Energy Based Temperature Profile for Heat Transfer Analysis of Concrete Section Exposed to Fire on One Side

Pattamad Panedpojaman

Abstract—For fire safety purposes, the fire resistance and the structural behavior of reinforced concrete members are assessed to satisfy specific fire performance criteria. The available prescribed provisions are based on standard fire load. Under various fire scenarios, engineers are in need of both heat transfer analysis and structural analysis. For heat transfer analysis, the study proposed a modified finite difference method to evaluate the temperature profile within a cross section. The research conducted is limited to concrete sections exposed to a fire on their one side. The method is based on the energy conservation principle and a pre-determined power function of the temperature profile. The power value of 2.7 is found to be a suitable value for concrete sections. The temperature profiles of the proposed method are only slightly deviate from those of the experiment, the FEM and the FDM for various fire loads such as ASTM E 119, ASTM 1529, BS EN 1991-1-2 and 550 $^{\circ}$C. The proposed method is useful to avoid incontinence of the large matrix system of the typical finite difference method to solve the temperature profile. Furthermore, design engineers can simply apply the proposed method in regular spreadsheet software.

Keywords—temperature profile, finite difference method, concrete section, one-side fire exposed, energy conservation

I. INTRODUCTION

REINFORCED concrete (RC) members are impacted under fire due to raising temperatures in their cross sections. The high temperature significantly reduces the mechanical properties of concrete and steel [1]. Fire resistance of RC members are generally specified in codes and standards such as AS 3600 [2], Eurocode 2 [3] and ACI 216.1 [4]. The provisions specify minimum cross-section dimensions and minimum clear cover to the reinforcing bars based on experimental tests or pre-determined analysis under specific fire curves such as ASTM E 119 [5], ISO 834[6] etc. As a result, they are prescriptive and cannot evaluate the fire resistance under different fire scenarios and conditions. For different fire scenarios, engineers are in need of alternative design tools.

To investigate the structural behavior of concrete structures, both heat transfer analysis and structural analysis are required. For heat transfer analysis, the finite element method (FEM) has proven to be a powerful method to predict the temperatures in reinforced concrete sections during fire exposure [7, 8]. Difficulty of using the FEM or cost of finite element software makes it impractical for design engineers.

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The finite difference method (FDM) is considered as a simpler method for evaluating temperature profile within a cross section exposed to fire. The FDM is widely developed and adopted for RC sections during fire exposure [9-12]. Note that the effect of the reinforcing steel of RC sections on the heat transfer analysis is neglected due to its small area relative to concrete area [9].
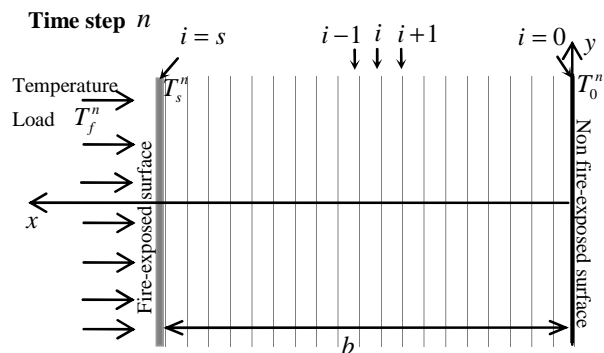

Fig. 1 Nodal network of a section exposed to a fire on its one side

The previous researches [9-12] were conducted based on the typical FDM. In the FDM, the physical system of a cross-section exposed to a fire on its one side is represented by a nodal network as shown in Fig. 1. The FDM replaces the governing equations and corresponding boundary conditions of heat transfer analysis by a set of algebraic equations. When the temperatures of all nodal points are known at any particular time t, the temperatures after a time increment Δt can be computed [13]. Size of the grid spacing Δx and the time increment Δt depends on geometry of cross sections, accuracy of the solution and the stable condition of the method. To compute the temperature profile, the method establishes a matrix of the temperatures of all nodal points at each time step. For a case of the large matrix generated, the method may not be convenient to find its solution corresponding to the stable condition.

To avoid the incontinent of the large matrix to solve the temperature profile, this study modified the FDM based on the pre-determined shape function of the temperature profile and the energy conservation. The research conducted in this paper is limited to concrete sections exposed to a fire on their one side.

