International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:6, No:1, 2012

Equivalent Transformation for Heterogeneous Traffic Cellular Automata

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***Abstract*—**Understanding driving behavior is a complicated researching topic. To describe accurate speed, flow and density of a multiclass users traffic flow, an adequate model is needed. In this study, we propose the concept of standard passenger car equivalent (SPCE) instead of passenger car equivalent (PCE) to estimate the influence of heavy vehicles and slow cars. Traffic cellular automata model is employed to calibrate and validate the results. According to the simulated results, the SPCE transformations present good accuracy.

***Keywords*—**traffic flow, passenger car equivalent, cellular automata

I. Introduction

IN recent years, the traffic demand in metropolitan areas has largely exceeded the vehicular capacity, which induces problems of increasing pollution and growing frequency of accidents. Traffic flow involves complex phenomena, such as acceleration, deceleration, dawdling, lane-changing and multiple driving behaviors. Therefore, various models are developed to understand and predict traffic flow phenomena so as to evaluate and adopt traffic control strategies. Mostly, traffic flow models are categorized to macroscopic, mesoscopic and microscopic models. Macroscopic models regard the whole traffic flow as a continuous medium [1]-[4]. Mesoscopic analysis is an intermediate method, which does not distinguish individual vehicles, but specifies the behavior of individuals; for instance, in probabilistic terms. Microscopic models simulate individual behavior of each vehicle. The rapid development of computer capacity enhances the studies of microscopic models [5]-[9]. Among the models, traffic cellular automata (TCA) have been increasingly used in simulations of traffic flow on account of simplicity and flexibility of the model [5]-[14].

In CA, a road is represented as a string of cells, which are either empty or occupied by exactly one vehicle. Movement takes place by propagating along the string of cells. Traffic problems in real world involve many aspects, such as geometric design, different driving behavior, different types of vehicles, weather, lane usage, network topology and so on. There are diverse research topics of traffic flow. Generally, multilane multiclass traffic is a common situation in the real world. Multilane traffic involves acceleration, deceleration, lane-changing, and passing of vehicles.

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Multilane traffic is not the only dilemma; there are different types of vehicles and driving behavior on a road. Each type of vehicle and driving behavior has its own acceleration, deceleration, lane-changing and passing characteristics. To improve the performance of traffic controls, describing and predicting the influence of each type of user is necessary. Therefore, this study tries to develop a dynamic multilane multiclass macroscopic traffic flow model and its numerical simulation. Although there are a large number of studies about the traffic CA rules, only a few of them deal with a systematic analytical description [15]-[16]. To provide dynamic traffic information on the roads, rapid computation is necessary. Therefore, how to simulate traffic flow fast and accurately is an importation issue of traffic related researches.

In TCA, different driving behaviors are described by the setting of parameters, such as maximum speed, dawdling probability, lane-changing probability and so on. Different types of vehicles, such as trucks, cars and motorcycles, are described by different sizes of particles [3]-[4] or are transformed to the same type by passenger car equivalent (PCE) [17]. Of course, different types of vehicles may present different moving characteristics. Passenger car equivalent is a metric used to assess traffic-flow rate on a highway. A passenger car equivalent is essentially the impact that a mode of transport has on traffic variables (such as headway, speed, density) compared to a single car. Typically, the PCE of a heavy vehicle (trucks or buses) is 2 and the PCE of a motorcycle is 1/3. Transforming different types of vehicles to the same type by PCE is the simplest and most used method. Using this method, the value of PCE plays an important role because the estimation of density and flow depends on the value of PCE. In Taiwan, the PCE of motorcycle is not always equal to 1/3. It varies from 0.2 to 0.5 according to different traffic conditions. Therefore, using PCE to transform different types of vehicles to the same type should be carefully. According to the results of Lo and Chu [18], PCE of heavy vehicles is a function of density, if the driving behavior of heavy vehicles is the same as it of passenger cars. It is not so realistic because the acceleration, deceleration and maximal speed of heavy vehicles and passenger cars are quite different. Therefore, a behavior-based PCE method for different types of vehicles and different driving behavior is proposed in this study. The paper is organized as follows. In Sec. 2, an introduction of traffic CA models are reviewed briefly. Then, the results of simulations and equivalent transformation are presented in Sec.3. Finally, Sec. 4 concludes with a short summary and discussion of our findings.

