frac{3}{\left(\frac{\mathrm{~L}_{\mathrm{i}}}{2} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)}\left[\frac{1}{\sin \left(\mathrm{~L}_{\mathrm{i}} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)}-\frac{1}{\left(\mathrm{~L}_{\mathrm{i}} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)}\right]  \tag{26}\\
& \mathrm{V}_{\mathrm{i}}=\frac{3}{\left(\frac{\mathrm{~L}_{\mathrm{i}}}{2} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)}\left[\frac{1}{\left(\mathrm{~L}_{\mathrm{i}} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)}-\frac{1}{\tan \left(\mathrm{~L}_{\mathrm{i}} \sqrt{\frac{\mathrm{p}_{\mathrm{ci}}}{\mathrm{EI}_{\mathrm{i}}}}\right)}\right] \tag{27}
\end{align*}
$$

Where $\mathrm{i}=1,2$ or 3

$$
\begin{align*}
\mathrm{p}_{\mathrm{C} 1} & =\mathrm{WOB}-\frac{\mathrm{W}_{\mathrm{C} 1} \mathrm{~B}_{\mathrm{C}} \mathrm{~L}_{1}}{2} \cos \phi  \tag{28}\\
\mathrm{p}_{\mathrm{C} 2} & =\mathrm{WOB}-\left(\mathrm{W}_{\mathrm{C} 1} \mathrm{~B}_{1} \mathrm{~L}_{1}+\frac{\mathrm{W}_{\mathrm{C} 2} \mathrm{~B}_{\mathrm{C}} \mathrm{~L}_{2}}{2}\right) \cos \phi  \tag{29}\\
\mathrm{p}_{\mathrm{C} 3} & =\mathrm{WOB}-\left(\mathrm{W}_{\mathrm{C} 1} \mathrm{~B}_{1} \mathrm{~L}_{1}+\mathrm{W}_{\mathrm{C} 2} \mathrm{~B}_{\mathrm{C}} \mathrm{~L}_{2}\right. \\
& \left.+\frac{\mathrm{W}_{\mathrm{C} 3} \mathrm{~B}_{\mathrm{C}} \mathrm{~L}_{3}}{2}\right) \cos \phi \tag{30}
\end{align*}
$$

in these equations, $1_{2}$ is hole clearance around 2 nd stabilizer, $\mathrm{p}_{\mathrm{ci}}$ are compressive load on the collars ( $\mathrm{i}=1,2$ and 3 ), $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$ are respectively distance between the bit and first stabilizer and distance between the 1st and 2nd stabilizer, $\mathrm{W}_{\mathrm{C} 1}$ and $\mathrm{W}_{\mathrm{C} 2}$ are respectively unit weight of 1st and 2nd stabilizer.

## III. ARTIFICIAL NEURAL NETWORK

The human beings brain anatomy, considering the thinking process, has always been one of the extreme mysteries to scientists. Researchers have exerted efforts aiming at mechanically and electronically imitating the reactions of
human beings. The invention of computers and the affordability of personal computers with significant processing speeds and huge capacities have encouraged researchers worldwide to tackle problems that previously been out of the scope of their imagination. ANNs are one of these tools that can be considered as problem solving programs modelled on the structure of the human brain where the neural network technology mimics the brains own problem-solving process. The neural network can suit pattern recognition problems, while other problems are best solved with conventional methods. Tracing human's behavior, a neural network takes previously solved examples to build a system of neurons that makes new decisions, classifications, and forecasts [7]. ANNs look for patterns in training sets of data, learn these patterns, and develop an ability to correctly classify new patterns, or to make forecasts and predictions. ANNs excel at problem diagnosis, decision making, prediction, and other classifying problems where pattern recognition is important while precise computational answers are not required.

In a supervised network, the network is shown how to make predictions, classifications, or decisions by giving it a large number of correct classifications or predictions from which it can learn. Back propagation networks (BPNs), general regression neural networks (GRNNs), and probabilistic neural networks (PNNs) are examples of supervised network types.

