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Estimation of Structural Parameters in Time Domain Using One Dimensional Piezo Zirconium Titanium Patch Model

N. Jinesh, K. Shankar

Abstract**—**This article presents a method of using the one dimensional piezo-electric patch on beam model for structural identification. A hybrid element constituted of one dimensional beam element and a PZT sensor is used with reduced material properties. This model is convenient and simple for identification of beams. Accuracy of this element is first verified against a corresponding 3D finite element model (FEM). The structural identification is carried out as an inverse problem whereby parameters are identified by minimizing the deviation between the predicted and measured voltage response of the patch, when subjected to excitation. A non-classical optimization algorithm Particle Swarm Optimization is used to minimize this objective function. The signals are polluted with 5% Gaussian noise to simulate experimental noise. The proposed method is applied on beam structure and identified parameters are stiffness and damping. The model is also validated experimentally.

Keywords**—**Structural identification, PZT patches, inverse problem, particle swarm optimization.

I. Introduction

SYSTEM Identification (SI) or structural identification of structures is typically an inverse problem whereby structural parameters such as stiffness and damping are identified from the input excitation as well as output responses. SI algorithms are generally categorized into frequency domain and time domain algorithms. Piezo Zirconium titanium (PZT) based smart structure today's have vital role in structural health monitoring applications because of its high sensitivity and response.

Kang et al. [1] presented a time domain algorithm to estimate the stiffness and damping parameters of the structure using measured acceleration. The proposed SI algorithm estimates structural parameters through the minimization of an error function defined by the time integral of the least-squared error between the measured and the calculated accelerations. Doebling et al. [2] reviewed literature on detection, location and characterization of damage in structural and mechanical system. The changes in modal frequencies, changes in measured mode shapes and their derivatives, changes in measured flexibility coefficients, structural property (mass, damping and stiffness) matrix updating, non-linear response detection were also discussed. Varghese and Shankar [3] used sub structural identification based on the concept of simultaneous measurement of power flow balance and acceleration matching. It is based on multi objective function with weighted aggregation approach and identified crack parameters with better accuracy than the conventional acceleration matching methods. Now a days PZT has gained significant attention for potential application as sensors for structural health monitoring because of its high reliability, sensitivity and electro mechanical coupling property. Sadilek and Zemcik [4] formulated one dimensional hybrid PZT element with structure using bilinear Lagrangian interpolation polynomial for electric potential and is carried out for modal and frequency response analysis. Bendary and Raid [5] developed one dimensional integrated beam element based on the concept of hermit cubic and Lagrangian interpolation function and carried out static and dynamic analysis.

In this paper, concept of Structural Identification(SSI) with one dimensional hybrid PZT patch model has been done for the stiffness snd damping identification. The objective function is made up of mean square of the deviation between measured and estimated voltage from the PZT Patches.

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II. Constitutive Equations

The piezo electric materials are used to transform the mechanical displacement into an electrical field (voltage potential) in which case the piezoelectric material acts as a sensor (direct effect), and its converse effect acts as an actuator. The constitutive equations for the transversely isotropic piezo electric medium which define the interaction between the stress (σ), strain (ε), electric displacement (D), and electric field (E) in the form

$$\sigma_j = C_{jk}\varepsilon_k - e_{jm}E_m \tag{1}$$

$$D_l = e_{lj}\varepsilon_k + \epsilon_{lm}E_m \tag{2}$$

where C_{jk}, ϵ_{lm} and e_{lj} $(j,k = 1, ..., 6$ and $l,m = 1,..,3)$ are elastic, dielectric and piezo electric coupling coefficient respectively.

