

Encrypter Information Software Using Chaotic Generators

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Abstract—This document show a software that show different chaotic generator, as continuous as discrete time. The software give the option for obtain the different signals, using different parameters and initial condition value. The program show then critical parameter for each model. All theses models are capable of encrypter information, this software show it too.

Keywords—cryptography, chaotic attractors, software.

I. INTRODUCTION

THE chaos theory describes the behavior of certain dynamical systems that may exhibit dynamics that are highly sensitive to initial conditions, popularly referred to as the butterfly effect. As a result of this sensitivity, the behavior of chaotic systems appears to be random. The future dynamics of these systems are completely defined by their initial conditions. This behavior is known as deterministic chaos, or simply chaos.

Chaotic behavior is also observed in natural systems, such as the weather. This may be explained by a chaos-theoretical analysis of a mathematical model of such a system, embodying the laws of physics that are relevant for the natural system.

The chaotic behavior occurs in many areas of practical engineering, i.e., in communications, the information transmission plays a crucial role, where an ever-growing

capacity for communication services is required. Two of the major requirements in communication systems are privacy and security.

The chaotic systems [1]-[9] have been greatly motivated by the possibility of encoding information by using a chaotic carrier.

Hence, we are interesting in software that shows the behavior of different chaotic signal. In this paper we present software for different chaotic attractor, for continuous-time and discrete-time systems.

II. CHAOTIC SIGNAL GENERATION

A. Continuous

Different models exist for chaotic dynamics in continuous time. They use differential equations that exhibits chaotic dynamics associated with the fractal properties of the attractor.

1) Lorenz

Lorenz wrote a remarkable article in 1963, he described a three parameter of the nonlinear first-order ordinary differential equation that, when integrated numerically on a computer, appeared to have extremely complicated solutions.

This set of ordinary differential equations that would model some of the unpredictable behavior that we normally associate with the weather [10]. They are

$$\begin{aligned}\dot{x}_1(t) &= \sigma(x_2(t) - x_1(t)), \\ \dot{x}_2(t) &= rx_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) &= x_1(t)x_2(t) - bx_3(t)\end{aligned}\quad (1)$$

Where $\sigma=10$, $b=8/3$, and $r=28$.

- $0 < r < 1$. There is only stable equilibrium point at the origin.
- $1 < r < 1.346$. Two new stable nodes are born and the origin becomes a saddle with a one-dimensional, unstable manifold.
- $1.346 < r < 13.926$. At the lower value the stable nodes become stable spirals.
- $13.926 < r < 24.74$. Unstable limit cycles are born near each of the spiral nodes, and the basins of attraction of each of the two fixed points become intertwined. The steady-steady notion is sensitive to initial conditions.
- $24.74 < r$. All three fixed points becomes unstable. Chaotic motions result.

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2) Chua

The second model is Chua's circuit is a simple electronic circuit that exhibits classic chaos theory behavior. It was introduced in 1983 by Leon O. Chua, who was a visitor at Waseda University in Japan at that time [11]. The ease of construction of the circuit has made it a ubiquitous real-world example of a chaotic system, leading some to declare it "a paradigm for chaos [12].

$$\begin{aligned}\dot{x}_1 &= \alpha[x_2 - x_1 - h(x_1)] \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -\beta x_2,\end{aligned}\quad (2)$$

Where $G=2/R$ and a three-segment piecewise linear v_{C1-i} characteristics of nonlinear elements is defined by

$$h(x_1) = bx_1 + \frac{1}{2}(a-b)(|x_1+1| - |x_1-1|).$$

For $\alpha=10$, $\beta=14.87$, $a=-1.27$, and $b=0$. Chua's circuit operates on the chaotic double scroll attractors. Chua's circuits exhibits a chaotic attractor.

3) Rossler

The Rössler system is a system of three equations. These equations are a nonlinear ordinary differential equation. Some properties of the Rössler system can be deduced via linear methods such as eigen-vector. The original Rössler paper says the Rössler attractor was intended to behave similarly to the Lorenz attractor, but also be easier to analyze qualitatively. This attractor has some similarities to the Lorenz attractor, but is simpler and has only one manifold. Otto Rössler designed the Rössler attractor in 1976, but the originally theoretical equations were later found to be useful in modeling equilibrium in chemical reactions. The defining equations are [13]:

$$\begin{aligned}\dot{x}_1(t) &= -x_2(t) - x_3(t), \\ \dot{x}_2(t) &= x_1(t) - ax_2(t), \\ \dot{x}_3(t) &= b + x_3(t)(x_1(t) - c)\end{aligned}\quad (3)$$

Rössler studied the chaotic attractor with $a = 0.2$, $b = 0.2$, and $c = 5.7$.