II. ENERGY BASED HEAT TRANSFER ANALYSIS

Consider sections exposed to a fire on their one side which is infinite in the direction of the y-coordinate, with the thickness b in direction of the x-coordinate as shown in Fig.

1. For isotropic and homogeneous media, the conductive heat flux q in the x direction is given by Fourier's heat conduction law as

$$-k\frac{\partial T}{\partial x} = q \tag{1}$$

where k is the thermal conductivity and T is the temperature.

The governing differential equation of the heat conduction is considered based on the energy conservation for conduction through an elemental volume. The energy conservation consists of net rate of heat entering by conduction, rate of energy generated internally and rate of increase of internal energy. In one-dimensional Cartesian coordinates, the full conduction equation is derived as

$$k\frac{\partial^2 T}{\partial x^2} + g = \rho c\frac{\partial T}{\partial t} \tag{2}$$

where g is the heat generation and c is the specific heat.

For a system of the one dimensional with the constant thermal properties and without the heat generation (i.e., $g = 0$), unsteady-state conduction problem is governed by

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \tag{3}$$

This equation is employed to compute temperatures of the internal section as a function of time. Once a convection and radiation boundary condition exists such as in case of fire, the boundary has to be considered separately. For the one dimensional system, the boundary condition at $x = b$ is

$$-k\frac{\partial T}{\partial x} = q = -h\left(T_f - T_s\right) - \varepsilon\sigma\left(T_f^4 - T_s^4\right) \tag{4}$$

T_f is the fire temperature which is a function of time. Based on (3) and (4), the temperature profile in the section with a convection and radiation boundary condition can complicatedly be solved. The FEM can be used to evaluate the solution.

A. Finite Difference Method

The finite difference method is a numerical technique which can be applied to the partial differential equations. The finite difference of derivatives involves the approximation of a differential equation by algebraic equations. The equations are pointwise continuous which is applicable throughout the region and space considered.

To apply the FDM, a network of grid points by dividing x and t domains into small intervals of Δx, as shown in Fig. 1, and Δt. According to the network, T_i^n represents the temperature at location $x = i\Delta x$ at $t = n\Delta t$. i is the number of a grid point whereas n is the number of a time step. If the forward-difference approximation is used in (3), the finite-difference equation is represented as

$$\frac{T_{i+1}^n + T_{i-1}^n - 2T_i^n}{\left(\Delta x\right)^2} = \frac{k}{\rho c}\frac{T_i^{n+1} - T_i^n}{\Delta t} \tag{5}$$

The finite difference equation in (5) is employed to compute temperatures of the internal grid points as a function of time.

Note that from the forward-difference approximation of time derivative, the solutions are not stable for all situations. To avoid the violation of the second law of thermodynamics, the stable condition of $(k/\rho c)\left(\Delta t\Delta x^2\right) \leq 0.5$ has to be satisfied [13].

The finite difference form of the boundary condition at $i = s$, (4) may be expressed as

$$-k\frac{T_s^n - T_{s-1}^n}{\Delta x} = q^n \tag{6}$$

where

$$q^n = -h\left(T_f^n - T_s^n\right) - \varepsilon\sigma\left(\left(T_f^n\right)^4 - \left(T_s^n\right)^4\right) \tag{7}$$

Due to the intervals of Δx in the FDM, the effect of the heat capacity of the system next to the boundary must be included in (6) [13] as

$$-k\frac{T_s^n - T_{s-1}^n}{\Delta x} - q^n = \rho c\frac{\Delta x}{2}\frac{T_s^{n+1} - T_s^n}{\Delta t} \tag{8}$$

This equation can be rearranged as

$$T_s^{n+1} = \frac{2\Delta t}{\rho c\Delta x}\left[-k\frac{T_s^n - T_{s-1}^n}{\Delta x} - q^n\right] + T_s^n \tag{9}$$

Therefore, T_s^{n+1} can be solved based on temperature of the previous step.

If T_f is an increasing functions, the value of T_s^{n+1} must be larger than the value of T_s^n. As a result, another stable condition has to be satisfied:

$$-k\frac{T_s^n - T_{s-1}^n}{\Delta x} - q^n > 0 \tag{10}$$

To compute the temperature profile, the method employs a matrix of the temperatures of all nodal points at each time step. For a case of the large matrix, the method may be difficult to find its solution corresponding to the stable condition.

