Empirical Analytical Modelling of Average Bond Stress and Anchorage of Tensile Bars in Reinforced Concrete

Maruful H. Mazumder, Raymond I. Gilbert

Abstract—The design specifications for calculating development and lapped splice lengths of reinforcement in concrete are derived from a conventional empirical modelling approach that correlates experimental test data using a single mathematical equation. This paper describes part of a recently completed experimental research program to assess the effects of different structural parameters on the development length requirements of modern high strength steel reinforcing bars, including the case of lapped splices in large-scale reinforced concrete members. The normalized average bond stresses for the different variations of anchorage lengths are assessed according to the general form of a typical empirical analytical model of bond and anchorage. Improved analytical modelling equations are developed in the paper that better correlate the normalized bond strength parameters with the structural parameters of an empirical model of bond and anchorage.

Keywords—Bond stress, Development length, Lapped splice length, Reinforced concrete.

I. Introduction

THE design of the anchorage of tensile reinforcing bars is of immense importance to meet the fundamental requirements of both strength and ductility (robustness) of a reinforced concrete (RC) member. Codes of practices (such as AS3600 [1], ACI318 [2], Eurocode 2 [3], DIN 1045-1 [4] and others) specify a minimum development length, $l_{sy.t}$, for the tensile reinforcement on both sides of a critical section so that the tensile bars at the critical section will not only develop the yield stress (f_{sy}) but also will sustain it with increasing deformation. The fundamental design equation for the development length is based on the incorrect assumption that the development of bond stress along the length of an anchored bar is uniform regardless of the variations of anchorage lengths. Larger anchorage length requirements are generally specified by the codes of practices accounting for different factors that may develop non-uniform distribution of bond stress along an anchored bar. According to the current Australian codes of practice (AS3600-2009 [1]) the basic development length, $l_{sy.t}$, specified to meet the requirement of ductility of a reinforcing bar is at least 29 bar diameters.

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The design specifications of the anchorage lengths of tensile reinforcing bars have been based on empirical analytical models to calculate the basic development length using conservative estimates of the average bond stress along the length of an anchored bar. The nature and distribution of bond stress within shorter lengths of an anchored bar is expected to be more uniform due to fewer cracks crossing the length of an anchored bar. Therefore, the estimates of average bond stress by a general empirical analytical model may be overly conservative for shorter anchorage lengths of a bar. This paper describes part of a recently completed experimental research program in the Centre for Infrastructure Engineering and Safety (CIES) at the University of New South Wales, Sydney, Australia. The research was aimed at assessing the effects of different factors on the anchorage requirements of modern high strength steel reinforcing bars, including the cases of end development and lapped splices of bars in slabs. The average ultimate bond stresses measured experimentally on RC slab specimens containing reinforcing bars with different anchorage lengths are presented. The specimens were tested to determine the effects of various factors on the average ultimate bond stress, including the actual length of the anchorage of the tensile bars in the specimens. Based on regression analyses of the test results, this paper outlines improved equations that give reliable estimates of the average bond stresses for different anchorage lengths for two different bas sizes.

II. General Overview of a Typical Empirical Analytical Model of Bond and Anchorage Length

One of the very first attempts to develop empirical mathematical expressions of average bond stress and anchorage lengths of reinforcement was reported by Orangun et al. [5], who analysed test results of lapped splices of reinforcement in 62 RC beams that were previously tested by Chinn, Ferguson and Thompson [6]; Ferguson and Breen [7]; Chamberlin [8]; and Ferguson and Krishnaswamy [9]. The test results were grouped into three different ranges of the c_d/d_b ratio, where c_d is the smaller of the available concrete cover of a bar to the nearest concrete surface and d_b is the bar diameter. The three different ranges of the c_d/d_b ratio were as follows:
i) $0.95 < c_d/d_b < 1.25$, average 1.1
ii) $1.3 < c_d/d_b < 1.75$, average 1.5
iii) $1.9 < c_d/d_b < 2.3$, average 2.0

With the aim of retaining a simple equation for conversion to a design provision, the following equation was chosen by Orangun et al. [5] for the analyses of the test results.

$$f_{ub}/\sqrt{f'_c} = b_1 + b_2\, c_d/d_b + b_3\, d_b/l_s \qquad (1)$$

where l_s is the lapped splice length of the reinforcing bar, f_{ub} is the average (ultimate) bond stress within l_s, f'_c is the characteristic compressive strength of concrete and $\sqrt{f'_c}$ is a characteristic parameter for the tensile strength of the concrete. The constants b_1, b_2 and b_3 were determined from a non-linear regression analysis of the test results of the 62 beams. Only specimens in which the steel did not yield were included in the analyses. Test results within the three groups of c_d/d_b ratios were analysed, and subsequently simplified and converted into a single mathematical expression.