On the other hand, unsupervised networks can classify a set of training patterns into a specified number of categories through clustering patterns rather than being shown in advance how to categorize. Kohonen networks are unsupervised ones. Three basic entities specify ANNs models: namely, models of neurons themselves; models of the synaptic interconnections and structures; and the training or learning rules for updating the connecting weights. A group of neurons is called a slab. Neurons are also grouped into layers according to their connection to the outside world. Thus, a neuron receiving data from outside the network is in the input layer while that containing the networks prediction is in the output layer. Neurons in between the input and output layers are in the hidden layers. A layer may contain one or more slabs of neurons. Neural network "learns" by adjusting the interconnection weights between layers. Iterations take place until reaching an acceptable tolerance between the output results obtained by the network and the actual, correct output initially fed to the system. Eventually, if the problem can be learned, a stable set of weights adaptively evolves that will produce good answers for all sample decisions or predictions. The real power of ANNs is evident when the trained network is able to produce good results for data that the network has never seen before. Unlike statistical methods, ANNs "discover" relationships in the input data sets through the iterative presentation of the data and the intrinsic mapping characteristics of neural topologies "learning" [7].
Two main phases operate ANNs. First, the training or learning phases which is very time consuming since the data is repeatedly presented to the network, while weights are updated to obtain a desired response. The second phase is the recall or the retrieval phase, where the trained network with frozen weights is applied to data that it has never seen before. To the contrary of the training phase, the retrieval phase can be very fast.
It is worth mentioning that a professional experience is the time to stop training. In other words, training may be insufficient and consequently the network will not learn the patterns, while the training may also be excessive which results in the network learning the noise or memorizing the training patterns rather than generalizing well with new patterns. A practical guide to overcome such problems is to randomly extract about $20 \%$ of the patterns in the training set to be used for cross validation. The error should then be monitored in the training and validation set. When the error in the validation set increase, this is a signal to stop training where the point of best generalization has then been reached. Cross validation is amongst the most powerful methods to stop the training.
Generally, neural networks offer viable solutions when there are large volumes of data available for training. Moreover, ANNs are considered appropriate solution when field or experimental data is available and a problem is difficult, or impossible, to formulate analytically.
In this paper, back propagation neural network (BPN) has been used as a tool for the analysis

## IV. BACK PROPAGATION OF NEURAL NETWORK

ANN is a mathematical system which can model the ability of biological neural networks by interconnecting many simple neurons. The neuron accepts inputs from a single or multiple sources and produces outputs by a simple calculating process with a predetermined non-linear function. A typical threelayered network with an input layer (I), a hidden layer (H) and an output layer $(\mathrm{O})$ is shown in Fig.1. Each layer consists of several neurons and the layers are interconnected by sets of correlation weight. The neurons receive inputs from the initial inputs or the interconnections and produce outputs by transformation using an adequate non-linear transfer function. A common transfer function is the sigmoid function expressed by $f(x)=\left(1+e^{-x}\right)^{-1}$ which a characteristic of has $d f / d x=f(x)[1-f(x)]$. The training process of the neural network is essentially executed through a series of patterns. In the learning process, the interconnection weights are adjusted within the input and output values. Therefore, the primary characteristics of ANN can be presented as follows:
(1) The ability to learn; (2) distributed memory; (3) fault tolerance and (4) parallel operation. With these characteristics, ANN has been widely applied to various areas. Since the principal of ANN has been well documented in the literature, only a brief is given in this section.

BPN, developed by Rumelhart, Hinton and Williams [8], is the most prevalent of the supervised learning models of ANN. BPN used the gradient steepest descent method to correct the weight of the interconnectivity neuron. BPN easily solved the interaction of processing elements by adding hidden layers. In the learning process of BPN, the interconnection weights are adjusted using an error convergence technique to obtain a desired output for a given input.