B. Modified Finite Difference Method

To simplify the FDM, this study adopts the energy conservation principle and a pre-determined shape function of the temperature profile. Based on the energy conservation principle, the provided heat energy from fire load Q_p balances the received heat energy Q_r, which is the internal energy in the fire-exposed section. The internal energy is to take into consideration the heat capacity of the system and the temperature profile as shown in Fig. 2. The energy equations are describes as follows:

$$Q_p^n = Q_r^n \tag{11}$$

$$Q_p^n = \sum_{m=1}^n A\left(-q^m\right)\Delta t \tag{12}$$

$$Q_r^n = \int_0^b \rho cA\left(T^n(x) - T_r\right)dx \tag{13}$$

A is the surface area. Due to a uniform temperature load through out the fire-exposed surface is assumed, the surface

area can be considered as unit area (i.e., $A = 1$). $T^n(x)$ is the energy based temperature profile within the cross section.
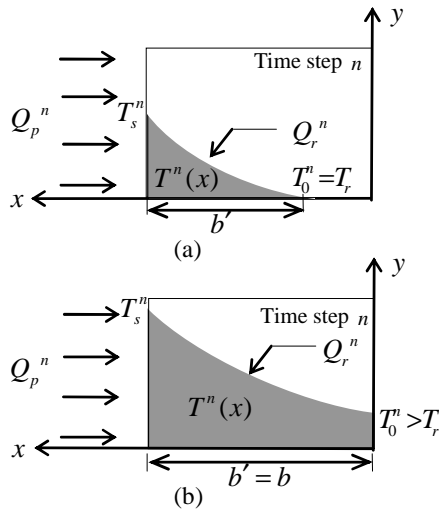

(a)

(b)

Fig. 2 Energy conservation of the fire-exposed section: (a) Low energy case; (b) High energy case

Through the energy conservation principle, T_s^{n+1} in (9) and the temperature profile can simply be solved when the shape function of $T^n(x)$ is known. In the study, the shape function is assumed to be a power function as

$$T^n(x) = Cx^\alpha + T_0^n \tag{14}$$

where

$$C = \frac{T_s^n - T_0^n}{b'^\alpha} \tag{15}$$

α is the power of the function; and
b' is described in Fig.2.

Equation (9) can be modified by substituting the term of $\left(T_{s-1}^n - T_s^n\right)/\Delta x$ with the derivative function of (14) as described in (16). The equation of T_s^{n+1} in (9) and the stable condition in (10) can be rewritten in (17) and (18), respectively.

$$\frac{T_{s-1}^n - T_s^n}{\Delta x} \approx \left.\frac{dT^n(x)}{dx}\right|_{x=b} = \alpha\frac{T_s^n - T_0^n}{b} \tag{16}$$

$$T_s^{n+1} = \frac{2\Delta t}{\rho c \Delta x}\left[-k\alpha\frac{T_s^n - T_0^n}{b'} - q^n\right] + T_s^n \tag{17}$$

$$-k\alpha\frac{T_s^n - T_0^n}{b'} - q^n > 0 \tag{18}$$

For each Δt, the difference between T_f^n and T_s^n varies. When the difference is narrow, the value of $-q^n$ in (7) may not enough to satisfy (18). The dissatisfaction can exist in any time step in both typical FDM and proposed method. Once the dissatisfaction is found, Δt should be increase otherwise such step should be neglected and postponed to the next step. Note

that due to the pre-determined shape function cannot violate of the second law of thermodynamics within the sections, the stable condition of $(k/\rho c)\left(\Delta t \Delta x^2\right) \leq 0.5$ is omitted in the proposed method. When the value of α is pre-determined, only T_s^n and T_0^n are unknown variables to solve T_s^{n+1} in (17). Therefore, the large matrix system of the FDM can be avoided.

