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Elastic Stress Analysis of Annular Bi-Material Discs with Variable Thickness under Mechanical and Thermomechanical Loads

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***Abstract*—**The closed form study deals with elastic stress analysis of annular bi-material discs with variable thickness subjected to the mechanical and thermomechanical loads. Those discs have many applications in the aerospace industry, such as gas turbines and gears. Those discs normally work under thermal and mechanical loads. Their life cycle can increase when stress components are minimized. Each material property is assumed to be isotropic. The results show that material combinations and thickness of profiles play an important role in determining the responses of bi-material discs and an optimal design of those structures. Stress distribution is investigated and results are shown as graphs.

***Keywords*—**Bi-material discs, elastic stress analysis, mechanical loads, rotating discs.

I. INTRODUCTION

STRESS is one of the fundamental concepts of engineering problems in order to give us some idea such as beginning of fracture transition from elastic region to plastic region.

Discs have many engineering applications such as in gas and steam turbine rotors, turbo generators, turbojet engines, internal combustion engines [1], [2].The analysis in rotating discs is an important issue because of their many practical applications in mechanical engineering.

Elastic stress analysis has been a popular field of study [3]-[12].Timoshenko and Goodier studied analytical solution for the stress analysis of a disc under pressure or mechanical loading [13]. Leopold investigated the stress analysis of rotating annular discs subjected to various temperature distributions [14]. Çallıoğlu examined a stress analysis for a rotating hollow disc made of rectilinearly glass-fiber/epoxy prepreg subjected to thermal loading [15]. Stanley and Garroch presented a new form of disc test, for the thermo-elastic analysis of a fiber-reinforced orthotropic circular disc under diametrical compression [16].Variations of radial, tangential and displacement of rotating disc subject to mechanical and thermal load are investigated in [17]. Jahed and Sherkatti analyzed thermo elastic analysis of inhomogeneous rotating disc with variable thickness [18]. Sayman examined stress analysis of a thermoplastic composite disc under uniform temperature distribution analytically and using a finite element method [19].Elastic-plastic stress analysis and changing stress situations with angular speed for rotating discs are investigated in [20]. Çallıoğlu and Karakaya presented stresses of bi-material disc subject to thermal load. According to another study, the disc contains isotropic materials and distribution of temperature is given as a parabolic function which is decreasing from inner surface to outer surface [21]. In a different study, Çallıoğlu et al analyzed variation of radial stress, circumferential stress and radial displacement under thermal and mechanical loads for rotating disc [22].

The purpose of this study is to obtain the solution for elastic stress analysis of annular bi-material disc with variable thickness subjected to thermal and mechanical loads by an analytical method. The disc consists of two different isotropic materials. Variations of radial, tangential and displacement of the disc are investigated.

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II. STRESS ANALYSIS

Governing differential equations of equilibrium for a rotating disc in cylindrical coordinates in 2D are [13]

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial_\theta}+\frac{\sigma_r-\sigma_\theta}{r}+R=0 \tag{1}$$

$$\frac{\partial\tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\sigma_\theta}{\partial_\theta}+\frac{2\tau_{r\theta}}{r}+R=0 \tag{2}$$

The stress component does not depend on θand is a function of r because of the symmetry. Additionally, $\tau_{r\theta}$ is equal to zero. Therefore the governing differential equation of equilibrium for a rotating discwith mass force can be written as

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}+\rho\omega^2 r=0 \tag{3}$$

$$\frac{d}{dr}\left(hr\sigma_r\right)+h\sigma_r-h\sigma_\theta+h\rho\omega^2r^2=0 \tag{4}$$

where σ_r and $\sigma_\theta$are the radial and circumferential stresses respectively, ρ is the density of the material and it is assumed as constant,$\omega$is the angular velocity, h is the thickness of the material.

Due to the rotational symmetry, the strain-displacement

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relation can be written as

$$\varepsilon_r = \frac{du}{dr}, \qquad \varepsilon_\theta = \frac{u}{r} \tag{5}$$

where $\varepsilon_r, \varepsilon_\theta$ and u are the strains in radial and tangential directions and displacement component in the radial direction, respectively.

Using Hooke's law for plane stress case, the stresses are given by

$$\sigma_r = \frac{E}{1-\nu^2}[\varepsilon_r + \nu\varepsilon_\theta - (1+\nu)\alpha T\ ] \tag{6}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}[\varepsilon_\theta + \nu\varepsilon_r - (1+\nu)\alpha T\ ] \tag{7}$$

where E, α, ν and Tare elastic modulus, thermal expansion coefficient, Poisson's ratio and temperature, respectively.

