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Estimation of Natural Frequency of the Bearing System under Periodic Force Based on Principal of Hydrodynamic Mass of Fluid

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***Abstract*—**Estimation of natural frequency of structures is very important and isn't usually calculated simply and sometimes complicated. Lack of knowledge about that caused hard damage and hazardous effects.

In this paper, with using from two different models in FEM method and based on hydrodynamic mass of fluids, natural frequency of an especial bearing (Fig. 1) in an electric field (or, a periodic force) is calculated in different stiffness and different geometric. In final, the results of two models and analytical solution are compared.

***Keywords*—**Natural frequency of the bearing, Hydrodynamic mass of fluid method.

I. Introduction

ESTIMATION of natural frequency of structures is very important and isn't usually calculated simply and sometimes complicated. Lack of knowledge about that caused hard damage and hazardous effects. For example famous bridge of Sanfransisco is explored because of having the same frequency as the wind!

Moreover because of not having a smooth manner for estimating natural frequency of structures, Calculating natural frequency is very complicated usually especially when the structure subject to interaction between a fluid (oil) and a solid. In these cases, the FEM methods are powerful methods because of having useful concept for analyzing complicated problems with fluid and solid interaction. The FSI (Fluid-Solid interaction) is widely investigated by many scientists. When the fluids move in a narrow region, the effects of hydrodynamic mass are usually higher than solid mass effects, although the immersed body is higher density than fluid [1].

Because of displacing of the fluid when moving a body into the fluid, a gradient pressure is produced in the fluid. Therefore, resultant force on the solid body is introduced from result of these forces.

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In this paper, this is assumed that the fluid is incompressible and frictionless so the force of fluid is related to relative acceleration of the solid body (hydrodynamic mass) essentially. The hydrodynamic concept is described by Lamb, Stokes, Batton & Birkhoff and others investigators.

In this paper, by using from two different models in FEM method and based on hydrodynamic mass of fluids, natural frequency of an especial bearing (Fig. 1) in an electric field (or, a periodic force) is calculated in different stiffness and different geometric. The results presented by Lamb for hydrodynamic mass is used in this paper [2].

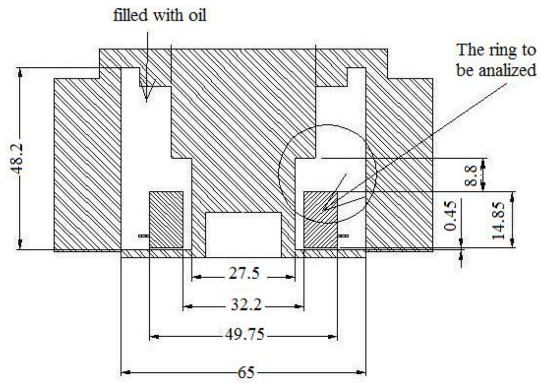

Fig. 1 The bearing to be analyzed in this paper

II. Theory

For the ideal situation, two long eccentric cylinders are considered with a fluid (oil) between them. The outer radius of inner cylinder and the inner radius of outer cylinder consider *a* and *b* respectively.

The potential of velocity φ is:

$$V_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} \quad ; \qquad V_r = -\frac{\partial \phi}{\partial r} \tag{1}$$

With assuming incompressible and frictionless, the fluid is ideal and irrigational flow, therefore the boundary conditions are

$$\frac{\partial \phi}{\partial r} = \dot{x}_1 \cos\theta \qquad at \qquad r=a \tag{2}$$

$$\frac{\partial \phi}{\partial r} = \dot{x}_2 \cos\theta \qquad at \qquad r=b \tag{3}$$

And continuity equation is

$$\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right)+\frac{1}{r}\frac{\partial^2 \phi}{\partial \theta^2}=0 \tag{4}$$