II. Heterogeneous Traffic Flow Modeling

Nagel and Schreckenberg propose the first TCA model which is named the NaSch model [8]. The NaSch model is implemented by the following four steps:

(Step 1) Acceleration. If time step is less than total simulation time, let each vehicle with speed, which is smaller than its maximum speed v_{max}, accelerate to a higher speed, i.e. v = min (v_{max}, v+1).

(Step 2) Deceleration. If the speed is greater than the distance gap d to the preceding vehicle, the vehicle will decelerate: v = min (v, d).

(Step 3) Dawdling. With slow-down probability p, the speed of a vehicle decreases spontaneously: v = max (v–1,0).

(Step 4) Propagation. Let each vehicle move forward v cells and let time step increase one. Then, repeat the procedure: acceleration, deceleration, dawdling and propagation.

In multi-lane traffic, overtaking maneuver uses an extended neighborhood behind and ahead the vehicle on both lanes. Technically, one can say that there must be a gap of size d^-+1+d^+. The label + (−) belongs to the gap on the target lane in front of (behind) the vehicle that wants to change lanes. In the following we characterize the security criterion (no accident condition) by the boundaries $[-d^-, d^+]$ of the required gap on the target lane relative to the current position of the vehicle considered for changing lanes. If one vehicle has enough gaps to change lane, it still has a probability p_{cl} to keep in the same lane so as to avoid the ping-pong phenomenon. The overtaking step is inserted in front of the acceleration step.

Multi-class users traffic are also been studied. Slow-to-start rules are mostly studied. Takayasu and Takayasu (the TT model or the T2 model) [11] suggested a CA model with a slow-to-start rule firstly. According to the TT model, a standing vehicle (i.e., a vehicle with the instantaneous speed v = 0) with exactly one empty cell in front accelerates with probability $q_t = 1 - p_t$, while all other vehicles accelerate deterministically. The other steps of the update rule (Step2 – Step4) of the NaSch model are kept unchanged. The TT model reduces to the NaSch model in the limit $p_t = 0$.

Another slow-to-start CA model is the BJH model proposed by Benjamin, Johnson and Hui [12]. Their slow-to-start rule is different to the TT model. According to the BJH model, the vehicles which had to brake due to the next vehicle ahead will move on the next opportunity only with probability $1 - p_s$. Step 1, 3, 4 of are the same as the NaSch model. An extra step (Step 1.5) is introduced and Step 2 is modified as follows:

(Step 1.5) Slow-to-start. If flag = 1, then let v = 0 with probability p_s.

(Step 2’) Blockage. v = min (v, d) and, then, flag = 1 if v = 0, else flag = 0.

Here flag is a label distinguishing vehicles which have to obey the slow-to-start rule (flag = 1) from those which do not have to (flag = 0). Obviously, for p_s = 0 the above rules reduce to the NaSch model. Velocity dependent randomization (VDR model) is a generalized BJH model, which considers a larger slow-down probability while the velocity is zero in the last time step. Fukui and Ishibashi [14] proposed a high speed CA model, which is so-called FI model. The FI model considers that a driver would not dawdle unless he is driving at the maximum speed.

Lo and Chu [18] find that if heavy vehicles and passenger cars with different behaviors are simulated first then transforming the results to passenger car-based density or occupancy, the speed and flow are unreasonable. The reason is that the simulated speed of TCA is obtained by the stochastic process, which depends on the number of vehicles and dawdling probability. Since one truck is equal to two cars, the number of trucks is only a half of the number of cars under a given density. Thus, unreasonably high speed and flow are obtained. To correct the unreasonable results, they considered the PCE of truck is a function of density. By the PCE function, the multiple-type vehicles traffic flow is simulated successfully.