III. One Dimensional Beam Geometry

The Euler-Bernoulli beam theory is applied for one dimensional beam with piezo electric patch, which neglects the shear effect. The model is assumed to be plane stress and width in the y direction is stress free. Therefore, it is possible to set $\sigma_y = \sigma_z = \tau_{xy} = \tau_{xz} = \tau_{yz} = \gamma_{xy} = \gamma_{xz} = \gamma_{yz} = 0$

while $\varepsilon_y \neq 0$; $\varepsilon_z \neq 0$. The polarization axis z is aligned with the thickness direction of the beam, thus only D_z is taken and thus for electric field $E_x = E_y = 0$. Applying these conditions, it is reduced in the form

$$\begin{bmatrix} \sigma_x \\ D_z \end{bmatrix} = \begin{bmatrix} \hat{C} & -\hat{e} \\ \hat{e} & \hat{\epsilon} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ E_z \end{bmatrix} \tag{3}$$

where $\hat{C} = Q_{11} - \frac{(Q_{12})^2}{Q_{22}}$ and $Q_{ij} = C_{ij} - (\frac{C_{i3}C_{j3}}{C_{33}})(i,j = 1,2)$; $\hat{e} = \bar{e_{31}} - \bar{e_{32}}(\frac{Q_{12}}{Q_{22}})$ and $\bar{e_{3i}} = e_{3i} - e_{33}(\frac{C_{13}}{C_{33}})(i = 1,2)$; $\hat{\epsilon} = \bar{\epsilon}_{33} + \frac{(\bar{e}_{32})^2}{Q_{22}}$ and $\bar{\epsilon}_{33} = \epsilon_{33} + \frac{(e_{32})^2}{C_{33}}$. Here, $\hat{C}$, $\hat{\epsilon}$ and $\hat{e}$ are reduced elastic, dielectric and piezo electric coupling coefficient respectively and this calculated values are shown in the Table I. This reduced property is used for further numerical study of one dimensional beam with PZT patch in MATLAB ®.

TABLE I
MATERIAL PROPERTIES OF THE BEAM

Material	Properties
Aluminium	E=71 GPa, $\nu = 0.3$, $\rho = 2210$ kg m^{-3}
PZT 5H [9]	$C_{11} = C_{22}$= 126 GPa ,C_{12}=79.5 Gpa
	$C_{13} = C_{23}$= 84.1 GPa ,C_{33}=117 Gpa
	$C_{44} = C_{55}$= 23 GPa ,C_{66}=23.25 Gpa
	$e_{31} = e_{32} = -6.5$ C m^{-2}, $e_{33} = 23.3$ C m^{-2}
	$e_{15} = e_{24} = 17$ C m^{-2}
	$\epsilon_{11} = \epsilon_{22} = 1.503 \times 10^{-8}$F m^{-1}
	$\epsilon_{33} = 1.3 \times 10^{-8}$F m^{-1}
	$\rho = 7500$ kg m^{-3}
Reduced Properties	$\hat{C} = 60.013$ GPa, $\hat{e} = -16.4921$ C m^{-2}
	$\hat{\epsilon} = 2.5885 \times 10^{-8}$F m^{-1}

IV. Analytical and FEM Formulation

The beam element is based on Euler-Bernoulli theory and element has two nodes. The two independent polynomial is used for interpolation of mechanical and electrical field variable. First hermit cubic polynomial is used for the interpolation of mechanical quantities of vertical displacement (w) and rotation (θ) as shown in Fig. 1. Let the vertical displacement (w) is approximated across the length as

$$w(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 = [N_w]\{w^e\} \tag{4}$$

where N_w is the shape interpolation functions and structural nodal degree of freedom per element is arranged as

$$w^e = [w_1, \theta_1, w_2, \theta_2]^T \tag{5}$$

$$w(x) = N_1 w_1 + N_2 \theta_1 + N_3 w_2 + N_4 \theta_2 = \sum_{j=1}^{4} N_j w_j \tag{6}$$

The axial displacement can be written as

$$u(x,z) = -z\theta(x) = -z\frac{\partial w}{\partial x} \tag{7}$$

Thus bending strain is

$$\varepsilon(x,z) = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} = [B_w]\{w^e\} \tag{8}$$

where $[B_w]$ is the strain-displacement matrix consisting of derivatives of shape functions.