Fig. 1 Structure of artificial neural network
In general, the error at the output layer in the BPN model propagates backward to the input layer through the hidden layer in the network to obtain the final desired output. The gradient descent method is utilized to calculate the weight of the network and adjusts the weight of interconnections to minimize the output error. The error function at the output neuron is defined as:
$\mathrm{E}=\frac{1}{2} \sum_{\mathrm{k}}\left(\mathrm{T}_{\mathrm{k}}-\mathrm{A}_{\mathrm{k}}\right)^{2}$
In which $T_{k}$ and $A_{k}$ represent the actual and predicted values of output neuron, respectively, and $k$ is the output neuron.

The gradient descent algorithm adapts the weights according to the gradient error, which is given by:
$\Delta \mathrm{W}_{\mathrm{ij}}=-\eta \times \frac{\partial \mathrm{E}}{\partial \mathrm{W}_{\mathrm{ij}}}$
Where $\eta$ is the learning rate and the general from of the $\frac{\partial \mathrm{E}}{\partial \mathrm{W}_{\mathrm{ij}}}$
term is expressed by the following from:
$\frac{\partial \mathrm{E}}{\partial \mathrm{W}_{\mathrm{ij}}}=-\delta_{\mathrm{j}}^{\mathrm{n}} \cdot \mathrm{A}_{\mathrm{i}}^{\mathrm{n}-1}$
Substituting equation (33) into equation (32), we have the gradient error as:

$$
\begin{equation*}
\Delta \mathrm{W}_{\mathrm{ij}}=\eta \cdot \delta_{\mathrm{j}}^{\mathrm{n}} \cdot \mathrm{~A}_{\mathrm{j}}^{\mathrm{n}-1} \tag{34}
\end{equation*}
$$

In which $A_{j}^{n-1}$ is the output value of sub-layer related to the connective weight $\left(\mathrm{W}_{\mathrm{ij}}\right) \cdot \delta_{\mathrm{j}}^{\mathrm{n}}$ is the error signal, which is computed based on whether or not neuron $j$ is in the output layer.

If neuron j is one of the output neurons, then:
$\delta_{j}=\left(T_{j}-Y_{j}\right) . Y_{j} \cdot\left(1-Y_{j}\right)$
If neuron j is a neuron of hidden layer:
$\delta_{\mathrm{j}}=\left[\sum_{\mathrm{j}} \delta_{\mathrm{j}} \cdot\left(\mathrm{W}_{\mathrm{hy}}\right)_{\mathrm{hj}}\right] \cdot \mathrm{H}_{\mathrm{h}} \cdot\left(1-\mathrm{H}_{\mathrm{h}}\right)$
Where $\mathrm{H}_{\mathrm{h}}$ is the value of hidden layer. Finally, the value of weight of the interconnectivity neuron can be expressed as follows:

$$
\begin{equation*}
\mathrm{W}_{\mathrm{ij}}^{\mathrm{m}}=\mathrm{W}_{\mathrm{ij}}^{\mathrm{m}-1}+\Delta \mathrm{W}_{\mathrm{ij}}^{\mathrm{m}}=\mathrm{W}_{\mathrm{ij}}^{\mathrm{m}-1}+\eta \cdot \delta_{\mathrm{j}}^{\mathrm{n}} \cdot \mathrm{~A}_{\mathrm{i}}^{\mathrm{n}-1} \tag{37}
\end{equation*}
$$

To accelerate the convergence of the error in the learning procedure, Jacobs [9] proposed the momentum term with momentum gain, $\alpha$, in equation (37).
$\mathrm{W}_{\mathrm{ij}}^{\mathrm{m}}=\mathrm{W}_{\mathrm{ij}}^{\mathrm{m}-1}+\alpha \cdot \Delta \mathrm{W}_{\mathrm{ij}}^{\mathrm{m}-1}+\eta \cdot \delta_{\mathrm{j}}^{\mathrm{n}} \cdot \mathrm{A}_{\mathrm{i}}^{\mathrm{n}-1}$
in which the value for $\alpha$ is between 0 and 1[10], [11].

## V. Analysis

ANNs have been used by many researchers in the field of mechanical engineering. In the current study, several neural networks have been developed by using the MATLAB [12] software. This software implements different neural network algoritthms such as BPN, PNN (Probabilistic Neuaral Network), GRNN (General Regression Neural Network) and many other network. To use the MATLAB, a set of inputs must be defiend, and a suitable trianing set has to be developed.