Fig. 1 (a) shows the test results for c_d/d_b in the range between 1.3 and 1.75. According to the best fit curve for all the bar sizes, the $f_{ub}/\sqrt{f'_c}$ appears to become almost constant for the l_s/d_b greater than about 35. However, in the range of lapped splice lengths below $25d_b$, there exist significant differences in the average measured ultimate bond stress in specimens with different lapped splice lengths. This phenomenon is not accounted for in the anchorage provisions in codes of practice, where the estimated average ultimate bond stress for an anchored bar is the same regardless of its anchorage length.

There also appears to be significant differences between the values of f_{ub} predicted by the best fit curve and the actual test results for different bar sizes, particularly for the shorter lapped splice lengths. The deviation of the actual test results from that represented by the best fit curve is more significant for bar N10 and N20 (Fig. 1 (b)). These deviations further increased when the best fit curves of the test results for the three different ranges of c_d/d_b were unified into a single empirical expression. This indicates that a designer may not get a reliable estimate of bond strength for the shorter lapped splice lengths using the specified best fit curve or the unified single expression of (1).

Orangun et al. [5] proposed the same empirical expression for calculating the development and lapped splice lengths of reinforcement as they observed similar behaviour in terms of cracking and splitting in the tests for development and lapped splice lengths.

With few modifications, the empirical analytical model proposed by Orangun et al. [5] served as the basis for the design recommendations of ACI Committee 408, first published in 1979 [10] and subsequently adopted in the later ACI code and specifications (e.g. ACI 318-05 [11]). The fundamental assumption that is generally reflected in all the design codes is that the reinforcement will yield at the critical section and considering this, the calculated basic development length is generally greater than $30d_b$. At or beyond this anchorage length, the average ultimate bond stress is almost constant according to the empirical analytical model that has been discussed.

Hence, the estimated average ultimate bond stress specified by the code may be overly conservative for shorter anchorage lengths of bars. Gilbert et al. [12], [13] and Mazumder et al. [14]-[16] recently reported test results of different development and lapped splice length specimens where the anchorage length variations were 10, 15 and $20d_b$ for two different bar sizes (d_b= 12, 16mm). It has been reported that there exist significant variation of the average bond stress for different anchorage lengths and different bar diameters. This paper discusses and analyses some of the selected test results reported in [12]-[16] according to the empirical analytical modelling approach proposed by Orangun et al. [5]. Further analyses have been undertaken to develop representative expressions of the average ultimate bond stress and the corresponding anchorage length for the two different bar diameters that were considered in the experiments.
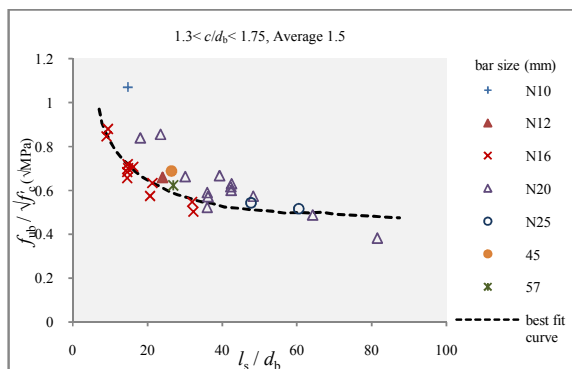
(a) Test results of different bar sizes (for $1.3 < c/d_b < 1.75$)
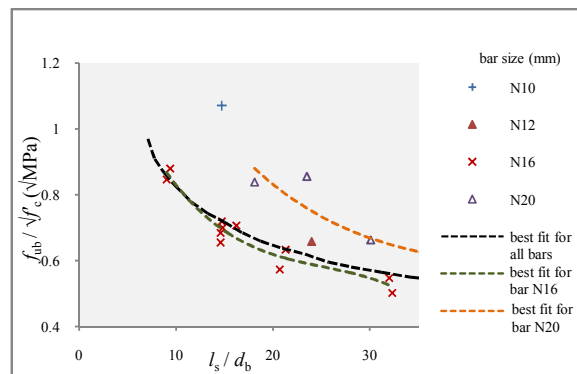
(b) Best fit of test results for individual bar sizes ($l_s/d_b \le 35$)