III. Simulations and Equivalent Transformation

In traffic cellular automata simulation, the length of cell is chosen according to the real world. In Taiwan, the upper speed limit of the No. 3 National Freeway is 110 km/h (kilometer per hour). Therefore, the length of a cell is considered as 7 m, the maximum speed v_{max} is 5 (i.e., 126.2 km/hr). The CA results are obtained from simulation on a chain of 1,000 sites, which is 7 km. A periodic boundary condition is assumed so that both the total number and density are conserved at each simulated point. For each initial configuration of vehicles, results are obtained by averaging over 10,000 time steps after the first 10,000 steps, so that the long-time limit is approached. This criterion was found to be sufficient to guarantee a steady-state being reached. The density considered herein is dimensionless density, which is defined as $k' = k/k_j$, where k is the density and k_j is the jam density. In this study, k_j = 142 veh/km (vehicle per kilometer) per lane. Figures 1-3 show the simulation results of heavy vehicles and passenger cars by the NaSch model. Generally, the average length of a car is about 4 to 5 meters and the average length of a truck or a bus is about 7 to 10 meters. Therefore, a car occupies one cell and a heavy vehicle occupies two cells in our simulation.

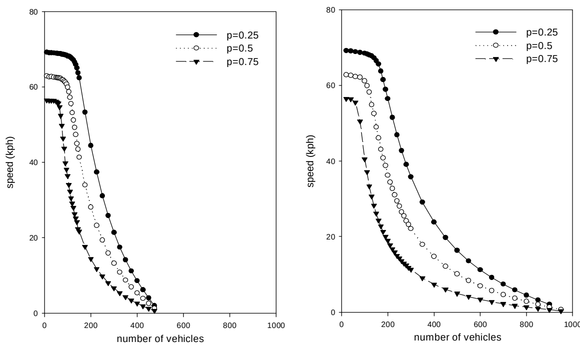
Lo and Chu [18] propose that PCE of heavy vehicles is a function of traffic density. They show that Eq. (1) is a good approximation of PCE for single-lane and two-lane traffic under v_{max}=5 and p=0.25. In traffic flow theory, speed is a function of density. Therefore, the basic assumption of their study is that if the average speeds of traffic flow on different roads are the same, the densities of the roads must be the same.

$$PCE(k') = 1.028k' + 0.98 \quad (1)$$

In this study, we follow the assumption and examine Eq. (1) by further combination of parameters. Figure 4 illustrates the PCE of heavy vehicle under nine combinations of simulation coefficients, that is, combinations of v_{max}=3, 4, 5 and p=0.25, 0.5, 0.75. PCE varies largely when density is low. Therefore, we omitted the data, which density is smaller than 0.1. The PCE value does not influence traffic flow much when density is smaller than 0.1 because in that regime traffic flow is under free-flow situation. The PCE function calibrated by simulation results of v_{max}=3, 4, 5 and p=0.25, 0.5, 0.75 is given by

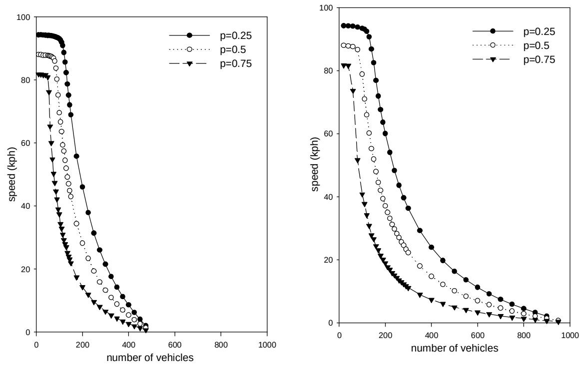
$$PCE(k') = 1.0162k' + 0.991, \quad (2)$$

which is close to Eq. (1). The R-square value of Eq. (2) is 0.9853. Table I gives the mean absolute percentage error (MAPE) of Eq. (2). All of the MAPE values are smaller than 6%, which are highly accurate estimation. According to the results, we can conclude that Eq. (2) is a good equivalent transformation of heavy vehicles to passenger cars or single-lane and two-lane traffic under all combination of simulation parameters of the NaSch model. However, the equation can only transform heavy vehicles to passenger cars with the same driving behavior. Therefore, we propose a concept of standard passenger car equivalent (SPCE) to transform heavy vehicles to passenger cars. Furthermore, the SPCE can be employed to estimate the effect of slow passenger cars because the influence of a slow car on traffic flow is more than the influence of a standard car. If there are slow cars in the traffic flow, the density should be larger. In this study, the fastest car is considered as the standard passenger car and its SPCE is set to be one. The SPCE of slow car will be larger than one. Two transformation procedures are considered. The first one is transforming influence of dawdling probability first then transforming influence of maximum speed. For example, transforming a car with v_{max}=3 and p=0.5 to standard car, whose v_{max}=5 and p=0.25. According to the procedure, we transform the slow car to v_{max}=3 and p=0.25 firstly and then transform it to v_{max}=5 and p=0.25. The second procedure is transforming influence of maximum speed first then transforming influence of dawdling probability. According to the procedure, we transform the slow car to v_{max}=5 and p=0.5 firstly and then transform it to v_{max}=5 and p=0.25.Figure 5 illustrates the SPCE-density relationship of different dawdling probabilities. When density is small, SPCE reduces largely. The SPCE of p=0.75 is quite different to that of the other two. Figure 6 illustrates the SPCE-density relationship of different maximum speeds. When density is larger than 0.2, SPCE has a steady trend.