B. Discrete

1) Hénon map

The Hénon map is a discrete-time dynamical system. It is one of the most studied examples of dynamical systems that exhibit chaotic behavior. The Hénon map takes a point (x_1, x_2) in the plane and maps it to a new point [14]

$$\begin{aligned}x_1(k+1) &= c - ax_2(k) + x_2(k), \\ x_2(k+1) &= bx_1(k)\end{aligned}\quad (4)$$

The map depends on two parameters, a , b , and c , which for the canonical Hénon map have values of $a = 1.4$, $b = 0.3$, and $c = 1$. For the canonical values the Hénon map is chaotic. For other values of a and b the map may be chaotic, intermittent, or converge to a periodic orbit. An overview of

the type of behavior of the map at different parameter values may be obtained from its orbit diagram.

2) Logistic map

The logistic map is a polynomial mapping of degree 2, often cited as an archetypal example of how complex, chaotic behavior can arise from very simple nonlinear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May, in part as a discrete-time demographic model analogous to the logistic equation first created by Pierre François Verhulst [5], the logistic map is written [16]

$$x(k+1) = \alpha x(k) + \beta x^2(k), \quad (5)$$

where:

$\alpha x(k)$ represent a growth or birth effect, whereas $\beta x^2(k)$ accounts for the limits to growth such as availability of energy or food. If $\beta=0$, we obtain the equation $x(k+1) = \alpha x(k)$ with explicit solution

$$x(k) = x(0)\alpha^k, \quad (6)$$

This nonlinear difference equation is intended to capture two effects.

- Reproduction where the population will increase at a rate proportional to the current population when the population size is small.
- starvation (density-dependent mortality) where the growth rate will decrease at a rate proportional to the value obtained by taking the theoretical "carrying capacity" of the environment less the current

However, as a demographic model the logistic map has the pathological problem that some initial conditions and parameter values lead to negative population sizes.

III. SIMULATION

A. Chaotic generator

Firstly, we develop software for resolve and graphic different chaotic generator in the previous section, this program is made in Matlab. The principal screen present two options, continuous and discrete models (see Fig. 1). In the fig.2 show of the three different model for continuous and the Fig.3 two discrete model. In this page it is necessary select the option selected.

The Figure 4 show the Lorenz Model, in this screen show the initial and final time, initial conditions, parameters, and the equation of this model. In this screen is possible to change the values, so, is possible to see the behavior of the x_1 , x_2 , and x_3 .

When finish the calculate, we can graph the variable with time or the phase between two or three variable. The Figure 5 shows a example.

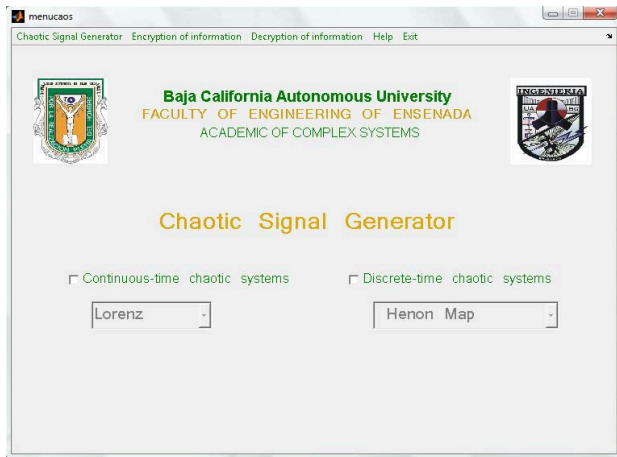


Fig. 1. Principal page.