A solution for □ in this condition is

$$\phi = f(r)\cos\theta \tag{5}$$

With solving (4) and (5) we have:

$$r^2 f'' + r f' - f = 0 \tag{6}$$

With solving (6) the velocities is found

$$V_r = \left(\frac{B}{r^2} - A\right)\cos\theta \tag{7}$$

$$V_\theta = \left(\frac{B}{r^2} + A\right)\sin\theta \tag{8}$$

Where constants A and B are

$$A = \frac{\dot{x}_1 a^2 - \dot{x}_2 b^2}{b^2 - a^2} \tag{9}$$

$$B = \frac{b^2 a^2}{b^2 - a^2}(\dot{x}_1 - \dot{x}_2) \tag{10}$$

Since the velocity of each particle in generalized coordinate is defined separately so Lagrange's equation can be written as

$$F_{fi} = -\frac{d}{dt}\frac{\partial T_f}{\partial \dot{x}_i} + \frac{\partial T_f}{\partial x_i} \tag{11}$$

Where x_i and T_f are generalized position and kinetic energy of the fluid and x is the translational motion of i^{th} cylinder and F_{fi} is the reaction force on that. If displacement of cylinder is small to compare with fluid thickness, last term in (11) can be neglected. Therefore,

$$F_{fi} = -\frac{d}{dt}\frac{\partial T_f}{\partial \dot{x}_i} \tag{12}$$

And the kinetic energy is:

$$T_f = \int_a^b \int_0^{2\pi} \frac{1}{2}\rho r L dr d\theta (V_r^2 + V_\theta^2) \tag{13}$$

From equations (7), (8), (12) and (13):

$$F_{f1} = -M_H \ddot{x}_1 + (M_1 + M_H)\ddot{x}_2 \tag{14}$$

$$F_{f2} = (M_1 + M_H)\ddot{x}_1 - (M_1 + M_2 + M_H)\ddot{x}_2 \tag{15}$$

Where F_{f1} and F_{f2} are the reaction force of fluid on the inner and outer cylinder respectively, and M_1 $(M_1=\pi a^2 L\rho)$ and M_2 $(M_2=\pi b^2 L\rho)$ are the mass displaced by inner cylinder and filling mass of outer cylinder in absence of inner cylinder respectively.

III. Calculation of Forces on Two Bodies

The same of Lamb model, when the motion of fluid is introduced by the motion of floated body, the fluid energy is a 2 degree function as:

$$2T_f = A_{11}\dot{x}_1^2 + A_{22}\dot{x}_2^2 + \ldots + 2A_{12}\dot{x}_1\dot{x}_2 + \ldots \tag{16}$$

Or (in matrix form)

$$2T_f = \dot{x}^T A \dot{x} \tag{17}$$

In case having two bodies

$$2T_f = A_{11}\dot{x}_1^2 + 2A_{12}\dot{x}_1\dot{x}_2 + A_{22}\dot{x}_2^2 \tag{18}$$

From (12) and (18):

$$F_{f1} = -A_{11}\ddot{x}_1 - A_{12}\ddot{x}_2 \tag{19}$$

$$F_{f2} = -A_{12}\ddot{x}_1 - A_{22}\ddot{x}_2 \tag{20}$$

Where F_{f1} and F_{f2} are reaction forces on bodies 1 and 2. With assuming the body 2 surround body 2, $\ddot{x}_1 = \ddot{x}_2$ and the fluid acceleration in each point of the incompressible fluid equals to $\ddot{x}_2$ and in result pressure gradient is in the fluid.

$$-\frac{\partial P}{\partial x} = \rho.\ddot{x}_2 \tag{21}$$

This pressure gradient produces the forces:

$$F_{f1} = -(A_{11} + A_{12})\ddot{x}_2 = M_1\ddot{x}_2 \tag{22}$$

$$F_{f2} = -(A_{12} + A_{22})\ddot{x}_2 = -M_2\ddot{x}_2 \tag{23}$$

And

$$A_{11} + A_{12} = -M_1 \tag{24}$$

$$A_{12} + A_{22} = M_2 \tag{25}$$

Equations (24) and (25) are two equations for three unknowns constant. With assuming $\ddot{x}_2 = 0$ (body 2 is fixed), from (19):

$$F_{f1} = -A_{11}\ddot{x}_1 = -M_H\ddot{x}_1 \tag{26}$$

The equation (26) defines MH and with assuming that the velocity of body 1 is known, can be calculated and then from continuity flow, the velocity of fluid can be estimated.