The develop networks in this paper are shown in table 1, that consists of many input parameres and 3 different output parametes, each one depending on the BHA materail
properties, hole, weight of bit, stabilizer loction, borehole inclination, direction and contact length.

Each net is composed of four slabs; an input slab, two hidden layer and an output layer. The nodes at input and output layer are determined by the number of predictor and predicted variable.

In this resech, there are 8,12 and 25 nodes in input layers due to the number of input variable, and 1 node in output layer, for similar reasons. There are no rules given to determine the exact number of hidden layers and the number of nodes in hidden layers.

A large number of hidden-layer nodes will lead to an overfit intermediate points, which can slow down the operation of NN. On the other hand an accurate output may not be achived if too few hidden layer nodes are included in the neural network. The number of nodes in the first and scond hidden layers are chosen 6-6, respectivily. Table I presents the topology of the built network.

TABLE I


The activation function in the input and the hidden layers is sigmoid fuction and linear fuction in the output layer.

For a proper working of the neural network a preprocessing of the input and output data is performed. The input values are normalized between -1 and 1 , since the activation fuction is a sigmoid fuction in the input layer. The output values are normalized between 0 and 1 and a linear function in the output layer. Finally the network is ready to be trained. This process is expianed in the section 7.

For the training of the network the MATLAB neural network toolbox is used. The Levenberg-Marquardt algorithm is chosen to perform the training with default values suggested in [12].

The stopping criteria is adjusted, that mean square error should be less then $10^{-9}$ and number of epochs (iterations) should be less then 5000 .

The output values estimated by the NN are compared with the target values calculated with analyticaly method (see
tables II, III and IV). In these table Tr.T and T.T are the training time and test time of the neural network, respectivily.

TABLE II
BIT SIDE FORCE IN SLICK BHA (lbf)

| BIT SIDE FORCE IN SLICK BHA (lbf) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. | $\mathrm{F}_{\mathrm{B}}$ <br> (Analytical) | $\mathrm{F}_{\mathrm{B}}(\mathrm{NN})$ | Tr.T(s) | T.T(s) | Error <br> $(\%)$ |
| 1 | -133.21 | -131.42 | 24.74 | 0.06 | 1.34 |
| 2 | -113.94 | -111.72 | 13.34 | 0.11 | 1.98 |
| 3 | -93.96 | -90.41 | 28.11 | 0.16 | 3.77 |
| 4 | -73.14 | -70.30 | 19.47 | 0.21 | 3.88 |
| 5 | -51.62 | -51.1 | 15.73 | 0.03 | 1 |
| 6 | -29.24 | -27.76 | 11.56 | 0.16 | 5.06 |
| $\mathrm{~F}_{\mathrm{B}}=$ | bit side force, $\mathrm{NN}=$ neural network, Tr.T $=$ training time, T.T $=$ test |  |  |  |  |

time, $\mathrm{s}=$ second, $\mathrm{No}=$ number, $\mathrm{lbf}=$ pound force.

TABLE III
Bit side force in single stabilizer BHA (lbf)

| BIT SIDE FORCE IN SINGLE STABILIZER BHA (lbf) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. | $\mathrm{F}_{\mathrm{B}}$ <br> (Analytical) | $\mathrm{F}_{\mathrm{B}}(\mathrm{NN})$ | Tr.T(s) | T.T(s) | Error <br> $(\%)$ |
| 1 | 3031.11 | 2984.24 | 37.11 | 0.81 | 1.54 |
| 2 | 3044.49 | 2829.17 | 20.01 | 0.79 | 7.07 |
| 3 | 3058.33 | 3000.45 | 42.16 | 0.75 | 1.89 |
| 4 | 3073.14 | 3010.74 | 29.22 | 0.85 | 2.03 |
| 5 | 3087.38 | 3044.23 | 23.59 | 0.77 | 1.39 |
| 6 | 3101.23 | 3012.18 | 17.34 | 0.77 | 2.91 |
| $\mathrm{~F}_{\mathrm{B}}=$ | bit side force, $\mathrm{NN}=$ neural network, Tr.T = training time, T.T = test |  |  |  |  |
| time, $\mathrm{s}=$ second, No = number, lbf= pound force. |  |  |  |  |  |

