Energy Loss at Drops using NeuroSolutions

Farzin Salmasi

Abstract—Energy dissipation in drops has been investigated by physical models. After determination of effective parameters on the phenomenon, three drops with different heights have been constructed from Plexiglas. They have been installed in two existing flumes in the hydraulic laboratory. Several runs of physical models have been undertaken to measured required parameters for determination of the energy dissipation. Results showed that the energy dissipation in drops depend on the drop height and discharge. Predicted relative energy dissipations varied from 10.0% to 94.3%. This work has also indicated that the energy loss at drop is mainly due to the mixing of the jet with the pool behind the jet that causes air bubble entrainment in the flow. Statistical model has been developed to predict the energy dissipation in vertical drops denotes nonlinear correlation between effective parameters. Further an artificial neural networks (ANNs) approach was used in this paper to develop an explicit procedure for calculating energy loss at drops using NeuroSolutions. Trained network was able to predict the response with R^2 and RMSE 0.977 and 0.0085 respectively. The performance of ANN was found effective when compared to regression equations in predicting the energy loss.

Keywords—Air bubble, drop, energy loss, hydraulic jump, NeuroSolutions

I. INTRODUCTION

A vertical drop or free over fall is a common feature in both natural and artificial channels. Natural drops are formed by river's bed erosion while artificial drops are built in irrigation systems to reduce the channel slope to designed slope [1]. Due to more energy dissipation drops are also applied in stepped spillways. The more energy dissipation is caused by mixing of the jet with the pool at downstream of each drop. As a result, it will be the cause of the reduction in the size of the energy dissipater that is generally provided at the toe of drops and stepped spillways [2]. A vertical drop located in a rectangular channel is illustrated in Fig. 1, where h is the height of drop, q is the discharge per unit width of the channel, y_c is the critical depth, L_p is distance between hitting jet location on stilling basin and vertical drop wall and y_p is average pool depth behind the falling jet. The upstream flow is subcritical and the flow immediately downstream of the drop is supercritical.
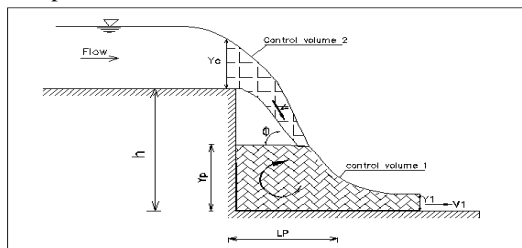

Fig. 1 Characteristics of flow over a vertical drop

F. Salmasi is with the water engineering department, faculty of agriculture, Tabriz university, Tabriz, IRAN (e-mail: Salmasi@Tabrizu.ac.ir).

Moore [3] performed an experimental study with drops of two heights and found that the energy loss at the drop could be significant depending upon the relative height of the drop h/y_c. As h/y_c increases from 1 to 12, the relative energy loss varies from zero to 0.53. Moore [3] also found that the Equation 1 is good predictor for the depth of the pool behind the falling jet, y_p:

$$\frac{y_P}{y_c} = \sqrt{\left(\frac{y_1}{y_c}\right)^2 + 2\left(\frac{y_c}{y_1}\right)} - 3 \tag{1}$$

In a discussion of Moore's paper [3], White [4] developed a method to predict the energy loss at the drop based on a number of assumptions. Using the momentum equation, White [4] obtained equation 2:

$$\frac{y_1}{y_c} = \frac{\sqrt{2}}{1.061 + \sqrt{h/y_c + 1.5}} \tag{2}$$

Energy at downstream of drop and before hydraulic jump can be written as equation 3:

$$H_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{y_c^3}{2y_1^2} \Rightarrow$$
$$\frac{H_1}{y_c} = \frac{y_1}{y_c} + \frac{1}{2}\left(\frac{y_c}{y_1}\right)^2 \tag{3}$$

By substitution of equation (2) in equation (3), White [4] showed that:

$$\frac{H_1}{y_c} = \frac{\sqrt{2}}{1.061 + \sqrt{h/y_c + 1.5}} +$$
$$\frac{1}{4}(1.061 + \sqrt{h/y_c + 1.5})^2 \tag{4}$$

since the energy upstream of the drop (Ht) can be written as equation 5:

$$H_t = h + 1.5y_c \tag{5}$$

then it can be shown that energy dissipation $(\Delta H = H_t - H_1)$ is a function of h/y_c. Gill [5] attempted to modify the theory of White [4] by refining his assumptions. Then Gill [5] obtained equation 6 for the depth below the drop y_1.

$$\frac{y_1}{y_c} = \frac{1}{\sqrt{\begin{array}{l} 0.5(1+\cos\phi)^2 \\ (h/y_c +1.5 - y_p / y_c) + \\ 2(y_p - y_1)/ y_c \end{array}}} \qquad (6)$$