IV. Dynamically Coupled Fluid Element (FLUID 38)

This element is used for showing of relation of two points of a structure that are separated with the fluid. The center points are cylinder axis and fluid fills space between them. Each element has two degrees of freedom in each node. For example motion in x and y directions and cylinder axis lies in z direction (see Fig. 1). This element is used in dynamic analyzed in this paper.

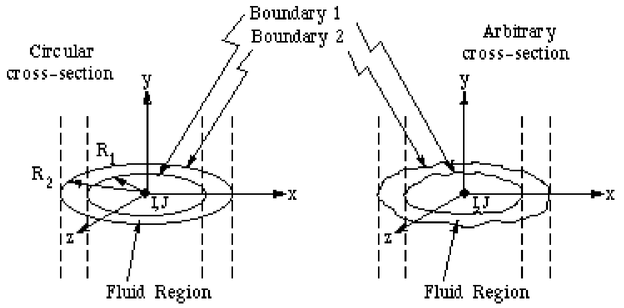

Fig. 2 FLUID38 dynamic fluid coupling element

V. Asymmetric Harmonic Fluid Element (FLUID 81)

This element is a modification of the PLANE25 element for modeling a fluid that is surrounded in regions that don't having net flow. This element has 4 nodes with 3 DOF (in x, y and z directions for each node) and as named is used in axial symmetric problems. Loading in this element can be nonsymmetric. (see Fig. 3).

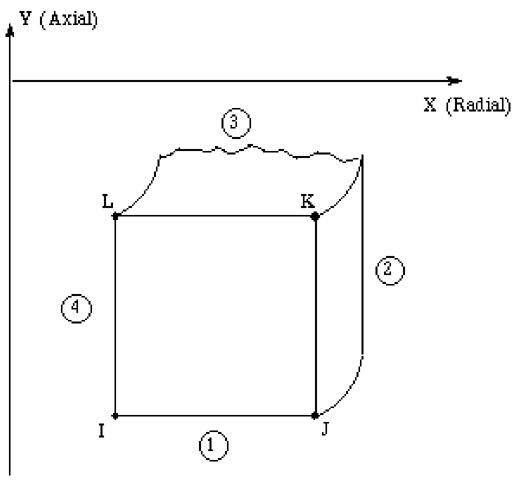

Fig. 3 FLUID81 Axisymmetric - Harmonic contained fluid element

VI. Principal of Operation: Servo Fluid External Pressure Bearing (SFEPB)

In this bearing, oil layer is introduced by external pressure in system. The stiffness of the bearing is introduced by the pressure gradient in different regions of the bearing. Therefore, when the more free motion of the shaft is mean lower stiffness of the bearing. As known journal bearing are two kind hydrodynamic and hydrostatic bearings. The hydrodynamic one is working according to dynamic principles and making fluid film between bearing and shaft. Another one is working according to static principles and is using external pressure for working. The SFEPB works with two concepts assigned, but they modified the design.

VII. Calculation of Stiffness and Damping Coefficients

Stiffness and damping coefficients in hydrodynamic bearing are [5]:

$$K_{BD} = \frac{\eta \Omega d L^3}{c^3} \frac{K_1 \varepsilon}{\left(1-\varepsilon^2\right)^{5/2}} \left(N/M, Ib/in\right) \tag{27}$$

$$D_{BD} = \frac{\eta d L^3}{c^3} \frac{D_1}{\left(1-\varepsilon^2\right)^{3/2}} \left(\frac{N.\sec}{m}, \frac{Ib.\sec}{in}\right) \tag{28}$$

And for the hydrostatic bearing

$$K_{BS} = \frac{2P_S}{cL} \frac{d}{L-a} C_0 \tag{29}$$

Where

K_{BD} , D_{BD} = Radial stiffness and damping of bearing
K_1, D_1 = *Constants*
c, d = Diametric clearance and journal diameter respectively
L = Bearing length
η, ε = Dynamic viscosity and eccentricity ratio
Ω = Rotational velocity
a = Kafshak length
C_0 = Dimensionless stiffness that is defined as

$$C_0 = \frac{7.65\beta(1-\beta)}{2-\beta+\gamma(1-\beta)} \approx 1 \tag{30}$$

$$\beta = \frac{P_P}{P_S} \approx 0.5 \tag{31}$$

$$\gamma = \frac{na(l-a)}{\pi d b} \approx 0.5 \tag{32}$$

P_p = Pressure
n = Number of
b = circumferential length (fig. 4).

