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Energy Recovery from Swell with a Height Inferior to 1.5 m

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Abstract**—**Renewable energy recovery is an important domain of research in past few years in view of protection of our ecosystem. Several industrial companies are setting up widespread recovery systems to exploit wave energy. Most of them have a large size, are implanted near the shores and exploit current flows. However, as oceans represent 70% of Earth surface, a huge space is still unexploited to produce energy. Present analysis focuses on surface small scale wave energy recovery. The principle is exactly the opposite of wheel damper for a car on a road. Instead of maintaining the car body as non-oscillatory as possible by adapted control, a system is designed so that its oscillation amplitude under wave action will be maximized with respect to a boat carrying it in view of differential potential energy recuperation. From parametric analysis of system equations, interesting domains have been selected and expected energy output has been evaluated.

Keywords**—**Small scale wave, potential energy, optimized energy recovery, auto-adaptive system.

I. Introduction

PResent project is in the continuity of many widespread existing projects. It has long been recognized that because oceans are covering 70% of Earth surface, marine energy in different forms will play an important role in coming energy transition [1]-[3]. Some of them such as wind [4] and tidal [5] turbines are subject to various constraints: They have to be located where the winds and/or currents are sufficient, which greatly reduces the usable surfaces. Also, the needs of foundations require shallow depths and induce high installation and maintenance costs. Other marine power systems, though free from these constraints, encounter difficulties related to their large sizes and types of waves they need to function [6]-[13], typically a large swell only found in some remote areas of the shore with difficult access. As all these projects aim at an important energy efficiency and delivery, they only focus on large waves. Other approach has the objective to discuss the possibility of designing a system capturing wave energy based on smaller amplitude waves existing all over ocean surface, and which can be useful as local energy sources [14]. Even if it looks random on small space and short time scales, swell Fourier analysis gives a stationary energy spectrum for period of an hour commonly represented by semi-empirical function [15], [16]. When expressed in kW/m evaluation of transported power gives typical 2,5 kW/m for small swell, about ten times larger value for regular oceanic swell and up to hundred times larger in case of big hurricane. For a shore of 10^3 km long in front of Atlantic Ocean, France could expect to collect 420 TWh/year, to be compared to 450 TWh/year final electric power consumption. Aside these energy sources based on wave displacement with respect to a fixed point on ocean surface, there also exists the possibility to exploit without specific fixed location reference the continuous up and down motion of ocean surface. This will be useful energy sources for non-fixed floating objects anywhere on ocean surface [17]. It is interesting in this case to determine to what extent smaller amplitude waves can as well be used even if their output is a modest amount of energy. Roughly for a wave of wavelength $\lambda \cong 1$ m and height H meters, the possible delivered energy is $E = MgH$ for an object of mass M newtons, and with a surface of 3.10^8 km^2 one would potentially get over the globe $E_{Tot} \cong 3.10^{16}$ Mh gigajoules which is extremely large. So, the recoverable energy amount with low amplitude waves is not negligible. It is just dispersed: To be relatively manageable from local point of view (for instance a sailing boat on ocean surface), the exploitation system should be of modest size as presented in next paragraph. But it should also be optimized for improving enough collected energy as discussed in the following.

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II. Small Scale Wave Model

It is interesting to note that energy recuperation system is the opposite of car suspension system, see Fig. 1. On a road with bumps, the shocks must be controlled and reduced for the body car to stay as much as possible steady. Here, as it is expected to recover as much energy as possible from the wave, one should take advantage of its movement. Modeling is based on the opposite of vehicle body ups and downs on an undulating road reduced through a damped suspension.

Fig. 1 Car Dumper

The wheels vertical oscillations, due to the uneven ground and to car forward speed, can be expressed as an imposed time dependent displacement producing a forced oscillation of suspension mechanism and analysis can be developed out to find best absorption strategy depending on passive, semi-active or active nature of damping system fixed by energy consumption constraints [18], [19].