Fig. 1 Variation of $f_{ub}/\sqrt{f'_c}$ with l_s/d_b at an average c_d/d_b of 1.5 [5]

III. THE EXPERIMENTAL PROGRAM

A. Development Length Specimens and Loading Regime

The development length specimens were 2000mm long, 600 mm wide and 200mm deep. With supports 1200mm apart, these statically determinate members were cantilevered at one end by an amount of 700mm, as shown in Fig. 2 (a). The line load P was 600mm past Support 1. Each specimen contains four reinforcement bars in the top of the specimen, the outer

two bars being terminated with a 180° cog immediately past Support 1. For the two centrally placed bars that carry bending in the cantilever, the bond between the concrete and the bars was eliminated from the point of length, l_d past Support 1 through to the far end of the specimen by encasing each bar in a plastic sleeve. The bar within the plastic sleeve continues along the specimen, protruding from the right hand end. For convenience during testing, the specimen was inverted so that the anchored bars were located in the bottom of the specimen (see Fig. 2 (c)). A total of fourteen development length specimens were tested under short-term static loading. The static loading involved monotonically increasing the applied load on the specimen by controlling the rate of deformation at a suitably slow rate until failure occurs in the specimen, either by bond failure or yielding of the reinforcement.

(a) Elevation of specimen

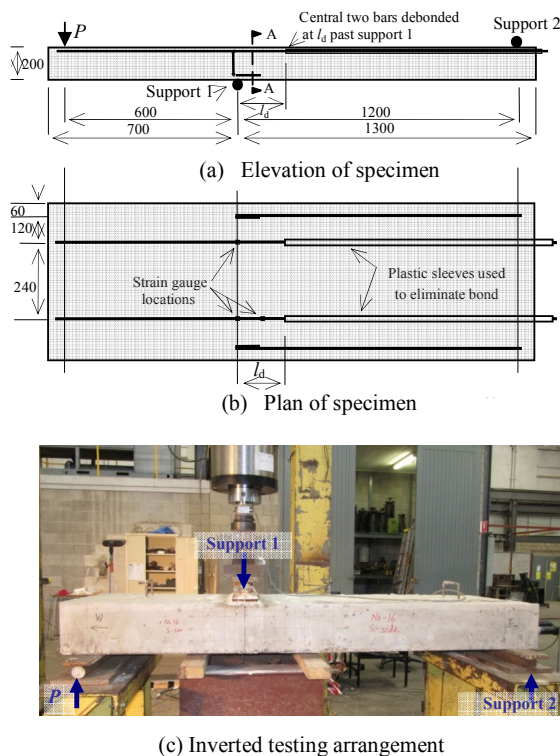(b) Plan of specimen

(c) Inverted testing arrangement

Fig. 2 Dimensions and loading arrangements of development length specimens

The deflection of the specimen at the point of load application, together with the slip at the end of the debonded bars, was measured throughout the test using LVDTs. Also measured throughout the test were the location of the primary cracks and crack widths. The variables considered were the bar diameter d_b = 12 or 16mm; the development length l_d = $5d_b$, $10d_b$, $15d_b$ and $20d_b$; and the bottom concrete cover c= 25 or 40mm. Strain gauges were used to monitor steel strains in the developing bar at the critical cross-section and mid-way along the length l_d, as shown in Fig. 2 (b). When interpreting the results of the static load tests, cracked section analysis can be readily undertaken to determine the stresses that develop in

the developing bars at the critical cross-section (Support 1) at all levels of applied loading after first cracking up to and including bond failure.