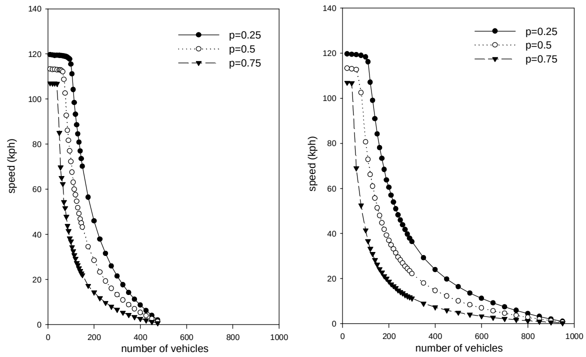
(a) (b)

Fig. 1 Speed-density relationship of (a) heavy vehicles and (b) passenger cars under v_{max}=3 and different dawdling probabilities



(a) (b)

Fig. 2 Speed-density relationship of (a) heavy vehicles and (b) passenger cars under v_{max}=4 and different dawdling probabilities



(a) (b)

Fig. 3 Speed-density relationship of (a) heavy vehicles and (b) passenger cars under v_{max}=5 and different dawdling probabilities

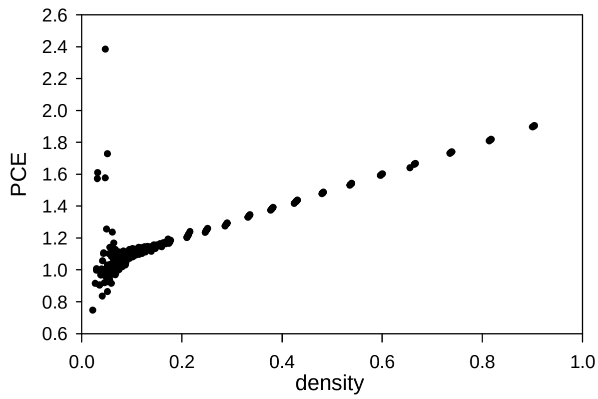


Fig. 4 PCE of heavy vehicle and density relationship for combination of v_{max}=3, 4, 5 and p=0.25, 0.5, 0.75

According to the results, we have two regression equations. Equation (3) is the transformation of p=0.5 to p=0.25 and Eq. (4) is the transformation of p=0.75 to p=0.25. The R-square values of Eqs (3) and (4) are 0.8779 and 0.9979, respectively. Let $SPCE_{v,p}$ be the $SPCE$ value of a car with speed is v and dawdling probability p. $SPCE_{v,p}$= $SPCE^p \times SPCE^v$, where $SPCE^p$ is the standard passenger car equivalent of dawdling probability p and $SPCE^v$ is the standard passenger car equivalent of maximum speed *v*. A general form of $SPCE^p$ is given by Eq. (5), where $a(p)$ and $b(p)$ are *p* dependent coefficients. In this study, we can only determine $a(p)$ and $b(p)$ by two data points. More simulations are left for further studies.