Where ϕ is the angle of the jet where it hits the pool. Gill [5] also performed experiments on four drops with heights of 48.3, 74.0, 99.4 and 176.5 mm and measured primarily the pool depths and the angle of the impinging jet. He found that his method predicted values of H1/yc somewhat larger than those predicted by White [4].Rand [6] performed an experimental study on drops and developed a set of empirical equations for the drop characteristics in terms of the Drop Number $(D = q^2 / gh^3 = (y_c / h)^3)$.Chamani and Rajaratnam [7] performed an experimental study on two drops of heights 0.62 and 0.25 m were built in a rectangular channel of width of 0.46 m, 6.55 m length and depth of 0.91 m. For the third drop with height 0.25 m, a slot was cut in the back wall, so that no pool was formed behind the jet, thereby simulating an inclined jet.Chamani and Rajaratnam [7] presented a method to predict drop characteristics. Their analysis was based on a number of assumptions to simplify the problem. The effect of air entrainment on flow characteristic was ignored. With reference to Fig. 1, applying the momentum equation to the control volume 1, they obtained equation 7.

$$\rho q V \cos\phi + 0.5\gamma y_p^2 = \rho q V_1 + 0.5\gamma y_1^2 \qquad (7)$$

Where γ, ρ are respectively the specific weight and mass density of the fluid, V is average hitting jet velocity with stilling pool and ϕ is its angle with horizontal (Fig. 1). It is assumed that the shear force at the bed is negligible. For the subcritical flow that approaches the drop structure at critical depth, the momentum and energy equation for the control volume 2 reduce to equations 8 and 9 respectively.

$$0.5\gamma y_c^2 + \rho q V_c = \rho q V \cos\phi \qquad (8)$$

$$h + 1.5 y_c = V^2 / 2g + y_p \qquad (9)$$

In fact the last relation is the continuity equation at supercritical condition of flow along the drop structure. As can be seen there are only four equations for five unknown including V, ϕ, y_p, V_1, y_1. To complete the equations, Chamani and Rajaratnam [7] used equation 10 that is an empirical equation.

$$y_p / h = 1.107(y_c / h)^{0.719} \qquad (10)$$

Artificial Neural Networks (ANNs)

ANNs are parallel information processing system. A neural network consists of a set of neurons or nodes arranged in layers and in the case that weighted inputs are used, these nodes provide suitable inputs by conversion functions [8]. Any layer consists of pre-designated neurons and each neural network includes one or more of these interconnected layers. Fig. 2 represents a three layered structure that consists of one input layer, I, one hidden layer, H, and one output layer, O. The operation process of these networks is so that the input layer accepts the data and the intermediate layer processes them and finally the output layer displays the resultant outputs of the model application. During the modeling stage, the coefficients related to the present errors in the nodes are corrected through comparing the model outputs with the recorded input data [9].
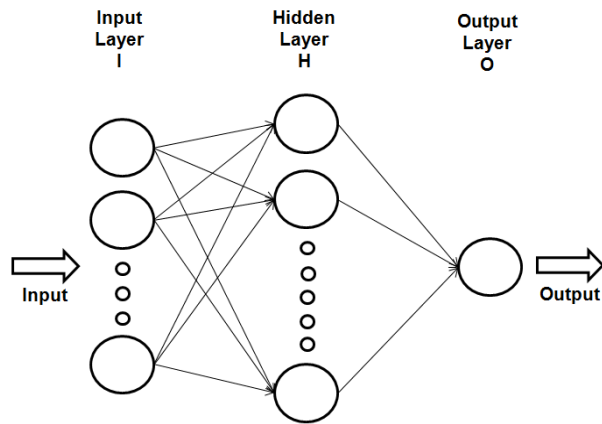

Fig. 2 Neuron Layout of Artificial Neural Networks (ANN)

In recent years, artificial neural network (ANN) models have attracted researchers in many disciplines of science and engineering, since they are capable of correlating large and complex datasets without any prior knowledge of the relationships among them. ANNs were applied by Yuhong and Wenxin [10] to predict the friction factor of open channel flow, by Zahiri and Dehghani [11] to determine flow discharge in straight compound channels, by Fadare and Ofidhe [12] for prediction of friction factor in pipe flow, by Ozgur [13] to predict mean monthly stream flow, by Nakhaei [14] for estimating the saturated hydraulic conductivity of granular material and by Landeras et al. [15] for forecasting weekly evapotranspiration.

II.MATERIALS AND METHODS

Air bubbles entrainment

Flow at downstream of drops contain air bubbles and it apparent like white-water. Existing of air bubbles in the flow causes some error in reading of flow depth and as a result,

reduction of the flow shear stress. Overall, this phenomenon affects the calculation of the flow hydraulic characteristics. To eliminate this error, Matos and Quintela [16] try to calculate the flow energy dissipation in over falls using conjugate water depth of hydraulic jump. They tried to reproduce the results obtained in investigation of Diez-Cascon et al. [17]. This indirect or nonintrusive method has already been referred by Stephenson [18] and Diez-Cascon et al. [17] and applied by Tozzi [19] and Pegram et al. [20]. The present study tries to cope with this problem to reduce the error of the developing models.