Here the two masses M_1 (the boat) and M_2 (the float) are carried by a wave through a vertical axis suspension. The latter is modeled by a spring of stiffness K_1 for the boat and K_2 for the floater, with damping F_1 for the boat and F_2 for the floater, see Fig. 2. Interaction between the boat and the float will be simply described by a spring of stiffness K.

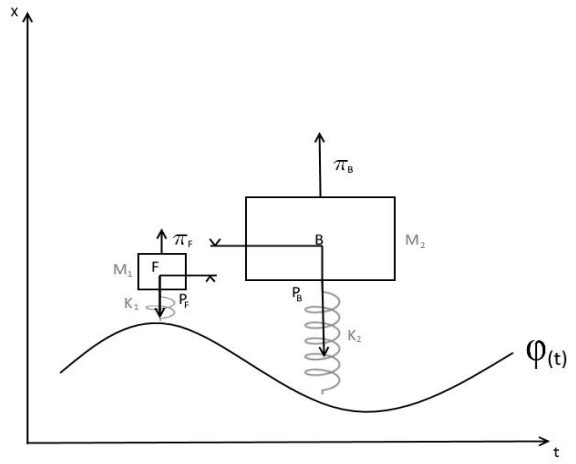

Fig. 2 Sketch of the Two Mass System ("B": The Boat, "F": The Floater)

With positions of the boat X_1 and the floater X_2, system equation is given in normalized form by

$$d^2X/dt^2 + \mathbf{f}\, dX/dt + \langle\boldsymbol{\Omega}\rangle^2 X = \boldsymbol{\Omega}^2\phi(t) \quad (1)$$

where $X = [X_1,X_2]^T$ is system state, $\mathbf{f} = \mathrm{diag}[f_1,f_2]$ the damping matrix, $\boldsymbol{\Omega}^2 = \mathrm{diag}[\omega_1^2,\omega_2^2]$ the stiffness matrix, $\langle\boldsymbol{\Omega}\rangle^2 = \boldsymbol{\Omega}^2 + \Gamma^2\mathbf{M}$, $\mathbf{M} = \mathrm{matrix}[m\ -m]$, $m = [1\ -\varepsilon]^T$, $\phi(t)$ the time dependent wave oscillation source. Normalized parameters are defined by $f_j = F_j/M_j$, $\omega_j^2 = K_j/M_j$, $\varepsilon = M_2/M_1$ and $\Gamma^2 = K/M_2$. Because the coefficients in left hand side of (1) are constant, its solution after Laplace transform is given by

$$X(s) = [s^2\mathbf{I} + s\mathbf{f} + \langle\boldsymbol{\Omega}\rangle^2]^{-1}\boldsymbol{\Omega}^2\Phi(s) \quad (2)$$

where $\Phi(s)$ is the Laplace transform of $\phi(t)$. This source term depends on the physical origin of the waves in the ocean, for instance travelling stationary ones bouncing from one side of the oceanic basin to the other one, or local Airy type one in specific location. In any case its determination is depending on the solution of a well-defined problem which will not be discussed.

It is now interesting to determine the value of system adjustable parameters so that the amplitude of the difference $D = |X_1 - X_2|$ is largest in typical frequency band corresponding to waves generation, as this corresponds to highest possible energy recuperation. After simplifications, the final result is:

$$D(s,\varepsilon,\Gamma,\Omega) = \Omega^2\{(S_1^2 + \lambda\Omega^2)(S_2^2 + \Omega^2) + \Gamma^2[S_1^2 + \varepsilon S_1^2 + \Omega^2(\lambda + \varepsilon)]\}^{-1}\{S_1^2 - \lambda S_2^2\}\Phi(s) \quad (3)$$

with $S_j^2 = s(s + f_j)$, $\omega_1^2 = \lambda\omega_2^2$ ($\lambda < 1$), and renaming $\Omega^2 = \omega_2^2$.