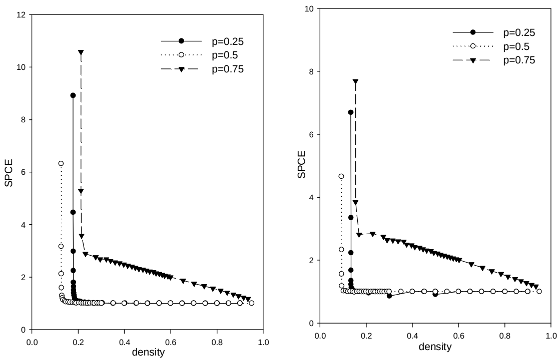
$$SPCE^{0.5}(k') = -0.705k' + 1.6722 \cdot \quad (3)$$

$$SPCE^{0.75}(k') = -2.3554k' + 3.4044 \cdot \quad (4)$$

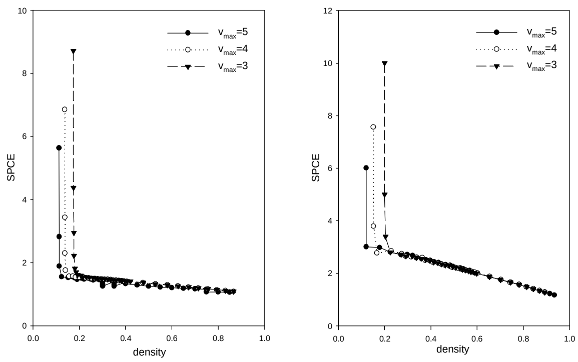
$$SPCE^{p}(k') = a(p)k' + b(p) \cdot \quad (5)$$

TABLE I
MAPE VALUE OF ACTUAL PCE AND ESTIMATED VALUE OF EQ. (2)

v_{max}	p	MAPE (%)
3	0.25	5.12
	0.5	3.41
	0.75	3.07
4	0.25	2.30
	0.5	1.92
	0.75	1.95
5	0.25	2.97
	0.5	1.74
	0.75	1.45



(a) (b)
Fig. 5 SPCE-density relationship of passenger cars under (a) v_{max}=3 and (b) v_{max}=4 with different dawdling probabilities



(a) (b)
Fig. 6 SPCE-density relationship of (a) *p*=0.5 and (b) *p*=0.75 under different maximum speed

Finally, the transformations of maximum speed are given by Eqs (6) – (7). k_c is the minimum *k'*, which makes $SPCE(k') = 1$. Also, a general form of $SPCE^v$ is given by Eq. (8), where $c(v)$ and $d(v)$ are *v* dependent coefficients. The general form of $SPCE_{v,p}$ is shown as Eq. (9). Table II shows the MAPE value of Eq. (9). Only MAPE of $SPCE_{3,0.25}$ and $SPCE_{3,0.5}$ are larger than 10%, the others are smaller than 10%. The results show that our equivalent transformations present good accuracy. By Eqs (2) and (9), we can find the equivalent value of heavy vehicles with different driving behavior so as to estimate their influence on the road accurately.

$$SPCE^{3}(k') = \begin{cases} -854.28k' + 118.69, & if\, k' < k_c \\ 1, & if\, k' \geq k_c \end{cases} . \quad (6)$$

$$SPCE^{4}(k') = \begin{cases} -853.49k' + 156.64, & if\, k' < k_c \\ 1, & if\, k' \geq k_c \end{cases} . \quad (7)$$

$$SPCE^{v}(k') = \begin{cases} c(v)k' + d(v), & if\, k' < k_c \\ 1, & if\, k' \geq k_c \end{cases} . \quad (8)$$

$$SPCE_{v,p}(k') = \begin{cases} [c(v)k' + d(v)] \cdot [a(p)k' + b(p)], & if\, k' < k_c \\ a(p)k' + b(p), & if\, k' \geq k_c \end{cases} . \quad (9)$$

TABLE II
MAPE VALUE OF ACTUAL PCE AND ESTIMATED VALUE OF EQ. (9)

v_{max}	p	MAPE (%)
3	0.25	15.70
	0.5	10.15
	0.75	8.78
4	0.25	6.21
	0.5	4.76
	0.75	3.23
5	0.5	7.53
	0.75	2.34

IV. CONCLUSION

In this study, a standard passenger car equivalent (SPCE) is proposed to estimate the influence of heavy vehicles and slow cars on the road. According to the results, our estimation presents a high accuracy of passenger car equivalent. More simulations are left for further researches so as to make our work more compact and robust.

ACKNOWLEDGMENT

The work was partially supported by the National Science Council (NSC), Taiwan, under Contract NSC 98-2221-E-216-024-MY3-3.

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