Experimental apparatus and measurements

The experiments were conducted at the hydraulic laboratory at Shahid Chamran University (SCU), Ahwaz, Iran[21]. Three drops were built from Plexiglas. Two drops with heights of 18.5cm and 31.6cm that were installed in a tilting rectangular flume of 0.25m width. Third drop with 70cm height was installed in a rectangular flume with 0.50m width. The length of the first flume is 12 m and its height is 0.48 m. The length of the second flume is 8m and its height is 1.5 m. Water is pumped into 4.5m constant head tank. Then, water is entered to the approach channel with a 10 inches pipe using a butterfly valve for adjusting the flow rate. Care was taken to minimize turbulence and swirl in the approaching channel. Three drops were installed at 4.0m downstream of the entrance section of the laboratory flume. At end of the laboratory flume, a vertical slide gate was installed for control of the water surface. In operation, the position of the slide gate was adjusted by a screwed rod to form a hydraulic jump in the basin and to locate the jump close to (but not drowning) the toe of the drops. The maximum discharge through the flumes was 60 L s^{-1}. The discharge was measured by a triangle weir with 53° installed in sidewall of a 1.5m * 2 m box at the downstream of flume. A total of 28 discharges were used for the three drops, providing a range of 0.02 to 0.94 for y_c/h.

Water levels, drops elevations, and stream-bed elevations were measured with a manually operated point gauge equipped with a vernier, readable to within 0.1mm accuracy. In each run test, water depth was measured at the upstream of drop, downstream before and after of hydraulic jump and above triangular weir. y_2 was measured where there were few bubbles in the tail water, and the precision of measurement of y_2 was achieved to within a repeatable range of 3mm for all flow conditions. All the measurements were made in the centre plain. The average flow velocity has been calculated using the measured flow rate and the depth. Tables I until IV present some primary experimental data for present study, Moore [3], Chamani and Rajaratnam [7] and Rand [6] in vertical drop.

TABLE I

EXPERIMENTAL DATA y_c/h VS. $\Delta H/H_t$ IN PRESENT STUDY [21]

Row	1	2	3	4	5	6	7	8	9	10	11	12	13	14
y_c/h	0.94	0.71	0.58	0.58	0.41	0.25	0.24	0.08	0.07	0.04	0.31	0.22	0.13	0.06
$\Delta H/H_t$	21.75	6.55	15.05	4.79	23.06	35.56	24.10	43.46	33.07	84.07	34.19	44.45	54.75	48.65
Row	15	16	17	18	19	20	21	22	23	24	25	26	27	28
y_c/h	0.02	0.18	0.17	0.17	0.17	0.16	0.16	0.15	0.14	0.12	0.10	0.08	0.05	0.04
$\Delta H/H_t$	94.43	49.02	49.26	53.02	56.76	52.95	54.30	54.61	58.41	61.59	76.33	83.42	91.71	88.42

TABLE II
EXPERIMENTAL DATA y_c / h VS. $\Delta H / H_t$ IN MOORE [3] STUDY

Row	1	2	3	4	5	6	7	8	9	10	11
y_c / h	0.07	0.09	0.11	0.17	0.2	0.25	0.255	0.325	0.335	0.5	1
$\Delta H / H_t$	57.02	53.1	52.22	37.04	33.13	27.84	23.91	20.11	20.92	13.11	7.82

TABLE III
EXPERIMENTAL DATA y_c / h VS. $\Delta H / H_t$ IN CHAMANI AND RAJARATNAM [7] STUDY

Row	1	2	3	4	5	6	7	8
y_c / h	0.1	0.12	0.14	0.16	0.2	0.25	0.3	0.35
$\Delta H / H_t$	47.03	43.91	41.81	37.04	37.82	28.73	23.9	20.02

TABLE IV
EXPERIMENTAL DATA y_c / h VS. $\Delta H / H_t$ IN RAND [6] STUDY

Row	1	2	3	4	5	6	7
y_c / h	0.11	0.17	0.23	0.28	0.31	0.35	0.4
$\Delta H / H_t$	48.3	21.82	17.81	19.14	17.03	17.02	16.1

III. RESULTS

Several runs of the physical models were performed and a wide range of hydraulic variables were measured. The measured data have been statistically analyzed. For improve head loss prediction, ANN was used and comparison between them performed. The following sections present these parts of the research program.

Effect of air bubbles entrainment

Energy at downstream before hydraulic jump calculated with y_1 and y_2. Where y_1 is depth before hydraulic jump and y_2 is depth after hydraulic jump. Energy at the toe of drop is calculated using equations 11 and 12.

$$H_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} \tag{11}$$

$$y_1 = \frac{1}{2} y_2 (\sqrt{1 + 8(y_c / y_2)^3} - 1) \tag{12}$$

Relative energy loss is expressed by equation 13.

$$\frac{\Delta H}{H_t} = \frac{H_t - H_1}{H_t} \tag{13}$$

Relative energy loss between upstream of drop and downstream of drop (before hydraulic jump), are calculated using two methods as follows:

(a)-Use of measured depth before hydraulic jump (y_1).