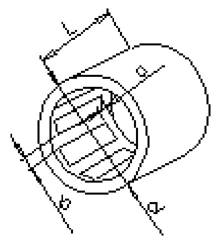

Fig. 4 Hydrostatic journal parameters

As shown in the equations, the stiffness and damping coefficient are related together so damping variation affect the stiffness coefficient.

VIII. Description of Problem and Assumption

As shown in Fig. 1, the bearing is filled with the fluid with environment pressure P_s =0. Because of small displacement of ring in z direction, the analysis is assigned axisymmetric and two dimensional. The following assumptions for solving the problem are considered.

1- With changing dimensions stiffness is not changed.
2- Motion is in radial direction essentially.

3- Fluid flows very smoothly and pressure gradient is negligible so $P_S \approx .001Pa$.

4- Using of formula (29), total stiffness of bearing is equal to 100 N/m and oil viscosity is equal to $1068\ Kg/m^3$.

IX. Solving of the Problem

With using The ANSYS software and two elements FLUID38 and FLUID81, the problem is solved (see Fig. 6(a), 6(b)).

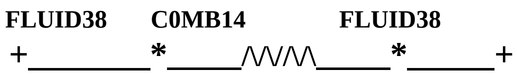

Fig. 6 (a) Modeling of the problem with FLUID38& COMB14

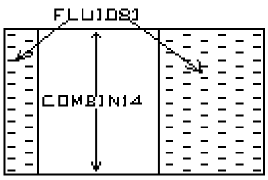

Fig. 6 (b) Modeling of the problem with FLUID81& COMB14

X. Results

The answer of numerical solution for the two modeling shows only 1% different (see Fig. 7). Moreover, the gradient of natural frequency of system vs. changing of others parameters is shown in Figs. 8-15.

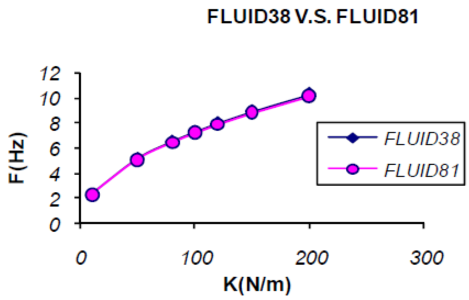

Fig. 7 F vs. K results for two modeling

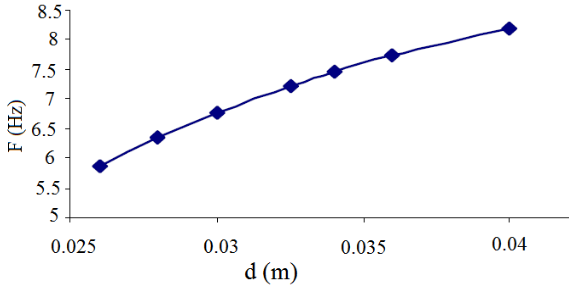

Fig. 8 The effect of changing of inner radius of outer cylinder on the natural frequency of system

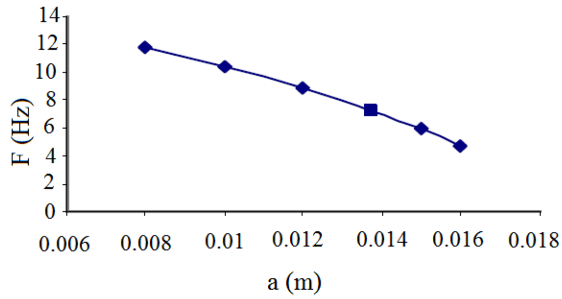

Fig. 9 The effect of changing of outer radius of inner cylinder on the natural frequency of system