To specify $T^n(x)$ in (14), T_s^n can be obtained from (17) of the previous step whereas T_0^n can be compute through the energy conservation equation (11) and (13). Two cases of the energy based temperature profile as shown in Fig. 2 are considered as follows:

1) Low energy case: $T_0^n = T_r$ and

$$b' = \frac{Q_p^n(\alpha+1)}{\rho c\left(T_s^n - T_r\right)} \tag{19}$$

2) High energy case: $T_0^n > T_r$, $b' = b$ and

$$T_0^n = \frac{1}{\alpha}\left[(\alpha+1)\left(\frac{Q_p^n}{\rho cb} + T_r\right) - T_s^n\right] \geq T_r \tag{20}$$

where T_r is the room temperature. T_0^n

The procedure to predict the temperature profile in the fire exposed section under a fire load can be summarized as illustrated in Fig. 3.
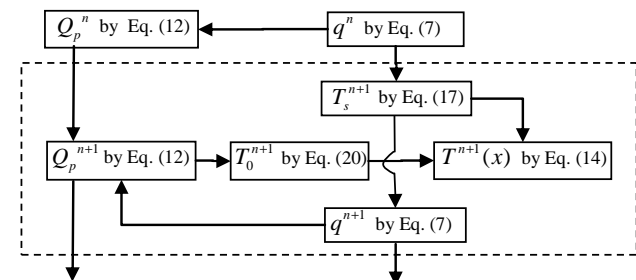

Fig. 3 Procedure to predict the temperature profile in a section at time step $n+1$

III. INVESTIGATION OF THE PRE-DETERMINED SHAPE FUNCTION

The research conducted is limited to concrete sections. To investigate the suitable value of the power α for concrete sections, the temperature profile of concrete sections analyzed by the FEM, ANSYS software, is compared with the proposed method with different values of the power. The concrete sections exposed to the standard fire of ASTM E 119 [5] on their one side as shown in Fig. 4 are specified in the comparison. The sections have a thickness of 100 mm and 300 mm. To analyze temperature profile in the section by the ANSYS model [14], the sections are modeled with three-dimensional solid elements, Solid70, having eight nodes with a single degree of freedom (i.e., temperature) at each node. The surface element, Surf152, is used to account for heat convection and radiation of the fire temperature.

The thermal properties of concrete as shown in Fig. 5 are specified in accordance with BS EN 1992-1-2 (2004) [15]. To simplify the computation and due to the value of c less varies with temperature, the value of c is assumed to be a constant value of 1×10^3 J/kg°K for the proposed method. The concrete density of 2400 kg/m^3 is specified to be constant for both FEM and proposed method. The coefficient of heat transfer-convection h of 25 W/m^2°K, Stefan Boltzmann constant of 5.67 x 10^{-8} W/m^2°K^4 and the resultant emissivity of 0.56 are used according to BS EN 1991-1-2 (2002) [3].
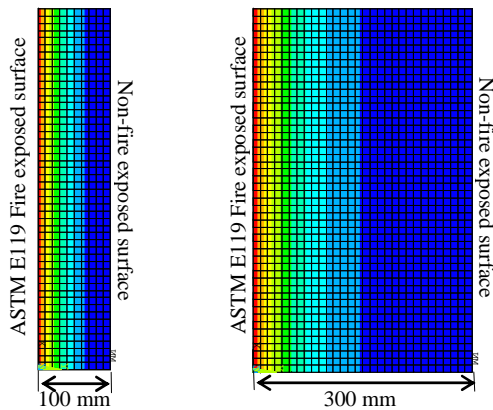
Fig. 4 Concrete sections exposed to the standard fire of ASTM E 119 and temperature profile by the FEM
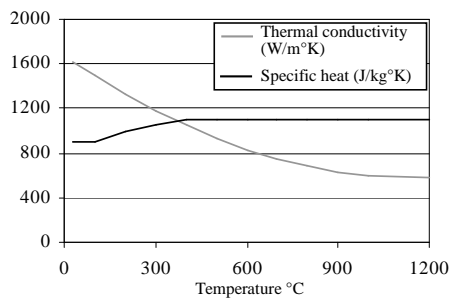
Fig. 5 Thermal properties of concrete

Size of the nodal spacing Δx = 10 mm and the time increment Δt = 2 min is specified in the computation of the proposed method and the FEM. For the proposed method, the stable condition at the fire exposed surface in (18) is checked at each time step. If the dissatisfaction exists in a time step, such step is neglected and postponed to the next time step.