(b)-Use of measured depth after hydraulic jump (y_2).

To apply the second method, depth of clear water (without air bubbles) after hydraulic jump was measured. Fig. 3 illustrates the results of the energy loss analysis by both methods for the drop with 70cm height.
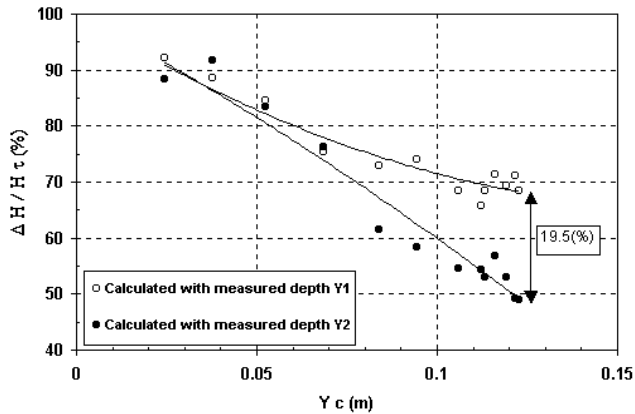
Fig. 3 Effect of air bubbles on relative energy loss at drop with 70cm height

As shown in Fig. 3, the relative energy loss is a function of discharge (or y_c). Increasing in discharge results the decreasing in relative energy loss and shows larger difference between calculated energy dissipation by two methods. The value of difference reaches 19.5% for the maximum discharge which denotes when the aerated-flow depth (y_1) is used to calculate the energy loss, it is significantly overestimated.

Comparison of different studies results

To find the accuracy of this work, authors have compared findings of the present study with the other reported studies in the literature. Fig. 4 has shown this comparative section. Overall, Fig. 4 shows that the relative energy loss at drop structures is decreased with increasing in h/y_c. Good agreement between the present study and Moore [3], Chamani and Rajaratnam [7] and Rand [6] exist. As shown in the Fig. 4, White's [4] result is more higher than the present study results and the other studies findings. It can be seen that the relative energy loss in this study varies from almost 8% (for $y_c/h=0.94$) to 94.3% (for $y_c/h=0.02$).Based on Fig. 4, Rand [6] results show lower energy dissipation and White [4] results show higher energy dissipation. USBR [22] suggested use of equation (2) for energy dissipation estimation for drop design. Average 11% energy dissipation for inclined jet in Chamani and Rajaratnam [7] experiment, show that the energy dissipation at a drop is mainly due to the mixing of the jet with the pool behind the jet.By application of EXCEL software for regression, two best curve fitness for data obtained as equations 14 and 15 for present study data.
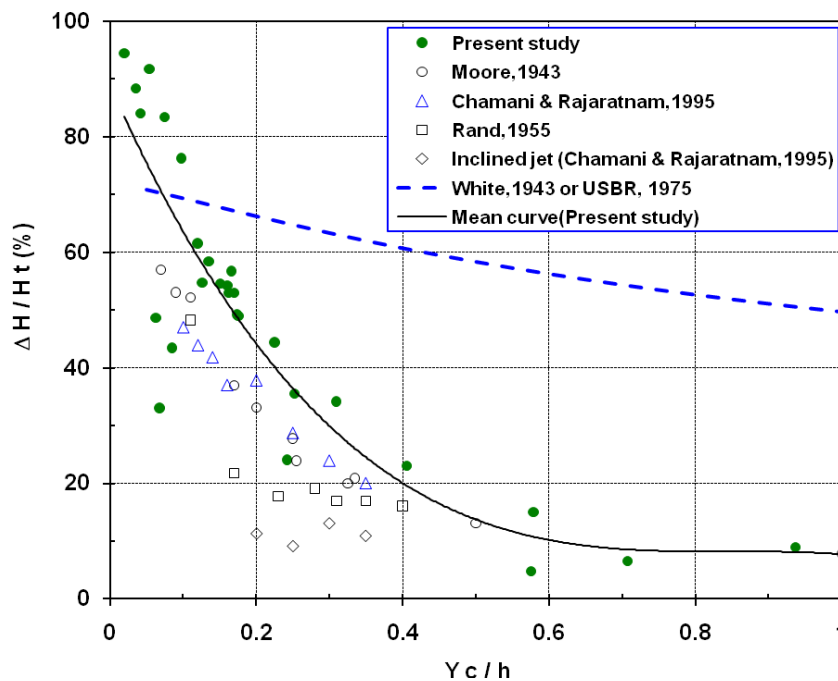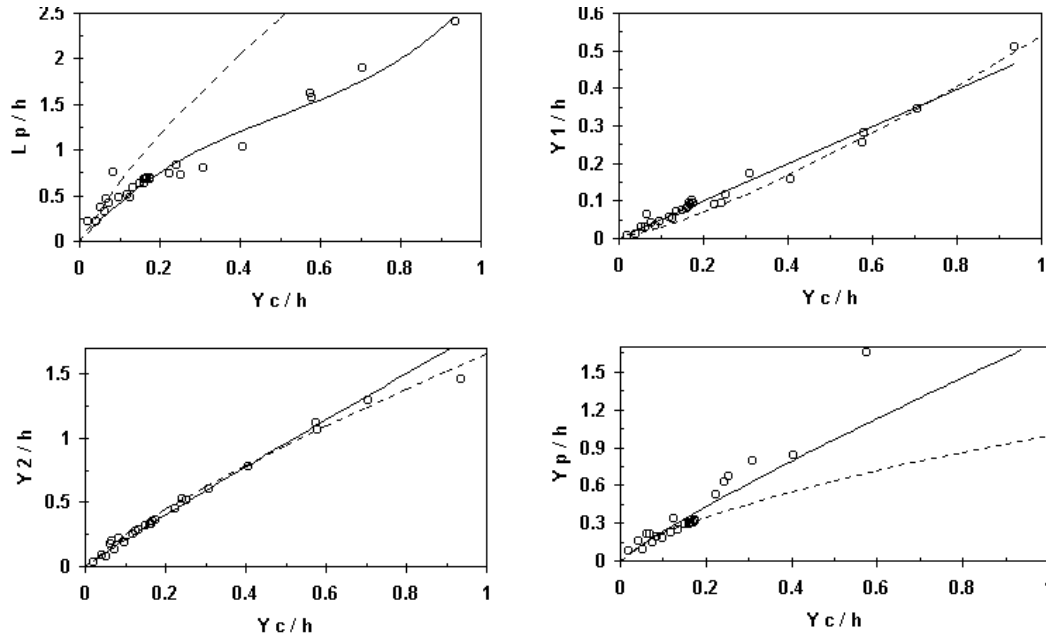