B. Lapped Splice Slab Specimens and Loading Regime

The lapped splice slab specimens were simply-supported members, 2000mm long, 850mm wide and 150mm deep, subjected to four point bending. Each specimen contained four 12mm diameter tensile reinforcing bars that were lap spliced in the middle third region where the moment was essentially constant. Details of the specimen with the lapped splices are shown in Fig. 3. The spliced bars were either in direct contact and lightly tied together with tie wire, or were separated by 28 mm to form a non-contact splice, as shown in Fig. 3 (b). The specimens were loaded slowly to failure in a deformation controlled testing frame, with failure initiated in all specimens by splitting cracks and bond failure at the lapped splice. Mid-span deflection together with the location and width of the primary cracks were measured throughout the test. The variables considered were the lapped splice length l_s= $15d_b$ and $20d_b$, and the clear spacing between the bars being spliced together, s_L= 0 and 28mm.

(a) Elevation of the specimen
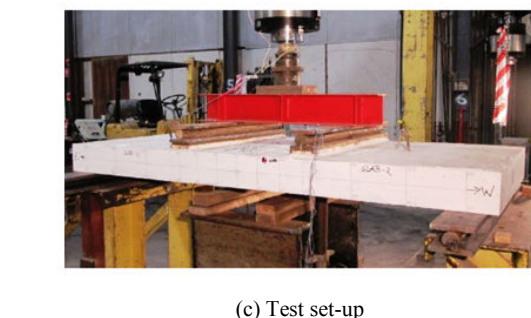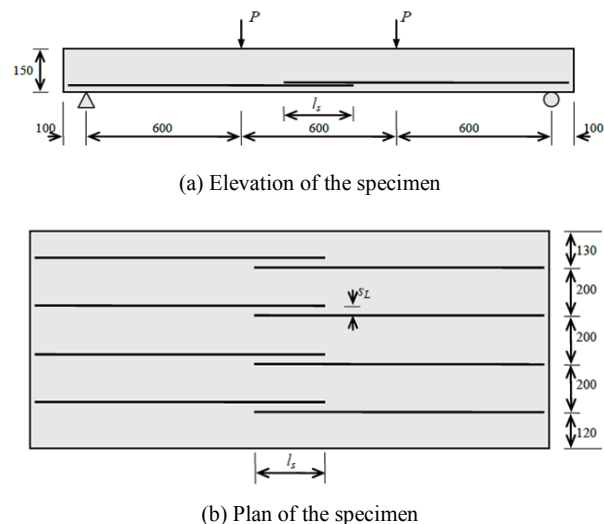

(b) Plan of the specimen

(c) Test set-up

Fig. 3 Dimensions and loading arrangements of lapped splice slab specimens

C. Test Results of Development Length Specimens

Test results of 11 selected development length specimens are shown in Table I. The maximum load P_{max} applied to the specimens during the tests is given in the table, together with the calculated (by cracked section analysis) maximum stress, σ_{st} in the monitored bars at the critical section and f_{ub} mobilized over the development length. The f_{ub} normalized to the characteristic parameter, $\sqrt{f'_c}$ for each specimen is also shown in Table I. Table II shows the material properties measured at the time of testing.

TABLE I
TEST RESULTS FOR DEVELOPMENT LENGTH SPECIMENS

Specimen no.	c_d/d_b	l_d (mm)	P_{max} (kN)	At section at Support 1 under P_{max}			AS3600-2009	
				σ_{st} (MPa)	f_{ub} (MPa)	$f_{ub}/\sqrt{f'_c}$	f_{ub} (MPa)	Factor of safety
DL-1	1.56	160	30.5	308	7.69	1.24	3.93	1.96
DL-2	1.56	240	40.5	403	6.72	1.08	3.93	1.71
DL-3	1.56	320	48.3	478	5.97	0.96	3.93	1.52
DL-6	2.08	120	27.2	477	11.92	1.92	4.45	2.68
DL-7	2.08	180	32.5	565	9.41	1.52	4.45	2.12
DL-8	2.08	240	30.8	537	6.72	1.08	4.45	2.51
DL-10	2.50	160	25.9	292	7.31	1.20	4.55	1.61
DL-11	2.50	240	34.8	387	6.44	1.06	4.55	1.42
DL-12	2.50	320	43.8	482	6.02	0.99	4.55	1.32
DL-16	3.33	120	21.4	422	10.54	1.76	5.21	2.03
DL-17	3.33	180	26.3	510	8.50	1.41	5.21	1.63