The comparisons of the temperature profile in the sections between the FEM and the proposed method with different values of α are illustrated in Fig 6. It is found that the temperature profiles can be approximately represented by power function. Furthermore, the good agreements between the temperatures at the fire exposed surface obtained from FEM and the proposed method are observed. The value of α less affects the temperature of the fire exposed surface. However, the value of α is not constant but approximately in range of 2 to 3.5.
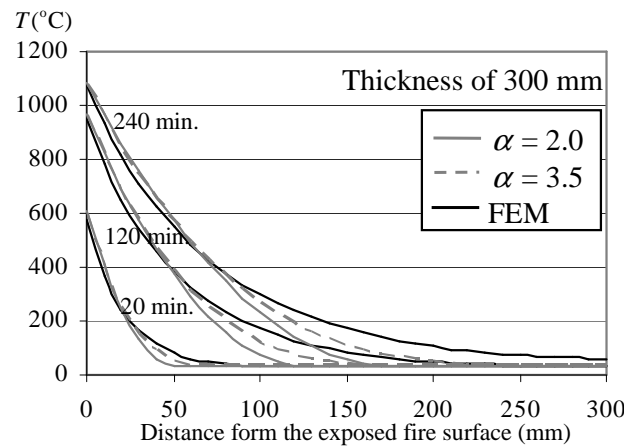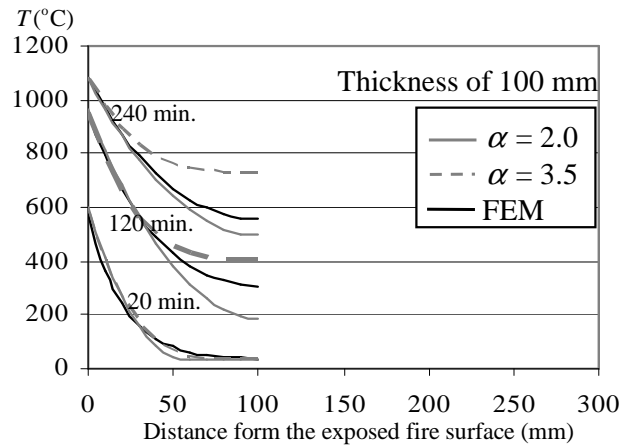
Fig. 6 comparison of the temperature profile between the FEM and the proposed method with different values of α

TABLE I
POSITIONS OF THE COMPARED TEMPERATURE AND THEIR SYMBOLS

Position	Symbol	
	Section of 100 mm thickness	Section of 300 mm thickness
Fire exposed surface	F-100	F-300
42 mm from the fire exposed surface	M-100	M-300
Non-fire exposed surface	N-100	N-300

To simply apply the energy based temperature profile for the heat transfer analysis, the α value is assumed to be a constant value of 2.7. Accuracy of the temperature predicted with the α assumption is investigated by comparing with the FEM. The temperature profiles of the sections (in Fig. 4) exposed to the standard fire of ASTM E 1529 [16], the standard fire of ASTM E 119 [5] and the temperature of 550 °C are investigated. The comparisons as shown in Fig 7 are illustrated in terms of the temperature variation with time at the specific points in the sections as described in Table 1. Note

344

that the temperature of 550 °C represents cases of low fire load. The position of 30 mm is specified as the general position of the reinforcing steel in RC members.

The good agreements of the temperature variation at the fire exposed surface are observed. Comparing with the FEM, the temperature variations of the proposed method at the other points, except the case of N-300, tends to slightly overestimate. In an overall picture, the proposed method with the α value of 2.7 can be used to predict the temperature profile of concrete sections under various fire exposures.

IV. VALIDATION

The proposed method is validated by comparing its prediction of the temperature variations with the experimental variation [17], the analytical variation of the FEM and the analytical variation of the FDM. The previous research [17] investigated the temperature in post-tensioned concrete slabs having a thickness of 160 mm and exposed to the standard fire of BS EN 1991-1-2 [15] on their one side. Thermocouples were used to measure temperature at the fire exposed surface, the non-fire exposed surface and the positions of 42 mm from fire exposed surface. The comparison of the temperature variation with time is illustrated in Fig. 8. From the illustration, it is seen that the results of the proposed method only slightly deviate from the results of the experiment, the FEM and the FDM. The energy based temperature profile can thus be used to approximate the temperature variation in concrete sections exposed to fire loads.