Fig. 4 Comparison of different studies results

Fig. 5 Relation between y_1/h, y_2/h, L_p/h and y_p/h vrs. y_c/h in drop

(———— Present experiment and - - - - - - - - Rand [6] experiment)

$$\Delta H / H_t = -245.15(y_c / h)^3 +$$
$$554.22(y_c / h)^2 - 390.92(y_c / h) + \qquad (14)$$
$$100 \ , n = 28 \ , \ R^2 = 0.74$$

$$\Delta H / H_t = 12.521(y_c / h)^{-0.6343}$$
$$n = 28 \ , \ R^2 = 0.65 \qquad (15)$$

In equations 14 and 15, n is number of experimental data and R^2 is deterministic coefficient.

In Fig. 5 the results of measured y_1, y_2, L_p and y_p with comparison the Rand's [6] experiment was presented.

Present experiment results show that there is good fitness for y_1/h and y_2/h value with Rand [6] experiments. But for L_p/h and y_p/h there are not fitness between present and Rand [6] experiment. Probably high oscillated impinging jet in pool behind drop wall reduces the accuracy of depth measurement. Results of application of regression analysis gave the best curve fitness for observation data as equations 16-19:

$$L_p / h = 4.181(y_c / h)^3 - 6.288(y_c / h)^2$$
$$+ 4.860(y_c / h) \ , \ n = 28 \ , \ R^2 = 0.945 \qquad (16)$$

$$y_p / h = 1.772(y_c / h)^{0.879}$$
$$n = 28 \ , \ R^2 = 0.881 \qquad (17)$$

$$(y_1 / h) = 0.496(y_c / h)$$
$$n = 28 \ , \ R^2 = 0.976 \qquad (18)$$

$$(y_2 / h) = 1.857(y_c / h)^{0.952}$$
$$n = 28 \ , \ R^2 = 0.970 \qquad (19)$$

In Fig. 6 the results of measured experimental for present study for 28 data point, Moore [3] for 11 data point, Chamani and Rajaratnam [7] for 8 data point and Rand [6] for 7 data point in vertical drop was presented.

Results of application of regression analysis gave the best curve fitness for all observation data referred in Fig. 6 as equation 20 (forth order polynomial) and equation 21(logarithm).

$$\Delta H / H_t = 278.7(y_c / h)^4 - 777.74(y_c / h)^3$$
$$+ 839.71(y_c / h)^2 - 419.15(y_c / h) + 92.506 \qquad (20)$$
$$n = 54 \ , \ R^2 = 0.7757$$

$$\Delta H / H_t = -24.64 Ln(y_c / h) - 1.8281$$
$$n = 54 \ , \ R^2 = 0.7621 \qquad (21)$$

ANN modeling

The ANN software program employed was NeuroSolutions. The first step to designing any neural network is to collect training data. By using the "Browse" button to select an input file, NeuralBuilder will scan the input file and present a list of the columns that it finds. The data for training the ANN model were generated using the experimental data on vertical drop described above. A dataset consisting of a total of 54 points resulting from the combination of different y_c/h as inputs and $\Delta H / H_t$ as output was used for training the ANN model.