Substituting (5) into (6) and (7) gives

$$\sigma_r = \frac{E}{1-\nu^2}[\frac{du}{dr} + \nu\frac{u}{r} - (1+\ \nu)\alpha T\ ] \tag{8}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}[\frac{u}{r} + \nu\frac{du}{dr} - (1+\nu)\alpha T\ ] \tag{9}$$

Substituting (8) and (9) into (4) and taking derivatives gives

$$\frac{d}{dr}\left(rh\sigma_r\right) + \frac{d}{dr}\left(\sigma_r rh\right) + h\sigma_r - h\sigma_\theta + h\rho\omega^2 r^2 = 0 \tag{10}$$

The disc is assumed as variable thickness, therefore

$$h = h_0\left(r/b\right)^n \tag{11}$$

where,h_0 is the thickness of the disc, b is outer radius and n is arbitrary constant.

Different cases of thickness of the disc are given in Table I.

TABLE I
DIFFERENT CASES OF THICKNESS PROFILES [21]

	n<0	n=0	n>0	n= -1
Thickness profiles	Case(a) Hyperbolic divergent	Case(b) Constant thickness	Case(c) Hyperbolic convergent	Case(d) Linear

Substituting (8), (9) and (11) into (10) and taking derivatives gives

$$r^2\frac{d^2u}{dr^2} + r\left(n+1\right)\frac{du}{dr} + \left(n\nu-1\right) = -\frac{1-\nu^2}{E}\rho\omega^2 r^3 + nr\left(1+\nu\right)\alpha T + \left(1+\nu\right)r^2\alpha\frac{dT}{dr} \tag{12}$$

$$r = e^t$$

$$\frac{dr}{dt} = e^t$$

$$\frac{du}{dr} = \frac{du}{dt}\frac{dt}{dr} = e^{-t}\frac{du}{dt} \tag{13}$$

$$\frac{d^2u}{dt^2} = e^{-2t}\frac{d^2u}{dt^2} - e^{-2t}\frac{du}{dt}$$

Substituting (13) into (12) gives

$$\frac{d^2u}{dt^2} + n\frac{du}{dt} + \left(n\nu-1\right)u = -\frac{1-\nu^2}{E}\rho\omega^2 e^{3t} + n\left(1+\nu\right)e^t\alpha T + \left(1+\nu\right)e^{2t}\alpha\frac{dT}{dr} \tag{14}$$

A. Stress Analysis under Mechanical Loads

For mechanical loads, (14) becomes

$$\frac{d^2u}{dt^2} + n\frac{du}{dt} + \left(n\nu-1\right)u = -\frac{1-\nu^2}{E}\rho\omega^2 e^{3t} \tag{15}$$

Equation (15) is the governing second order differential equation. Once this equation is solved for u, the components of stress can be found and radial displacement, u, can be written as

$$u = C_1 r^{\frac{n+k}{2}} + C_2 r^{\frac{n-k}{2}} + Ar^3 \tag{16}$$

where C_1 and C_2 are the integration constants and the positive constant k is

$$k = \sqrt{n^2 - 4\nu n + 4} \tag{17}$$

and the term A is;

$$A = \frac{-\frac{E}{1-\nu^2}\rho\omega^2}{9 + 3n + n\nu - 1} \tag{18}$$

under mechanical loads (8) and (9) can be written as

$$\sigma_r = \frac{E}{1-\nu^2}[\frac{du}{dr} + \nu\frac{u}{r}] \tag{19}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}[\frac{u}{r} + \nu\frac{du}{dr}] \tag{20}$$

The stress components can be obtained from the radial displacement, u, in (16) as,

$$\sigma_r = \frac{E}{1-\nu^2}[C_1\frac{n+k}{2}r^{\frac{n+k-2}{2}} + C_2\frac{n-k}{2}r^{\frac{n-k-2}{2}} + 3Ar^2 + \nu\left(C_1 r^{\frac{n+k-2}{2}} + C_2 r^{\frac{n-k-2}{2}} + Ar^2\right)] \tag{21}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[C_1 r^{\frac{n+k-2}{2}} + C_2 r^{\frac{n-k-2}{2}} + Ar^2 + \nu\left(C_1\frac{n+k}{2}r^{\frac{n+k-2}{2}} + C_2\frac{n-k}{2}r^{\frac{n-k-2}{2}} + 3Ar^2\right)\right] \tag{22}$$

The integration constants, C_1 and C_2 can be determined from the following boundary conditions. It is assumed that hollow disc is free-free.