V. CONCLUSION AND DISCUSSION

This paper simplifies the FDM to predict the temperature profile within concrete sections exposed to a fire on their one side. The proposed method is based on the energy conservation principle and a pre-determined shape function of the temperature profile. According to the energy conservation principle, the provided heat energy from the boundary balances the received heat energy in a fire-exposed section. The heat energy in the section is to take into consideration the heat capacity of the system and the temperature profile function. In the study, the shape function is assumed to be a power function. The specific function is derived based on the energy conservation principle.

To investigate the suitable value of the power α for concrete sections, the temperature profiles of the concrete sections computed by the proposed method using different power number is compared with those computed by the FEM. It is found that the value of α less affects the predicted temperature at fire exposed surface. The value of α is not constant but approximately in range of 2 to 3.5. However, the temperature profiles of the proposed method with the α value of 2.7 slightly deviate from those of the experiment, the FEM and the FDM for various fire loads.
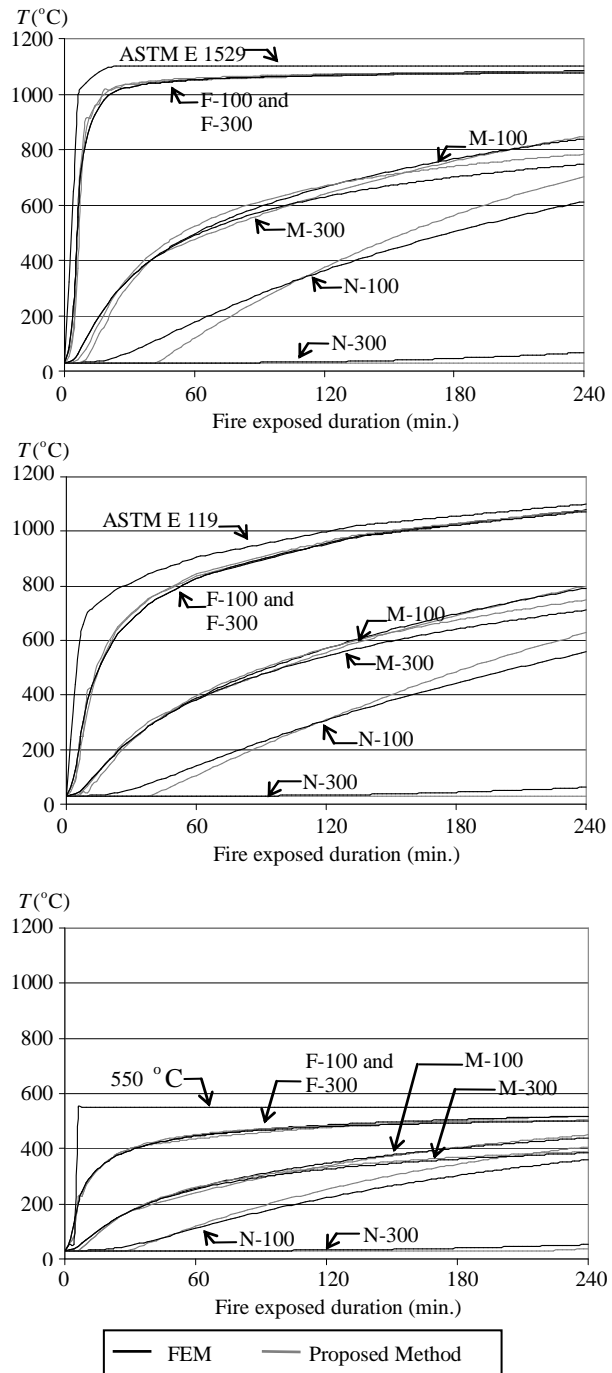
Fig. 7 Comparison of the temperature profile between the proposed method with the α value of 2.7 and the FEM
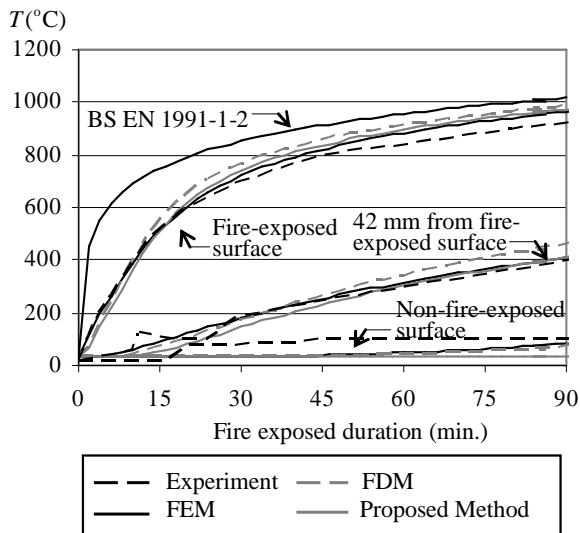
Fig. 8 Comparison of the predicted temperature variation with the experiments, the FEM and the FDM