TABLE II
MATERIAL PROPERTIES OF DEVELOPMENT LENGTH SPECIMENS

Specimen no.	f'_c (MPa)	$f_{ct,f}$ (MPa)	E_c (MPa)	f_{sy} (MPa)	f_{su} (MPa)
DL-1 to 3	38.5	3.75	34700	546	731
DL-6 to 8	38.5	3.75	34700	561	721
DL-10 to 12	36.9	3.60	29300	546	731
DL- 16 to 17	36.9	3.60	29300	561	721

Test results from the selected development length specimens shown in Table I indicate that the increase of development length generally results a reduction of the average bond stress (f_{ub}) as shown in Fig. 4.
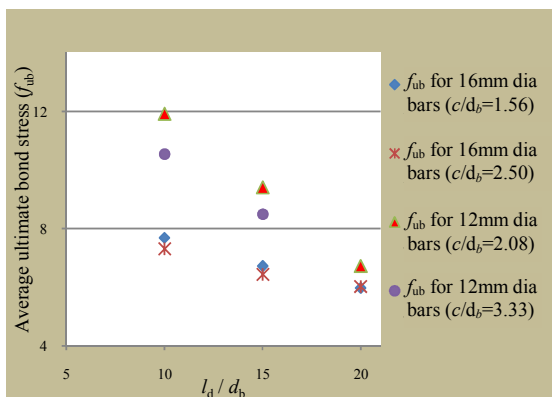

Fig. 4 f_{ub} vs. l_d for different development length specimens

However, the values of f_{ub} calculated according to the design model specified in AS3600-2009 are the same for the different development lengths (as shown in Table I). This, together with the higher factors of safety (taken as the ratio of the measured f_{ub} to that specified in AS3600-2009) for the shorter anchorage lengths, indicates the conservative estimates of the average bond stresses using the code specifications, particularly for the shorter development length specimens. It can also be seen in Fig. 4 that the effect on f_{ub} of the dimensionless parameter c_d/d_b becomes less significant with an increase of the development length and with an increase in the bar diameter.

B. Test Results of Lapped Splice Slab Specimens

Test results of 4 selected lapped splice slab specimens (SL-1, 2, 3 and 6) are shown in Table III. The material properties at the time of testing are shown in Table IV. Test results of the lapped splice slab specimens also indicate that the average ultimate bond stress is reduced with an increase of the lapped splice length. The difference in the measured value of f_{ub} for the contact and non-contact splices (SL-2 and 6) of the same length is insignificant.

TABLE III
TEST RESULTS OF LAPPED SPLICE SLAB SPECIMENS

Specimen no. and (s_L-mm)	c_d/d_b	l_d (mm)	P_{max} (kN)	At critical section under P_{max}			AS3600-2009	
				σ_{st} (MPa)	f_{ub} (MPa)	$f_{ub}/\sqrt{f'_c}$	f_{ub} (MPa)	Factor of safety
SL-1 (0)	2.08	120	27.0	27.0	8.71	1.41	3.53	2.47
SL-2 (0)	2.08	180	38.6	488	8.13	1.32	3.53	2.30
SL-3 (0)	2.08	240	44.9	563	7.04	1.14	3.53	1.99
SL-6 (28)	2.08	180	37.5	475	7.91	1.28	4.53	2.24

TABLE IV
MATERIAL PROPERTIES OF LAPPED SPLICE SLAB SPECIMENS

Specimen no.	f'_c (MPa)	$f_{ct,f}$ (MPa)	E_c (MPa)	f_{sy} (MPa)	f_{su} (MPa)
SL-1 to 3, 6	38.0	3.50	30500	561	731

IV. AN EXTENSION OF THE EMPIRICAL ANALYTICAL MODELLING OF BOND AND ANCHORAGE LENGTHS

Experimental test results of the development and lapped splice specimens generally indicate that the variations of f_{ub} within an anchored bar due to variation of the bar diameters are significant for shorter anchorage lengths ($l_d/l_s \leq 15d_b$). Regardless of the variation of the c_d/d_b or the bar diameter (d_b), the difference in f_{ub} decreases as l_d or l_s increases. Results indicative of this fact have also been discussed in the overview of a general empirical modelling approach of bond and anchorage where a single mathematical expression is typically used regardless of the variations of the anchorage lengths. However, the significant differences in f_{ub} for shorter anchorage lengths of different bar sizes cannot be accounted for by a single mathematical expression. Since the difference in f_{ub} decreases as l_d or l_s increases, a unified mathematical expression can be more appropriately adopted for longer anchorage lengths, which for the tested specimens seems to be at or beyond the anchorage lengths of $25d_b$. Therefore,