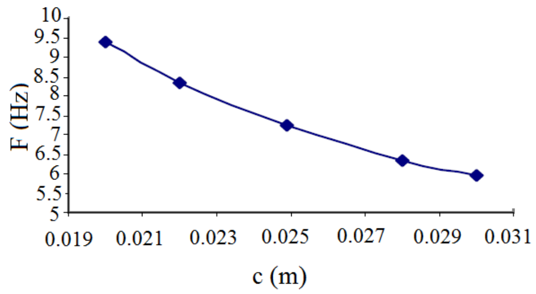

Fig. 10 The effect of changing of outer radius of ring in natural frequency of system

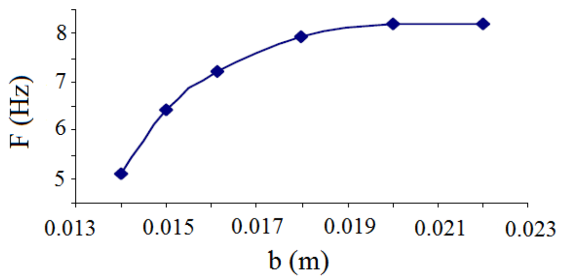

Fig. 11 The effect of changing of inner radius of ring on natural frequency of system

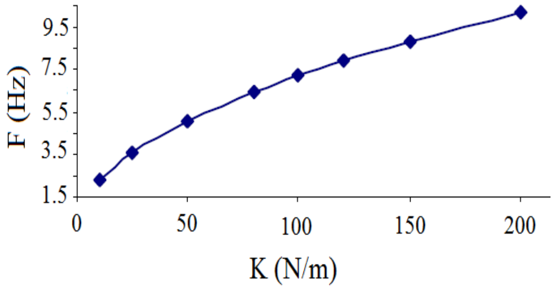

Fig. 12 The effect of bearing of stiffness on the natural frequency of system

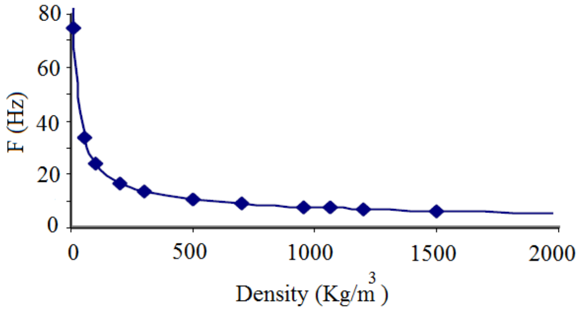

Fig. 13 The effect of oil density on the natural frequency of system

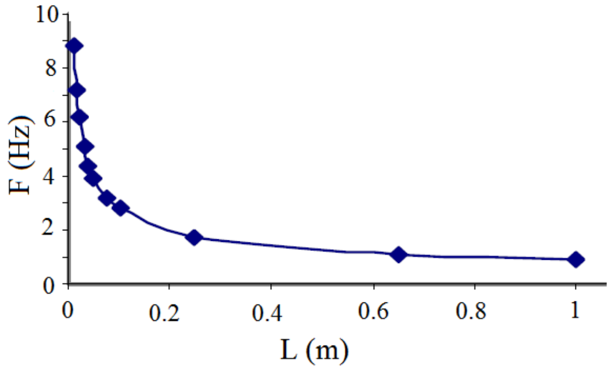

Fig. 14 The effect of ring thickness on the natural frequency of system

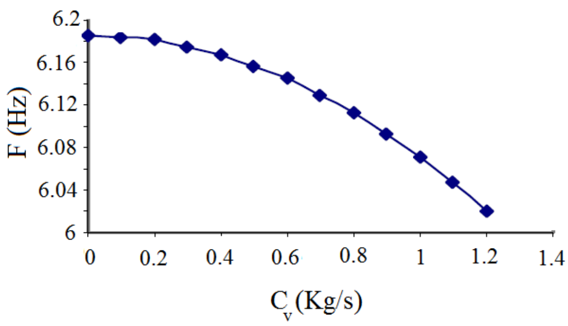

Fig. 15 The effect of damping on the natural frequency of system

XI. Conclusion

1- The first natural frequency is 7.22 HZ and the second one is 121 Hz.

2- The natural frequency is related to square root of stiffness as in theory.

3- The natural frequency is related to inverse of density.

4- As shown in Fig. 16, if damping is equal to zero then the maximum value for natural frequency is estimated.

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