10% of total input data was selected as test data and 10% of total input was selected as cross validation. Cross validation is a highly recommended method for stopping network training.

This method monitors the error on an independent set of data and stops training when this error begins to increase. This is considered to be the point of best generalization. Panel in Fig. 7, allows specifying both the cross validation and testing data sets.
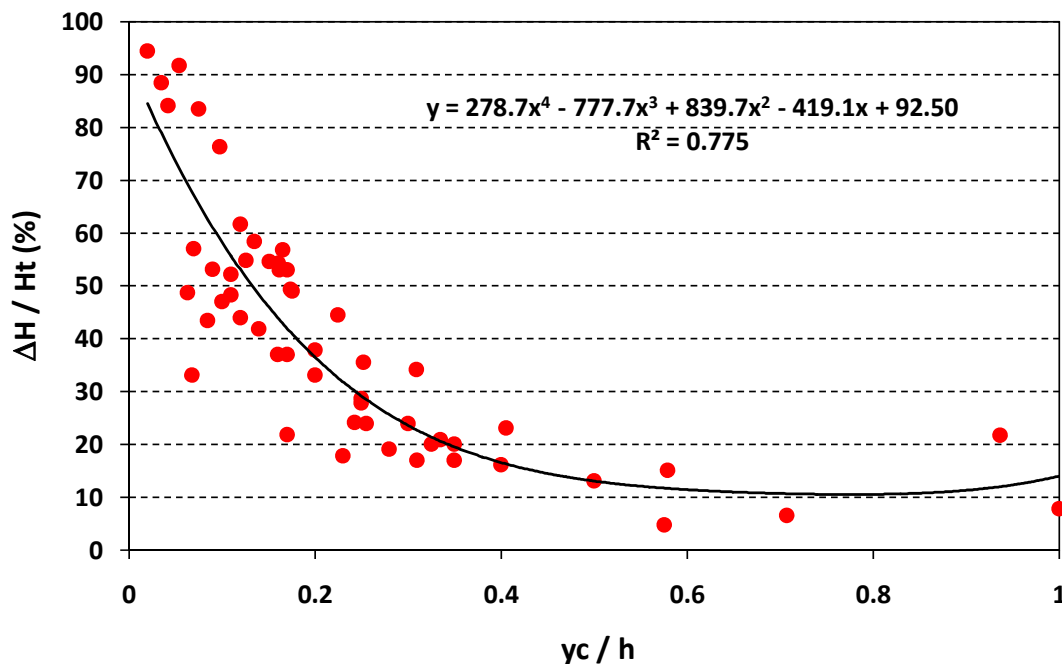

Fig. 6 Relation between $\Delta H / H_t$ vrs. y_c/h in vertical drop for all data (present study; Moore [3]; Chamani and Rajaratnam [7] and Rand [6])

TABLE V
PREDICTION ERRORS FOR THE TRAINING AND TESTING DATASET OF THE HEAD LOSS ASSOCIATED WITH DIFFERENT ANN CONFIGURATIONS

Transfer function	No. of hidden layers	No. of neurons/layer	Training		Test	
			RMS	R^2	RMS	R^2
Sigmoid	1	2	0.0384	0.978	0.0422	0.968
Sigmoid	1	3	0.0375	0.978	0.0406	0.974
Sigmoid	1	4	0.0383	0.977	0.0386	0.977
Sigmoid	**1**	**5**	**0.0085**	**0.977**	**0.0039**	**0.976**
Sigmoid	1	6	0.0388	0.977	0.0382	0.976

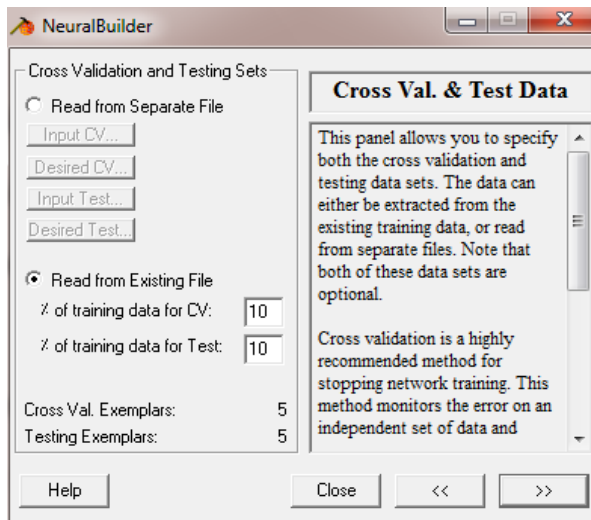| Sigmoid	1	8	0.0369	0.977	0.038	0.977
Sigmoid	1	10	0.0399	0.976	0.038	0.974
Hyperbolic Tangent	1	5	0.0372	0.978	0.0388	0.976
Gaussian	1	5	0.0404	0.974	0.0374	0.979
Sigmoid	2	2 , 2	0.0385	0.977	0.0411	0.973
Sigmoid	2	2 , 3	0.0401	0.974	0.0374	0.979