The electric potential is $\phi(x,z)$considered as a function of the thickness and the length of the beam. Hence, let Langrangian bi-linear function is estimated for the interpolation as

$$\phi(x,z) = a_4 + a_5 x + a_6 z + a_7 xz = [\Phi_\phi]\{\phi^e\} \tag{9}$$

As shown in Fig. 1, ϕ andψ are lower and upper electrical potential of PZT surface. The electrical nodal degree of freedom per element can be ordered as

$$\{\phi^e\} = [\phi_1, \psi_1, \phi_2, \psi_2]^T \tag{10}$$

Thus 9 can be written as

$$\phi(x,z) = \Phi_1\phi_1 + \Phi_2\psi_1 + \Phi_3\phi_2 + \Phi_4\psi_2 = \sum_{j=1}^{4} \Phi_j \phi_j \tag{11}$$

The electric field $E(x,z)$ and electric potential(ϕ) can be related as follows

$$E(x,z) = \frac{\partial \phi(x,z)}{\partial z} \tag{12}$$

Thus it can be written as follows

$$E(x,z) = [B_\phi]\{\phi^e\} \tag{13}$$

where $[B_\phi]$ is the Electrical field-potential matrix consisting of derivatives of shape functions. Here, homogeneous electrical boundary condition is imposed on the bottom surface of PZT patch to eliminate rigid body modes *i.e.* lower surface is grounded with $\phi = 0\ V$, while the upper surface was left open.

V. Evaluation of Elemental Matrices

Applying variational principle to potential, kinetic energy and external forces (mechanical and electrical loading)), it must be satisfied for any arbitrary variation of the displacements and electrical potentials and thus equation of motion of elemental matrices can be represented as

$$\begin{bmatrix} [M^e_{ww}] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{w}^e\} \\ \{\ddot{\phi}^e\} \end{Bmatrix} + \begin{bmatrix} [K^e_{ww}] & [K^e_{w\phi}] \\ [K^e_{w\phi}]^T & [K^e_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} \{w^e\} \\ \{\phi^e\} \end{Bmatrix} = \begin{Bmatrix} \{F^e\} \\ \{Q^e\} \end{Bmatrix} \tag{14}$$

Equation (14) is the elemental equilibrium in the discretized form, where $[M^e_{ww}]$ is the mass matrix, $[K^e_{ww}]$ is the stiffness matrix corresponding to the mechanical degree of freedom $[K^e_{w\phi}]$ is the stiffness matrix due to electromechanical coupling, $[K^e_{\phi\phi}]$ is the stiffness matrix due to the electrical degrees of freedom alone, $\{F^e\}$ is the mechanical load vector and $\{Q^e\}$ is the electrical charge load vector. The size of each element matrices are [4x4] and are given by

$$\begin{aligned} [M^e_{ww}] &= \int_V [N_w]^T \rho [N_w] dV \\ [K^e_{ww}] &= \int_V [B_w]^T \hat{C} [B_w] dV \\ [K^e_{w\phi}] &= \int_V [B_w]^T \hat{e} [B_\phi] dV \\ [K^e_{\phi\phi}] &= \int_V [B_\phi]^T \hat{\epsilon} [B_\phi] dV \end{aligned} \tag{15}$$

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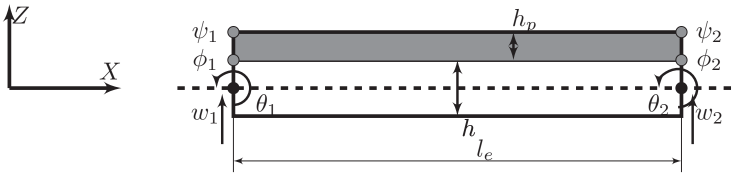

Fig. 1 One dimensional beam element: PZT and Supporting Structure sharing Common Nodes