TABLE IV
BIT SIDE FORCE IN TWO STABILIZER BHA (lbf)

| No. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{F}_{\mathrm{B}}$ <br> $($ Analytical) | $\mathrm{F}_{\mathrm{B}}(\mathrm{NN})$ | Tr.T(s) | T.T(s) | Error <br> $(\%)$ |  |
| 1 | -559.20 | -534.85 | 40.12 | 0.91 | 4.35 |
| 2 | -550.51 | -520.12 | 30.18 | 0.64 | 5.55 |
| 3 | -541.48 | -510.36 | 33.41 | 0.84 | 5.74 |
| 4 | -531.81 | -515.14 | 52.18 | 0.51 | 3.13 |
| 5 | -520.23 | -508.92 | 25.69 | 0.33 | 2.17 |
| 6 | -508.39 | -501.74 | 19.35 | 0.62 | 1.30 |
| $\mathrm{~F}_{\mathrm{B}}=$ bit side force, $\mathrm{NN}=$ neural network, Tr.T $=$ training time, T.T $=$ test |  |  |  |  |  | time, $\mathrm{s}=$ second, $\mathrm{No}=$ number, $\mathrm{lbf}=$ pound force .

## VI. Conclusion

According to the previous studies, it is evident that ANNs perform better than, or as well as, conventional methods used as a basis for comparison in many situations, whereas, they fail to perform well in a few.
The main objective of this paper has been to develop reliable BPNs that can be used to assess bit side force susceptibility. The developed nets can be used as standalone, simple, yet reliable.
The proposed study proved that using BPNs in the problem of bit side force susceptibility produces excellent results via an accurate, yet simple tool. The results show that the BPNs can be estimated the bit side force with high accuracy (the maximum error is below 5 percent). By using this method decreases the time of analysis.

The reduction of the required time was due to the fact that only the bit side force was used in the drill has not the complex equations. The BPN method also has the advantages of computational speed, low cost and ease of use by people with little technical experience.

## References

[1] T. B. Adam, K. K. Millheim, M. E. Chenevert, F. S. Young, "Applied Drilling Engineering", vol. 2, 1991, SPE Tex book Series, Dallas, TX, USA.
[2] S. G. Timoshenko, Theory of Elastic Stability, MCGraw-Hill, New York, 1936.
[3] B. Jiazhi, Bottom Hole Assembly Problems Solved by Beam- Column Theory, SPE Int. Meeting of Petroleum Engineering, Beijing SPE10561, 1986.
[4] H. B. Walker, Down hole assembly design increases ROP., World Oil, 1977, pp.59-65.
[5] K. K. Millheim, M. C. Apostal, The effect of bottom hole assembly dynamics on the trajectory of a bit, J. Pet. Technol., 1981, pp. 23232338.
[6] M. Agawani, S. S. Rahman, E. E. Maidla, "BHA Design Algorithm for Extended Reach Wells", SPE Petroleum Computer Conference, Dallas, TX, USA, 1996.
[7] S. Hayken, "Neural Networks: A Comprehensive Foundation", Macmillan College Publishing Co., New York, 1994.
[8] D. E. Rumelhart, G. E. Hinton, R. J. Williams, "Learning Representations by Back Propagating Error", Nature 323, 1986, pp. 533536.
[9] R. A. Jacobs, Increased rates of convergence through learning rate adaptation, neural networks, vol. 1, 1988, pp. 295-307.
[10] P. D. Wasserman, Neural Computing: Theory and Practice, Van Nostrand Reinhold, New York, 1989.
[11] J. A. Freeman, Simulating Neural Networks, Addison-Wesley Publishing Company, Inc., New York, 1994.
[12] H. Demuth, M. Beale, M. Hagan, Neural Network Toolbox for Use with MATLAB, The Math Works, Inc, 2006

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