Fig. 7 Panel in NeuroSolutions allows to specify both the cross validation and testing data sets

The optimal ANN configuration was selected from amongst various ANN configurations based on their predictive performance. The two error measures used to compare the performance of the various ANN configurations were: determination coefficient (R^2) and root mean square error (RMSE).In order to find the optimal network, several configurations were tried in which the number of hidden layers varied from one to two and the number of neurons within each hidden layer varied from two to 10 (Table V). Multilayer perceptrons (MLPs) are layered feed forward networks typically trained with static back propagation. These networks have found their way into countless applications requiring static pattern classification. Their main advantage is that they are easy to use, and that they can approximate any input/output map. The key disadvantages are that they train slowly, and require lots of training data (typically three times more training samples than network weights).Panel in Fig. 8 is used to specify the parameters a layer of processing elements (PEs). NeuroSolutions simulations are vector based for efficiency. This implies that each layer contains a vector of PEs and that the parameters selected apply to the entire vector. The parameters are dependent on the neural model, but all require a nonlinearity function to specify the behavior of the PEs. In addition, each layer has an associated learning rule

and learning parameters. The numbers of PEs and learning parameters are entered in the corresponding fields. The learning rule and nonlinearity are selected from a list of options contained within pull-down menus.
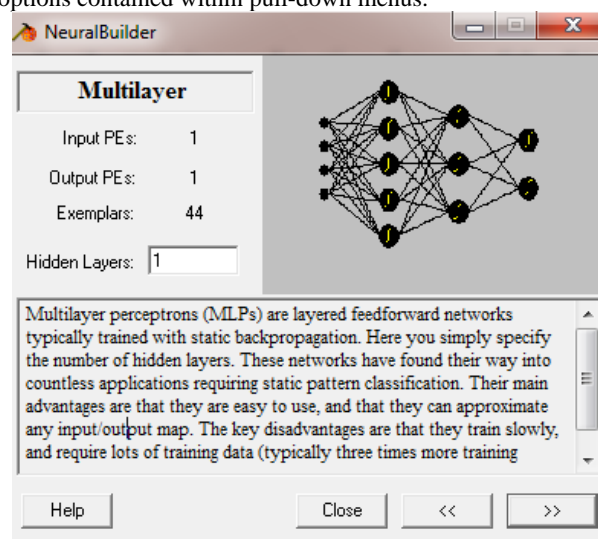

Fig. 8 Panel for specify the parameters a layer of processing elements (PEs)

Based on Table V, ANN with one hidden layer has enough accuracy and architecture 2,5,1 (input layer having two neurons, one hidden layer with 5 neuron; one output neuron) have minimum RMS and maximum R2. So architecture 2,5,1 with Sigmoid transfer function could be selected in this case. It is seems that using ANNs for head loss in drops or similar problems is due to researchers interesting to check how ANNs, work for nonlinear problems. In this way, we have tried also to handle ANNs capability in nonlinear relation between y_c/h and $\Delta H / H_t$. As shown in Fig. 9, result is acceptable in estimating head loss.Trained network was able to predict the response with R^2 and RMSE 0.977 and 0.0085 respectively (Table V). This ANN model is capable of predicting the values of head loss in vertical drop and was in close agreement with experimental results.
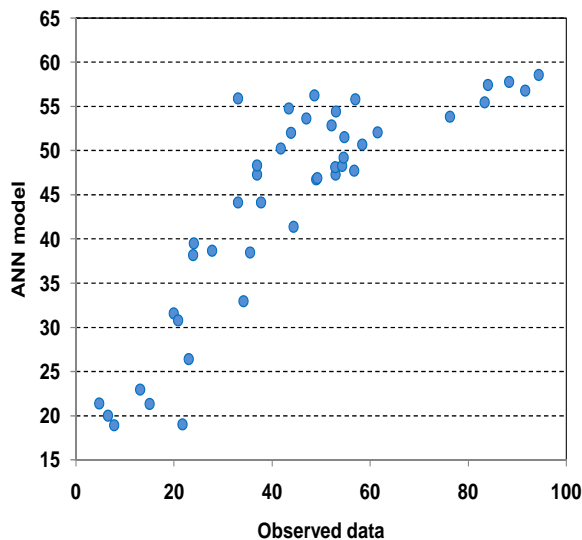
Fig. 9 Plot of *f* values as predicted by ANN and observed data

IV. CONCLUSION

This paper presents an experimental study the results of hydraulic laboratory study on the energy dissipation in three physical models. Relative energy dissipation on drops calculated by using both y_1 and y_2 (sequent depth). In section where y_1 measured, there was air bubble in the flow, while in section where y_2 measured there was little air entrainment. In addition ANN model has R^2 and RMSE values 0.977 and 0.0085 respectively and was capable of predicting the values of head loss in vertical drop and was in close agreement with laboratory tests.