In general, all structures are lightly damped. Thus, adding an artificial linear viscous damping to (14)

$$\begin{bmatrix}[M^e_{ww}] & [0] \\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\ddot{w}^e\} \\ \{\ddot{\phi}^e\}\end{Bmatrix}+\begin{bmatrix}[C^e_{ww}] & [0] \\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\dot{w}^e\} \\ \{\dot{\phi}^e\}\end{Bmatrix}+ \begin{bmatrix}[K^e_{ww}] & [K^e_{w\phi}] \\ [K^e_{w\phi}]^T & [K^e_{\phi\phi}]\end{bmatrix}\begin{Bmatrix}\{w^e\} \\ \{\phi^e\}\end{Bmatrix}=\begin{Bmatrix}\{F^e\} \\ \{Q^e\}\end{Bmatrix} \quad (16)$$

where the damping matrix $[C^e_{ww}]$ is defined as a proportional damping *i.e.*

$$[C^e_{ww}] = a[M^e_{ww}] + b[K^e_{ww}] \quad (17)$$

where a and b are Rayleigh's damping coefficients. The matrices of equations are then assembled to obtain the global dynamic system equation:

$$\{\phi\} = [K_{\phi\phi}]^{-1}\{Q\} - [K_{\phi\phi}]^{-1}[K_{w\phi}]^T\{w\} \quad (18)$$

The above equation is substituted in the fist part of the (18) and is written as

$$[M_{ww}]\{\ddot{w}\} + [C_{ww}]\{\dot{w}\} + [K_{ww}]^*\{w\} = \{F\}^* \quad (19)$$

where

$$[K_{ww}]^* = [K_{ww}] - [K_{w\phi}][K_{\phi\phi}][K_{w\phi}]^T, \\ \{F\}^* = \{F\} - [K_{w\phi}][K_{\phi\phi}]^{-1}\{Q\} \quad (20)$$

For the sensor problem, displacement histories are obtained by solving (19)and substituted in (18), and voltage vector across the sensor patch is obtained.

VI. Numerical Validation of One Dimensional Finite Element Model

A computer code in MATLAB® environment has been developed to study the dynamic analysis of PZT sensor bonded to the top surface of cantilever beam as shown in Fig. 2. The dimension of the rectangular beam is taken from Sandesh and Shankar [7] and PZT sensor dimensions are($50\times 25.4\times 1\ \mathrm{mm}^3$). The material properties of host structure (Aluminium) and PZT sensor(reduced properties) is used as shown in the Table I. As no commercial one dimensional PZT element is available, the present formulation is validated using ANSYS® 3D simulation. For ANSYS®, 3D simulation a mesh of SOLID 186 element is used for the host structure and a mesh of SOLID 226 element is used for sensor(PZT 5H) patch.

In time domain analysis, numerically simulated response histories of all DOFs of the model is calculated in terms of displacement, velocity, acceleration and voltage using Newmark-Beta method. An impulsive force of 2 N in time

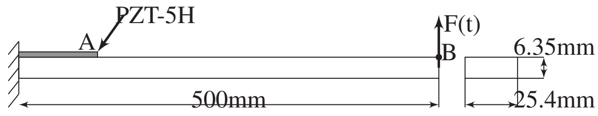

Fig. 2 Cantilever beam with PZT patch

period of 0.01 s in time step of 0.001 is applied in the free end of a cantilever beam. The effect of the damping was accounted by Rayleighs damping with modal damping ratio of 1% at its first two modes. The maximum voltage potential (near the clamped end) time history at PZT patch A are plotted in Fig. 3. As seen from the plot, the present MATLAB® code gives precise predictions as given by the ANSYS® simulation. From this plot, two well defined peaks are identified as first two modal frequencies. The result compares well with ANSYS® 3D simulation.