III. System Analysis and Optimization

Equation (3) gives the Laplace transform of the response difference between the boat and the floater to a wave of amplitude $\Phi(s)$ and characteristic s. Aside ε and f_k, its expression contains two important passive system parameters Γ and Ω describing system properties at present level of representation. In view of energy recuperation, optimization of D rests upon determination of their best values as system potential energy is given by $\boldsymbol{\mathcal{E}}_{pot} = M_2gD$. To simplify the analysis, it will be supposed that friction origin is the same for the two masses in which case specific frictions f_j (j=1,2) are equal $f_j = f$, so $S_j = S$ and (3) becomes in normalized form

$$D(\sigma,\varepsilon,\gamma,\lambda,\varphi) = (1 - \lambda)\Sigma^2(\Sigma^4 + \boldsymbol{\mathcal{L}}\Sigma^2 + \boldsymbol{\mathcal{M}})^{-1}\Phi(s) \quad (4)$$

by defining $\Sigma = \sigma(\sigma + \varphi)$ with $\sigma = s/\Omega$, $\varphi = f/\Omega$ and $\gamma = \Gamma/\Omega$, $\boldsymbol{\mathcal{L}} = 1 + \lambda + (1 + \varepsilon)\gamma^2$, $\boldsymbol{\mathcal{M}} = \lambda + (\lambda + \varepsilon)\gamma^2$. The denominator can also be written $\Delta = (\Sigma^2 + \Sigma_+^2)(\Sigma^2 + \Sigma_-^2)$ with $\Sigma_\pm^2 = .5(\boldsymbol{\mathcal{L}} \pm (\boldsymbol{\mathcal{L}}^2 - 4\boldsymbol{\mathcal{M}})^{1/2}) > 0$ because $\boldsymbol{\mathcal{L}}^2 - 4\boldsymbol{\mathcal{M}} = [(1 - \varepsilon)\gamma^2 + 1 - \lambda]^2 + 4\varepsilon\gamma^4 > 0$. When $\varepsilon << 1$, $\Sigma_+^2 \to 1 + \gamma^2$ and $\Sigma_-^2 \to \lambda$. Now the transforming factor Σ^2/Δ of D can be split into two parts $D_1 = \Sigma^2(\Sigma^2 + \Sigma_+^2)^{-1}$ and $D_2 = [\Sigma^2 + \Sigma_-^2]^{-1}$ the absolute value of which is to be maximized by choosing adequate values of γ and ε. $|D_2|$ takes the form

$$|D_2| = \{[\Sigma_-^2 - \omega^2]^2 + \omega^2\varphi^2\}^{-1/2} \quad (5)$$

with maximum value

$$|D_2|_{max} = \varphi^{-1}\{\Sigma_-^2 - \varphi^2/4\}^{-1/2} \quad (6)$$

for $\omega_{2,m}^2 = \Sigma_-^2 - \varphi^2/2$. On the other hand, the maximum $|D_1|_{max}$ of $|D_1|^2 = \omega^2(\omega^2 + \varphi^2)[(\Sigma_+^2 - \omega^2)^2 + \omega^2\varphi^2]^{-1}$ is occurring for

$$\omega_{1,m}^2 = \Sigma_+^2/2 + [\Sigma_+^2/4 + \varphi^2/2]^{1/2} \quad (7)$$

See Fig. 3. Owing to system structure, it is not possible to choose parameters $\varepsilon,\gamma,\lambda,\varphi$ so that equality $\omega_m = \omega_{1,m} = \omega_{2,m}$ holds, which would maximize $|D|$ for wave specific value ω_m. So, this optimization scheme cannot be applied to a single frequency wave Ω_w oscillating without damping at ocean surface like Airy-von Gerstner swell [20]. Instead, because the two peaks of $|D_1|$ and $|D_2|$ are split apart with typically $\omega_{1,m} \cong (1 + \gamma^2)^{1/2}$ and $\omega_{2,m} \cong \lambda^{1/2}$ for reasonable $\varepsilon << 1$, and $|D_1|(\Sigma_+/2) = 1$, see Fig. 3, one could try to have $\omega_{2,m} \cong \Omega_w$ as $|D_2|(\omega)$ is

very sharp. However the product $|D_1||D_2|(\omega_{2,m}) < 1$ for reasonable value of φ, and better strategy has to be found.