By using the energy based temperature profile, the temperature matrix of the FDM nodal network can be avoided. The method is also useful to manipulate the stable conditions of the FDM in the section and at the fire exposed surface. Therefore, design engineers can simply apply the proposed method to evaluate the temperature profile by using regular spreadsheet software. The proposed method facilitates design engineers analyzing the heat transfer which is a necessary part to investigate the fire resistance and the structural behavior of concrete structures under fire scenarios. The proposed method is potential to develop for other cases of fire exposure.

REFERENCES

[1] O. M.A. Youssef, M. Moftah, "General stress–strain relationship for concrete at elevated temperatures", Eng. Struct., 29(10), 2007, 2618–2634.
[2] AS 3600, Concrete structures. Australia: Committee BD-002, 2001.
[3] BS EN 1991-1-2, Actions on structures: Part 1-2 General actions—structures exposed to fire. Brussels (Belgium): European Committee for Standardization, 2002.
[4] ACI 216.1-07, Standard method for determining fire resistance of concrete and masonry construction assemblies. Detroit: American Concrete Institute; 2007.
[5] ASTM E 119, Standard methods of fire test of building construction and materials, Test Method E119a -08. American Society for Testing and Materials, West Conshohocken, PA, 2008.
[6] ISO 834, Fire-resistance tests—elements of building construction—Part 1: General requirements. International Standard, Geneva, 1999.
[7] S. Bratina, M. Saje, I. Planinc, "The effects of different strain contributions on the response of RC beams in fire", Eng. Struct., 29(3), 2007, 418–430.
[8] A. Law, J. Stern-Gottfried, M. Gillie, G. Rein, "The influence of travelling fires on a concrete frame", Eng. Struct., 33, 2011, 1635–1642.
[9] T.T. Lie, Structural fire protection. ASCE Manuals and Reports on Engineering Practice, No. 78, New York, NY, USA, 1992.
[10] V.R. Kodur, T.C. Wang, F.P. Cheng, "Predicting the fire resistance behaviour of high strength concrete columns", Cem. Concr. Compos., 26, 2004, 141–153.
[11] V.K.R. Kodur, M. Dwaikat, "A numerical model for predicting the fire resistance of reinforced concrete beams", Cem. Concr. Compos., 30, 2008, 431–443.
[12] S.F. El-Fitiany, M.A. Youssef, "Assessing the flexural and axial behaviour of reinforced concrete members at elevated temperatures using sectional analysis", Fire Saf. J., 44, 2009, 691–703.
[13] K. V. Wong, Intermediate Heat Transfer. New York: Marcel Dekker, INC., 2003, ch. 5.
[14] ANSYS, ANSYS multiphysics. Version 11.0 SP1. ANSYS Inc., Canonsburg (PA), 2007.
[15] BS EN 1992-1-2, Design of concrete structures. General rules. Structural fire design. Brussels (Belgium): European Committee for Standardization, 2004.
[16] ASTM E 1529, Standard Test Methods for Determining Effects of Large Hydrocarbon Pool Fires on Structural Members and Assemblies. ASTM Intl., West Conshohocken, PA., 2000.
[17] C.G. Bailey, E. Ellobody, "Fire tests on bonded post-tensioned concrete slabs", Eng. Struct., 31, 2009, 686-696.