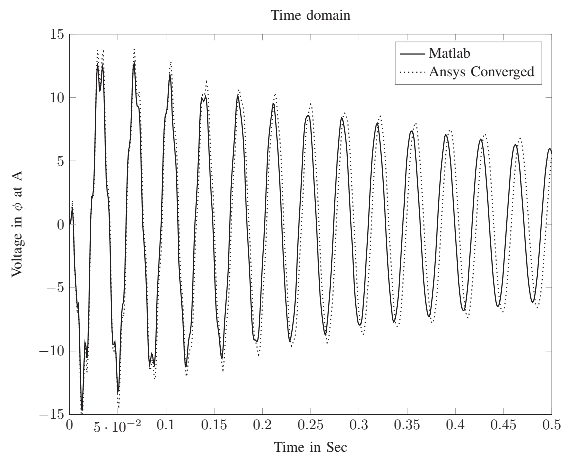

Fig. 3 Comparison of maximum Voltage time histories in Matlab and ANSYS

VII. Particle Swarm Optimization (PSO) Algorithm

A heuristic optimization technique referred to as PSO is used here which mimics the social behavior of swarms. It was first proposed by James and Ebehart [6]. It imitates the social behaviour of a swarm of birds. Each bird will tend to follow the general swarm direction in search of the target (food), but it has a component of its own intelligence and memory (*i.e.* local search) which influences its action. Each bird is visualized as a 'particle' which approaches the target (*i.e.* the global optima) with a 'velocity'. The number of particles (*i.e.* population) and their initial random positions are specified. As the particles progress to the global optima through many generations, their current position is updated using two parameters G_{best} which represent the historically best co-ordinate of all the particles in the population and $P_{best,i}$ the historically best co-ordinate of the i^{th} particle. The equations giving the velocity v and position x for the i^{th} particle in the $k+1$ generation are given by

$$v_i(k+1) = \varphi(k)v_i(k) + \alpha_1[\gamma_{1i}(P_{best,i} - x_i(k))] + \alpha_2[\gamma_{2i}[(G_{best} - x_i(k))] \quad (21)$$

$$x_i(k+1) = x_i(k) + v_i(k+1) \quad (22)$$

where i is the particle index, k the discrete time index, v the velocity of the i^{th} particle,x the position of the i^{th} particle/present solution. Here γ_1 and γ_2 represents two random numbers between 0 and 1, φ is an inertia term uniformly decreasing from 0.9 to 0.4 with generations and α_1 and α_2 are two acceleration constants set to two [8]. Several studies have pointed out the superiority of PSO algorithm over the more conventional heuristic algorithms such as Genetic Algorithm for inverse problem applications (GA) [7].

VIII. Structural Parameter Identification Using PZT Patch

The time domain based approach is used and voltage history is used as the main response quantity in the identification procedure. A few experimentally measured voltage potential responses are measured from PZT patches. The estimated voltage potential is obtained from mathematical model using (18). For exact identification, it has to match with experimentally measured responses. In this method, experimental responses are simulated from a known numerical model and polluted with Gaussian noise of zero mean and a certain standard deviation for practicality. Using Particle Swarm Optimization (PSO) algorithm the following fitness (objective) function is minimized, which is the sum of squares of deviations between the measured and estimated voltage.

$$f = \frac{\sum_{i=1}^{M}\sum_{j=1}^{L}|\phi^m(i,j) - \phi^e(i,j)|^2}{ML} \quad (23)$$

Here ϕ^m and ϕ^e denote measured and estimated voltage responses for fitness (f) estimation, M is the number of measurement sensors used and L is the number of time steps. Ideally, it must be minimized to zero, but usually it approaches a value close to zero. In order to validate the proposed method, numerical studies are carried out on beam structure. Experimentally measured responses are numerically simulated from fully defined model by MATLAB® using Newmarks constant acceleration scheme. The structure is excited by a harmonic force at mid point and responses are measured at PZT patches. The mass of the structure is assumed to be known a priori and the stiffness parameters are the unknowns for the first two cases. These are estimated using the inverse formulation with single objective approach. In order to simulate the effect of noise in experiments, Gaussian random noise level of 5% and zero mean is added to all the measured signals. In this numerical study two different PZT patch lengths are investigated and are PZT:5%, PZT:10% and PZT:20%. They respectively represent the length of the patch expressed as percentage of beam length. The width of PZT is same as host structure and a constant thickness of 1 mm is used for this study.