mathematical expressions are developed here that provide good agreement with the test results for two different bar diameters, correlating well with the normalized bond parameters ($f_{ub}/\sqrt{f'_c}$) and with the parameters c_d/d_b, d_b/l_d or d_b/l_s. The test results shown in Fig. 5 indicate that a linear regression analysis may be sufficient to develop good correlation with the test results. The linear regression analyses have been undertaken using the same form of the equation as that was proposed by Orangun et al. [5].
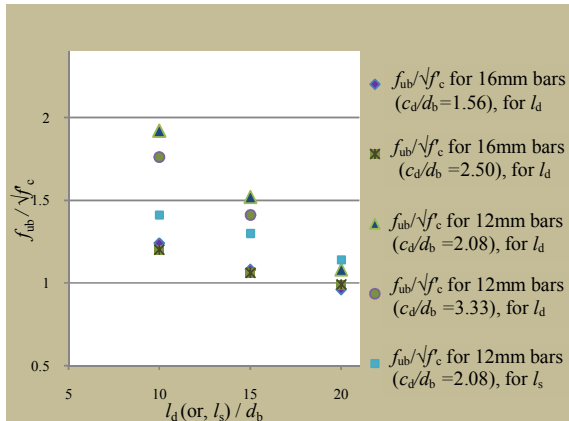

Fig. 5 $f_{ub}/\sqrt{f'_c}$ vs. l_d (or, l_s) for different c_d/d_b

After the linear regression analysis of the test results for development length specimens with 16mm diameter reinforcing bar, the following equation fits the test results with a high correlation coefficient ($R_2 = 0.98$):

$$f_{ub}/\sqrt{f'_c} = 0.759 - 0.011 \, c_d/d_b + 4.852 \, d_b/l_d \qquad (2)$$

The best fit mathematical equation for the development lengths specimens of 12mm diameter bars is given by (3) with $R_2 = 0.94$:

$$f_{ub}/\sqrt{f'_c} = 0.596 - 0.067 \, c_d/d_b + 14.51 \, d_b/l_d \qquad (3)$$

There is insignificant difference between the average bond stresses of the contact and non-contact splices of length $15d_b$. Therefore, an average of the two values of the f_{ub} has been taken as a representative value for this lapped splice length. The best fit equation for the test results of lapped splice slab specimens with 12mm diameter bars is given by (4) with $R_2 = 0.92$:

$$f_{ub}/\sqrt{f'_c} = 0.913 + 5.119 \, d_b/l_s \qquad (4)$$

For the anchorage lengths below $25d_b$, a better estimate of the average bond stress can be derived from (2)-(4). Alternatively, the required development or lapped splice length can be calculated from the equations for a desired average bond stress to be achieved within the anchorage.

V. CONCLUSIONS

The test results reported in this paper are from two stages of a recently completed research program at the University of New South Wales that determined the effects of varying the development lengths and lapped splice lengths of reinforcing bars on the average ultimate bond stress along an anchored bar. Full-scale reinforced concrete slabs were tested under monotonic static loads, and the reinforcing bar diameters within the specimens were either 12mm or 16mm with the anchorage lengths of the tensile bars limited at and below 20 times the bar diameter. The average ultimate bond stress for smaller diameter bars is significantly higher than the value for larger diameter bars and average ultimate bond stress decreases as the anchorage length is increased.

The test results are analysed according to a conventional empirical modelling approach that has been used for many years to develop the typical bond and anchorage length specifications that are contained in the different codes of practice. The significant differences in the average ultimate bond stress for shorter anchorage lengths of two different bar sizes cannot be accounted for satisfactorily by a single mathematical expression as adopted within the form of the conventional empirical model. Since the average ultimate bond stress decreases as the development or lapped splice length is increased, a unified mathematical equation can be more appropriately adopted for longer anchorage lengths, which for the tested specimens seems to be at or beyond the anchorage lengths of 25 times the bar diameter. Therefore, mathematical equations have been developed for shorter anchorage lengths with 12mm and 16mm diameter bars and these better correlate the normalized bond strengths and structural parameters of a typical empirical analytical model.

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