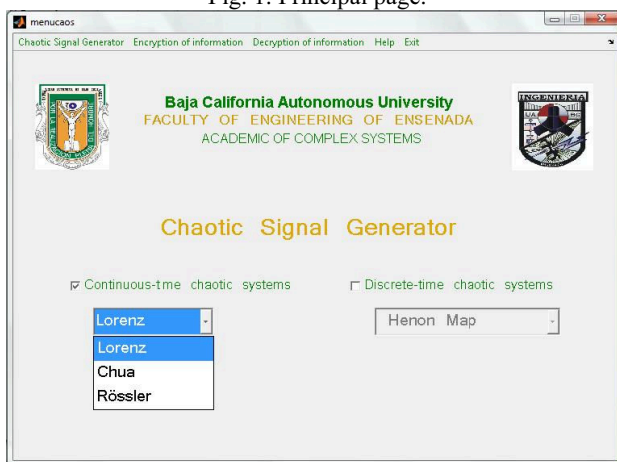


Fig. 2. Continuous chaotic system screen

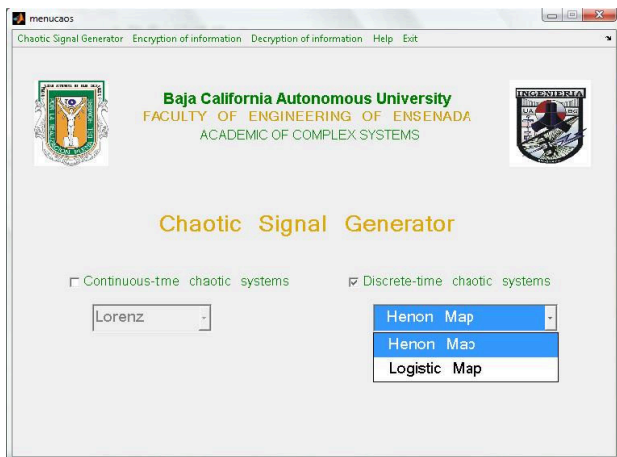


Fig. 3. Discrete chaotic system screen.

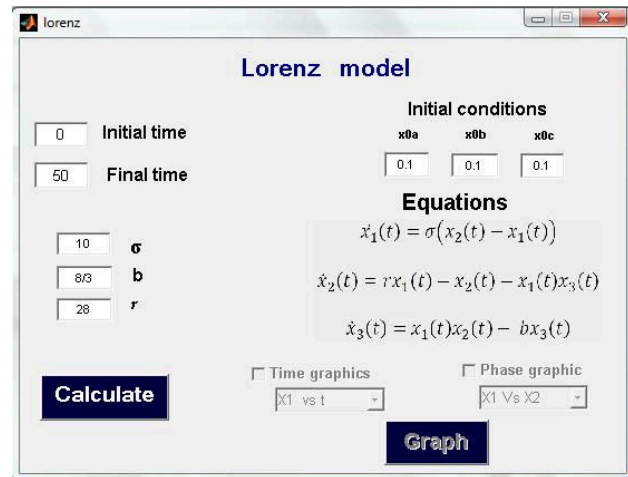


Fig. 4. Screen for model Lorenz.

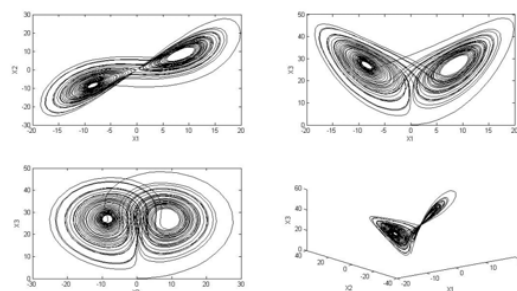


Fig. 5. Lorenz Attractors.

Like the Lorenz model, it is possible call the other five models. In all case we can the change the time, initial condition, parameters, and for each model show the own equations. For illustration only show the Lorenz model.

B. Chaotic generator Application: Secret Communication software

The models presented in previous section have applications to the secret communication. The principal page show the option for encrypts the information for continuous models (Fig. 6).

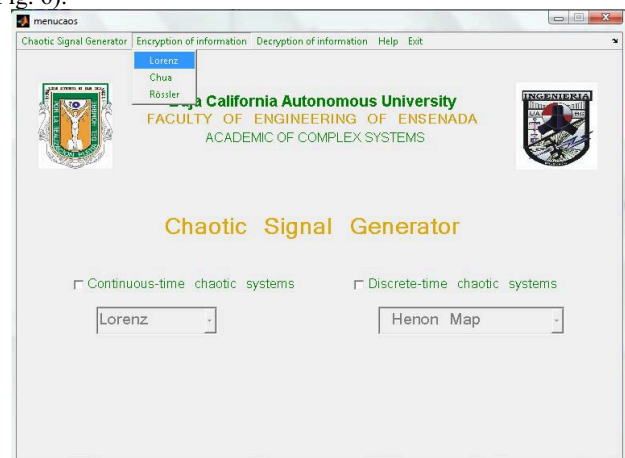


Fig. 6. Encryption information.

For example, we take the Lorenz model for show the ...

Then, appear the screen show in the Fig. 7. In this screen we have the option the parameters, time, and message. This message can be sinusoidal or rectangular signal.