A. Fixed-Fixed Beam Structure

The proposed method is applied to fixed-fixed beam as shown in Fig. 4. Model geometry and properties of host structure(Aluminium) are taken from [7]. In this study two PZT patches are bonded on either side of the structure and the beam is divided into 5 Euler finite elements. The Flexural rigidity (EI) of each element is 38.48 N.m^2. The first and second natural frequency of modes of vibration are 148.46 Hz and 415.06 Hz respectively. Rayleigh damping with the modal damping ratio of 3% is used for the first two modes of vibration. Rayleigh damping with the modal damping ratio of 3% is used for the first two modes of vibration. The beam is subjected to harmonic excitation of $F(t) = 2.3sin(2\pi 100t)$ N in the vertical (upward) direction at node 4. The displacement, velocity and acceleration time history data are calculated for each nodal point using Newmarks method with constant time step of 0.001 sec. Using the displacement response history, voltage responses are measured through two PZT patches.

Here, five unknown stiffness and two damping coefficients are obtained by PSO in the range of $\pm 50\%$ of actual value. In this PSO parameters are set to 100 particles (swarm size) and 200 generations. First, in this study we compare the identification of structural parameters by voltage matching with different lengths of PZT. The identified results are shown in Tables II and III respectively. From the tables, it can be seen that for noise free case Mean Absolute Error (MAE) of PZT-10% *i.e.*$(50 \times 25.4 \times 1 \text{ mm}^3$ PZT) is 0.123% and whereas PZT-5% *i.e.*$(25 \times 25.4 \times 1 \text{ mm}^3$ PZT) is 0.146%. The percentage increase of error compared with these two case is only 34.5%. Similarly for 5% noise case, it is 0.159% and 0.213% respectively, and percentage change of error of these two case is 30% only. The percentage of error in identifying damping constants α and β of PZT-10% are 0.035% and 0.08% for noise free case and is 0.109% and 0.242% for 5% noise respectively. Similarly for PZT-5% the results are 0.044% and 0.66% for no noise and 0.06% and 0.564% for 5% noisy case respectively. The objective function convergence study in identifying stiffness and damping coefficient for beam structure without and 5% noise contamination is presented in the Fig. 5. The figure shows convergence of voltage matching (ϕ) using different lengths of PZT sensor and It is clear that as length of PZT increases convergence is faster and thus identification error can be reduced. In the Fig. 5 and 6 shown, objective function of PZT:20% has minimized better than PZT:5% and PZT:10% and is most accurate in identification. However, it is not required to use such large patch length. PZT:5% error in identification is only 0.167% which is sufficient for practical use. As compared with pure acceleration matching, proposed method even with PZT:5% shows better convergence and thus identifies the parameters with better accuracy.

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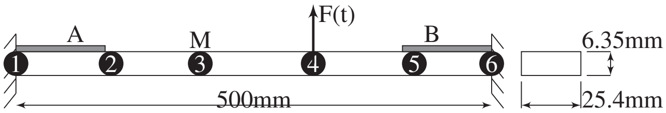

Fig. 4 Fixed-Fixed beam with PZT patches

TABLE II
IDENTIFIED STIFFNESS AND DAMPING PARAMETERS WITH PZT: 10%

Element	Exact EI $N.m^2$	Voltage Matching (V) Identified EI $N.m^2$(%error)	Voltage Matching (V) Identified EI $N.m^2$(%error)
		Noise Free	5% Noise
1	38.619	38.583(-0.093)	38.565(0.138)
2	38.479	38.539(-0.154)	38.549(-0.181)
3	38.479	38.413(0.173)	38.54(-0.157)
4	38.479	38.442(0.097)	38.522(-0.112)
5	38.619	38.658(-0.100)	38.699(0.208)
Mean absolute error (MAE)(%)		0.123	0.159
	Exact	Identified(%error)	
Damping α	41.12	0.034	-0.53
constants β	1.702×10^{-5}	-0.08	0.514