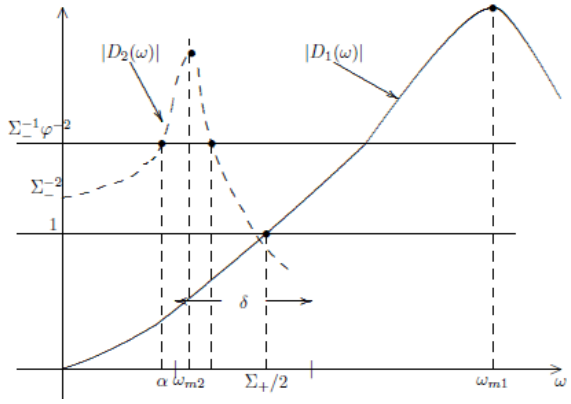

Fig. 3 Sketch of Filter Functions $|D_k(\omega)|$ with $\alpha = (\Sigma_-^{-2} - \varphi^2)^{1/2}$

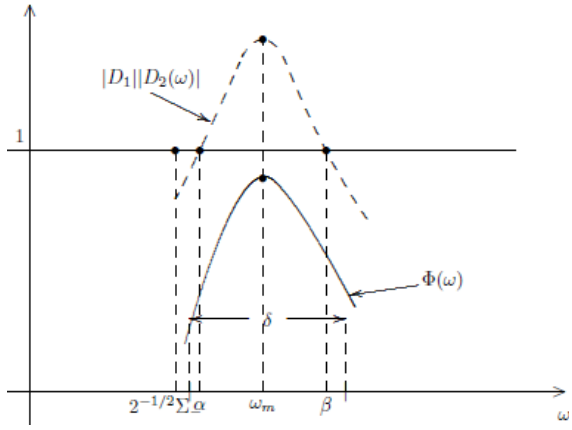

Fig. 4 Sketch of Localization of Wave Spectrum Φ(ω) and Simplified Filter Function $|D_{app}|$

Product $|D_1||D_2|(\omega)$ exhibits a maximum at $\omega_{Max} > \omega_{2,m}$, and is typically larger than 1 in the interval $[\alpha,\beta]$ where β is such that $||D_2|(\beta) \cong 1$. In realistic situation the wave oscillation $\phi(t)$ is a combination of different waves belonging to an interval $\delta = [\omega_-,\omega_+]$ with Fourier transform function Φ(ω), and it is interesting for maximizing |D| to choose floater and system parameters so that $|D_1||D_2|(\omega) = F(\Phi(\omega))$, where F(.) is a regular function. This implies that intervals δ and $[\alpha,\beta]$ should have non zero intersection. Best situation occurs when the maximum abscissa ω_M of Φ(ω) is exactly placed at ω_{Max}, see Fig. 4. When this is possible, second order expansion of $|D_1||D_2|(\omega)$ around its maximum, and inverse Fourier transform gives approximate displacement difference

$$D(t) \cong D_{app}(t) = (1-\lambda)\{D_{Max}\phi(t) + 2\omega_{Max}^2 (d^2D)_{Max}\phi''(t)\cos\omega_{Max}t\} \quad (8)$$

with $(d^2D)_{Max} = d^2D/dX^2(\omega_{Max})$ and $X = \omega_{Max}^2$, out of which lower bounds of potential energy $\mathcal{E}_{pot} = M_2gD(t)$ and kinetic energy $\mathcal{E}_{kin} = .5M_2[D'(t)]^2$ can be directly calculated. $D_{app}(t)$ contains a constant multiplicative term and an inertia term (the second derivative term) resulting from acceleration (deceleration) phases in oscillatory motion which increases (decreases) the apparent weight of mass M_2 and modifies potential energy $\mathcal{E}_{pot}$. To ride the system with largest potential energy value one should keep the equality $\omega_{Max} = \omega_M$ such that $d\Phi(\omega_M)/d\omega = 0$, which implies that floater system parameters have to be adaptively driven from measured actual swell conditions [21]. Present analysis is valid for single humped wave spectrum. When this not the case, numerical gradient calculations have to be developed directly on D(t) from (4).