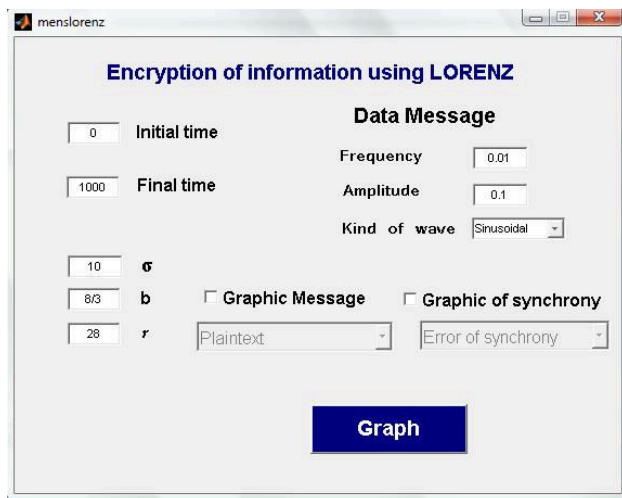
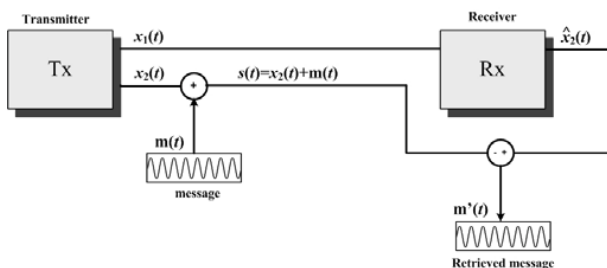


Fig. 7. Encrypt information for Lorenz model.

The schematic for encryption message is shown in the Fig. 8. We use one channel for synchronize transmitter and receiver. A second channel is used to add the message, for illustration we use a sinusoidal message. Finally, we subtract $\hat{x}_2(t) - s(t) = m'(t)$, that is, the recovery message



Finally we can select the desired graph. These can be the original and retrieved message, transmitted signal, and the error between original and retrieved message, Fig. 9. The other option is graph the synchrony error (Fig.10).

Finally, the Figure 11 shows the original message, transmitted signal, retrieval message, and error between original and retrieval message.

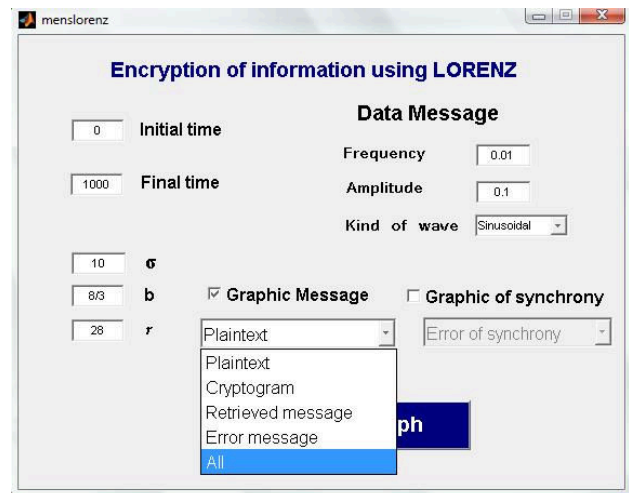


Fig.9. Screen for select message graph.

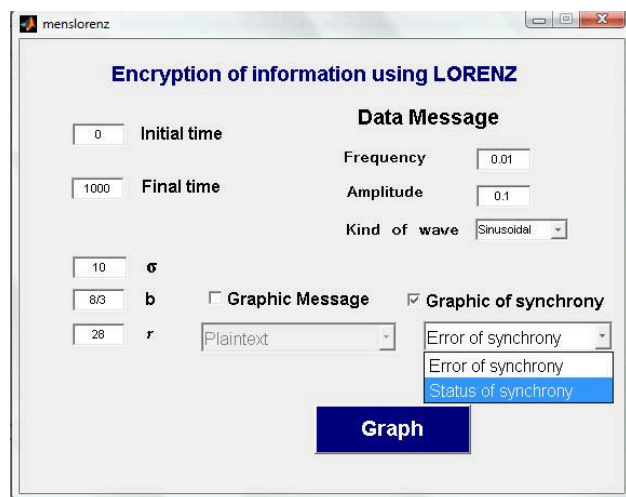


Fig.10. Screen for select synchrony graph.

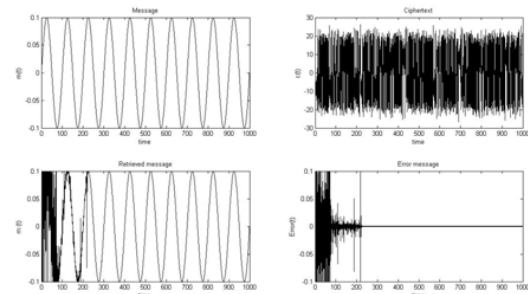


Fig. 11. Message encrypted.

IV. CONCLUSION

This software is a tool very useful for to show different chaotic generator. In this software is possible appreciate the attractor behavior when are changed the different parameters. The critical parameters are a option in the software, this parameter show value range for chaotic behavior. For encrypted message is possible select binary o digital

information. In a future work, we added the option for select the from file or write the message

APPENDIX

Appendixes, if needed, appear before the acknowledgment.

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