TABLE III
IDENTIFIED STIFFNESS AND DAMPING PARAMETERS WITH PZT: 5%

Element	Exact EI $N.m^2$	Voltage Matching (V) Identified EI $N.m^2$(%error)	Voltage Matching (V) Identified EI $N.m^2$(%error)
		Noise Free	5% Noise
1	38.619	38.656(-0.095)	38.559(0.155)
2	38.479	38.428(0.134)	38.539(-0.154)
3	38.479	38.465(0.038)	38.402(0.202)
4	38.479	38.442(-0.227)	38.578(-0.255)
5	38.619	38.648(-0.236)	38.517(0.264)
Mean absolute error (MAE) (%)		0.146	0.213
	Exact	Identified (%error)	
Damping α	41.12	0.045	-0.66
constants β	1.702×10^{-5}	-0.06	0.564

IX. EXPERIMENTAL WORK: FIXED-FIXED BEAM WITH PZT STRUCTURE

A fixed-beam made of acrylic material with dimensions $(450 \times 25 \times 12)$ mm is used for the experimental study. Here two PZT patch of dimensions $(25 \times 25 \times 1)$ mm each are bonded at the fixed ends of the structure as shown in Fig. 7. The modulus of Elasticity (E) was estimated to be 3.9 GPa from a simple bending test and the density was measured to be 1190 kg/m^3. The actual flexural rigidity (EI) of the beam is 14.04 N.m^2. The damping ratio (ζ) was calculated from a simple free vibration decay test using logarithmic decrement and estimated as 8%. The natural frequencies for the first two modes of the structure are calculated from the frequency domain as 110 Hz and 306 Hz. Assuming Rayleigh's proportional damping model, exact values of damping constants α and β are calculated as 81.64 and 6.12×10^{-5} respectively for two modes. The beam was divided into five element as shown in Fig. 4.