IV. Application

The specific case where weighting function $\Phi(\omega) = A\omega^{-5}\exp(-B\omega^{-4})$ with two constants A and B depending on considered wind wave model [22] will be used here. This is a narrow very peaked distribution allowing calculate all moments $\Phi_n = \int\omega^n\Phi(\omega)d\omega$ analytically as $\Phi_n = .25AB^{(n/4-1)}\Gamma(1-n/4)$, out of which in particular wave height $H = 4\Phi_0$ and mean wave period $T_1 = \Phi_0/\Phi_1$ can be found. Its maximum occurs for $\omega_M = (4B/5)^{1/4}$. Numerical calculations have been developed to maximize function $D(t,\varepsilon,\gamma,\varphi)$ with respect to system parameters ε,γ,φ which have been chosen to satisfy $\omega_{Max} = \omega_M$. Parameters A and B are directly related to wave height H and period T_1 by

$$A = BH;\ B = .25[\Gamma(1/4)]^4(\pi\Omega T_1)^{-4} \quad (9)$$

and can be adjusted to actual swell condition from measurement of T_1 and H. To optimize energy recovery, it is interesting to have variable system floater coefficients K and M_2 so that they satisfy condition (10) thanks to direct control chain $(T_1,H) \rightarrow (A,B) \rightarrow (K, M_2)$. With normalized system parameters given in Table I.

TABLE I
Parameters' Value

γ^2	λ	φ	ε
.05	.05	210^{-1}	3.10^{-3}

It is seen on Fig. 5 that the interval for which $|D_1||D_2|(X) > 1$ is $X \in \{[.5] \cup [2,3.6]\}$, and second interval [2.3,6] is more adapted to fit wave spectrum. Condition $X_{2,Max} = X_w$ fixes the value of scaling factor $\Omega^2 = K_2/M_2$ out of which all other system parameters can be determined by the relations $K_2 = \omega_W^2M_2/X$, $K_1 = \lambda\omega_W^2M_2/\varepsilon X$ and $K = \gamma^2K_2$, $M_1 = M_2/\varepsilon$ once M_2 (ie the desired potential energy per wave height unit) has been fixed. In present case, it is verified from (4) that highest value of amplification factor is equal to $(1-\lambda)|D_1||D_2|(X_{2max}) \cong .98$ i.e. is less than 1. This fully justifies optimality research proposed in the text.

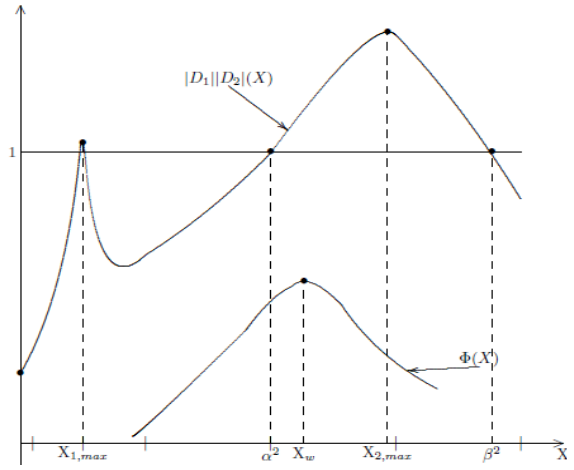

Fig. 5 Curves $|D_1||D_2|(X)$ and $\Phi(X)$ ($X = \omega^2$)

V. Conclusion

The recuperation of wave energy at the surface of the oceans with a system of a floater and a boat submitted to ocean surface oscillations has been modeled as an opposite problem to car body stabilization on a bumpy road. Analytical study of system equations allows derive a lower bound of position difference between floater and boat. Its expression can be optimized by a convenient choice of floater parameters which may be adapted to actual wave situation.

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