Results showed that:

- Application of measured depth y_1 instead of sequent depth (y_2), results more energy dissipation. In facs air entrainment in flow (reductions in viscasity), causes to redue shear stress and finlay redues the energy dissipation property. Also air entrainment causes to increase flow bulk and so flow depth. This problem causes erorr in calculating energy loss.

- Increasing in discharge, causes reduction in energy dissipation.

- Comparison between this study's results and results of Moore [3], Rand [6], White [4], Rajaratnam and Chamani [7], showed that White's [4] model over estimates the energy dissipation in drops and Rand's [6] model under estimates the energy loss in drops. The others can predict the energy dissipation in drops as same as the proposed statistical model.

- This work has also indicated that the loss at drop is mainly due to the mixing of the jet with the pool behind the jet.

- ANN model is capable of predicting the values of head loss in vertical drop and was in close agreement with experimental results.

REFERENCES

[1] W .H. Hager, "Hydraulic of plane free overfall", Journal of Hydraulic Engineering, 109(12): 1683-1697, 1983.
[2] H. Chanson, "Comparition of energy dissipation between nappe and skimming flow regims on stepped chutes". Journal of hydraulic research. 32(2): 213-218, 1994.
[3] W. L. Moore, "Energy loss at the base of free overfall", Transactions, SCE,Vol.108, pp1343-1360, 1943.
[4] M. P. White, "Discussion of Moore",Transactions, ASCE, Vol.108, pp 1361-1364, 1943.
[5] Gill, M.A. "Hydraulic of rectangular vertical drop", Journal of Hydraulic Research, Vol 17, No. 4,pp 289-302, 1979.
[6] W. Rand, "Flow geometry at straitght drop spillway". Proceedings, ASCE,Vol.81, Paper 791, pp1-13, 1955.
[7] M. R. Chamani, and N. Rajaratnam, "Energy loss at drops", Journal of Hydraulic Research, 7(2): 373-384, 1995.
[8] K. Ozgu, "Daily River Flow Forecasting Using Artificial Neural Networks and Auto-Regressive Models", Turkish J. Eng. Env. Sci., 29, 9-20, 2005.
[9] G. R. Rakhandehroo, M. Vaghefi, and M. M. Shafiee, "Flood forecasting in similar catchments using neural networks", Turkish J. Eng. Env. Sci., 34, 57-66, 2010.
[10] Z. Yuhong, and H. Wenxin, "Application of artificial neural network to predict the friction factor of open channel flow". Commun. Nonlinear Sci. Numer. Simulat. (14): 2373–2378, 2009.
[11] A. Zahiri and A. A. Dehghani, "Flow Discharge Determination in Straight Compound Channels Using ANNs", World Academy of Science, Engineering and Technology, (58): 12-15, 2009.
[12] D. A. Fadare and U. I. Ofidhe, "Artificial Neural Network Model for Prediction of Friction Factor in Pipe Flow" Journal of Applied Sciences Research, 5(6): 662-670, 2009.
[13] K. Ozgur, "River Flow Modeling Using Artificial Neural Networks", Journal of Hydrologic Engineering, 9(1): 60-63, 2004.
[14] M. Nakhaei, "Estimating the Saturated Hydraulic Conductivity of Granular Material, Using Artificial Neural Network", Based on Grain Size Distribution Curve. Journal of Sciences, Islamic Republic of Iran. University of Tehran. 16(1): 55-62, 2005.
[15] G. Landeras, A. Ortiz-Barredo, and J. Javier López, "Forecasting weekly evapotranspiration with ARIMA and Artificial Neural Network models". Journal of Irrigation and Drainage Engineering, 135(3): 323-334, 2009.
[16] J. Matos and A/ Quintela, "Discussion on Jet flow on stepped spillways". Journal of hydraulic engineering. 120(2): 443-444, 1994.
[17] J. Diez-cascon, I. L. Blanco, J. Reviua, and R. Garcia, "Studies on the hydraulic behaviour of stepped spillways", Int. water power and dam construction. pp 22-26, 1991.
[18] D. Stephenson, "Gabion energy dissipaters". Proc., 13th ICOLD Conger., New Delhi, India, 50(3): 33-43, 1979.
[19] M. J. Tozzi, "Flow characterization/behavior on stepped spillways". Ph.D. thesis, University of sao paulo- Brazil (in portuguese), 1992.
[20] G. G. S. Pegram, and A.K. Officer, and S.R. Mottram, "Hydraulic of skimming flow on modeled stepped spillways". Journal of hydraulic engineering. 125 (5): 500-509, 1999.
[21] F. Salmasi, Hydraulic investigation and physical modeling on stepped spillways, Ph.D. dissertation, Shahid Chamran University, Ahvaz, Iran, 2004.
[22] US Department of the interior, Bureau of Reclamation, USBR, "Design of small canal structures",1978.