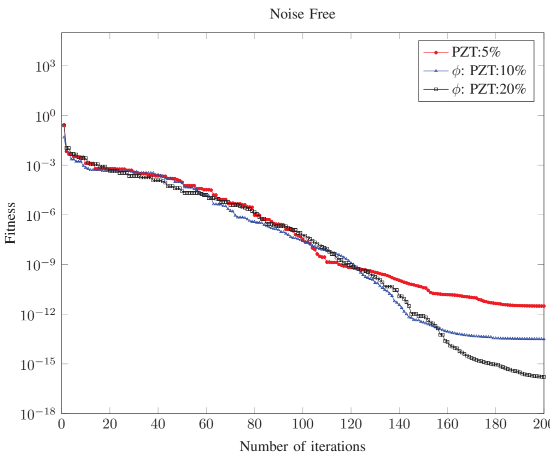

Fig. 5 Convergence plot for Noise-free case

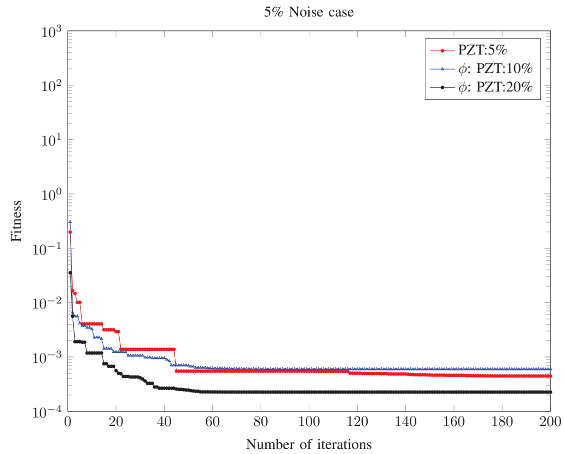

Fig. 6 Convergence plot for 5% Noise case

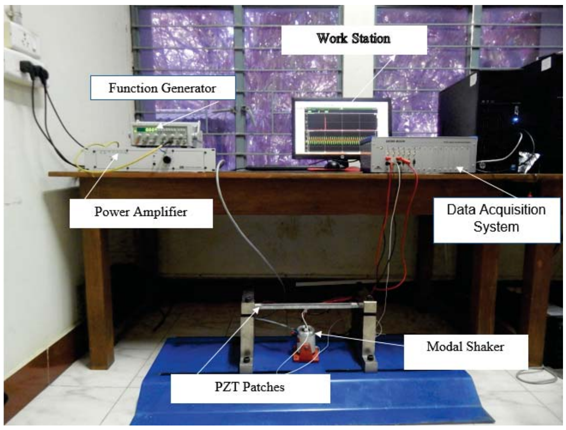

Fig. 7 Experimental set up of Fixed-Fixed beam with PZT Patch

The beam is excited by a sinusoidal force of $2.8sin(2\pi \times 60.7t)$ N at the middle of the structure by a LDS permanent magnet 20 N modal shaker with a maximum displacement

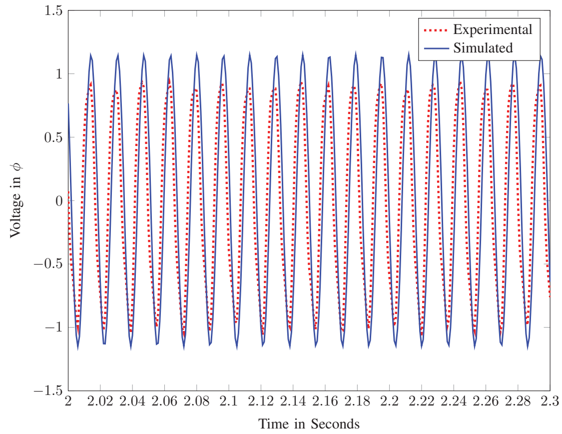

Fig. 8 Comparison of Experimental and Simulated voltage response at a PZT patch

of 5 mm with an operating frequency range of 5 Hz-13 kHz. The applied force is measured by using a KISTLER force transducer and is acquired with a sampling frequency of 1000 Hz using DEWE 43 DAQ system. The dynamic voltage response is measured through two piezo patches and it is sampled at the rate of 1000 Hz. From the acquired data, a portion of data length 5 sec is considered for parameter identification. The unknown structural parameter are identified by minimizing the mean square error between the measured and predicted response using 23. The PSO parameters are set to 100 particles (swarm size) and 500 generations. The search range for structural parameters is set as $\pm 50\%$ of actual value. The identified parameters are shown in Table IV. The structural stiffness parameters are identified with mean absolute error 5.49% and damping parameters are identified with maximum absolute error of 10.8%. The mean CPU time required for convergence is 125 sec. Fig. 8 shows the matching between the experimental voltage of the left patch and numerically predicted voltage at successful identification.

TABLE IV
EXPERIMENTALLY IDENTIFIED PARAMETERS OF FIXED-FIXED BEAM

Parameter	Exact $N.m^2$	Identified $N.m^2$	% of Error
EI_1	14.1656	15.24	-7.58
EI_2	14.04	13.35	4.91
EI_3	14.04	13.56	3.42
EI_4	14.04	14.96	-6.55
EI_5	14.1656	13.46	4.98
Mean Absolute Error (MAE) (%)			5.49
α	81.64	90.5	-10.8
β	6.12×10^{-5}	6.65×10^{-5}	-8.6

X. Conclusions

In this article, structural parameter identification based on voltage matching by using PZT patch sensor is presented. Mathematical model of the PZT sensor with host structure is developed on the basis of one dimensional geometry and reduced material property. The structural identification is done by the minimization of mean square error of the deviation between the measured and predicted voltage responses with unknown structural parameters as optimization variables using PSO. The effect of different PZT length is investigated with numerical example and convergence of the fitness function of those case is studied. The result shows that, identification using PZT: 5% (smallest patch length under study) is even sufficient for accurate structural parameter identification and it is successfully verified experimentally.

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