$$\sigma_r(r_i) = 0, \qquad \sigma_r(r_o) = 0 \tag{23}$$

By using these conditions, C_1 and C_2 can be obtained as

$$C_1 = \frac{\left(-3Ar_i^2 - \nu Ar_i^2\right)\left(\frac{n-k}{2}r_o^{\frac{n-k-2}{2}} + \nu r_o^{\frac{n-k-2}{2}}\right) - \left(-3Ar_o^2 - \nu Ar_o^2\right)\left(\frac{n-k}{2}r_i^{\frac{n-k-2}{2}} + \nu r_i^{\frac{n-k-2}{2}}\right)}{\left(\frac{n+k}{2}r_i^{\frac{n+k-2}{2}} + \nu r_i^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_o^{\frac{n-k-2}{2}} + \nu r_o^{\frac{n-k-2}{2}}\right) - \left(\frac{n+k}{2}r_o^{\frac{n+k-2}{2}} + \nu r_o^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_i^{\frac{n-k-2}{2}} + \nu r_i^{\frac{n-k-2}{2}}\right)} \tag{24}$$

$$C_2 = \frac{\left(-3Ar_o^2 - \nu Ar_o^2\right)\left(\frac{n+k}{2}r_i^{\frac{n+k-2}{2}} + \nu r_i^{\frac{n+k-2}{2}}\right) - \left(-3Ar_i^2 - \nu Ar_i^2\right)\left(\frac{n+k}{2}r_o^{\frac{n+k-2}{2}} + \nu r_o^{\frac{n+k-2}{2}}\right)}{\left(\frac{n+k}{2}r_i^{\frac{n+k-2}{2}} + \nu r_i^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_o^{\frac{n-k-2}{2}} + \nu r_o^{\frac{n-k-2}{2}}\right) - \left(\frac{n+k}{2}r_o^{\frac{n+k-2}{2}} + \nu r_o^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_i^{\frac{n-k-2}{2}} + \nu r_i^{\frac{n-k-2}{2}}\right)} \tag{25}$$

B. Stress Analysis under Thermomechanical Loads

For thermo-mechanical loads, (14) becomes

$$\frac{d^2u}{dt^2} + n\frac{du}{dt} + (n\nu - 1)u = -\frac{1-\nu^2}{E}\rho\omega^2 e^{3t} + n(1+\nu)e^t\alpha T \tag{26}$$

Equation (26) is the governing second order differential equation. Once this equation is solved for u, the components of stress can be found, radial displacement, u, can be written as

$$u = C_1 r^{\frac{n+k}{2}} + C_2 r^{\frac{n-k}{2}} + Kr^3 + Lr \tag{27}$$

where C_1 and C_2 are the integration constants and the positive constant k is

$$k = \sqrt{n^2 - 4\nu n + 4} \tag{28}$$

and the term K and L are;

$$K = \frac{-\frac{1-\nu^2}{E}\rho\omega^2}{8 + 3n + n\nu}, \qquad L = \frac{n(1+\nu)\alpha T}{n + n\nu} \tag{29}$$

The stress components can be obtained from the radial displacement, u, in (27) as,

$$\sigma_r = \frac{E}{1-\nu^2}\left[C_1\frac{n+k}{2}r^{\frac{n+k-2}{2}} + C_2\frac{n-k}{2}r^{\frac{n-k-2}{2}} + 3Kr^2 + Lr + \nu\left(C_1 r^{\frac{n+k-2}{2}} + C_2 r^{\frac{n-k-2}{2}} + Kr^2 + L\right) - (1+\nu)\alpha T\right] \tag{30}$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[C_1 r^{\frac{n+k-2}{2}} + C_2 r^{\frac{n-k-2}{2}} + Kr^2 + L + \nu\left(C_1\frac{n+k}{2}r^{\frac{n+k-2}{2}} + C_2\frac{n-k}{2}r^{\frac{n-k-2}{2}} + 3Kr^2 + Lr\right) - (1+\nu)\alpha T\right] \tag{31}$$

The integration constants, C_1 and C_2 can be determined from the following boundary conditions. It is assumed that hollow disc is free-free.

$$\sigma_r(r_i) = 0, \qquad \sigma_r(r_o) = 0 \tag{32}$$

By using these conditions, C_1 and C_2 can be obtained as

$$C_1 = \frac{\left(-3Kr_i^2 - Lr_i - \nu Kr_i^2 - \nu L + (1+\nu)\alpha T\right)\left(\frac{n-k}{2}r_o^{\frac{n-k-2}{2}} + \nu r_o^{\frac{n-k-2}{2}}\right) - \left(-3Kr_o^2 - Lr_o - \nu Kr_o^2 - \nu L + (1+\nu)\alpha T\right)\left(\frac{n-k}{2}r_i^{\frac{n-k-2}{2}} + \nu r_i^{\frac{n-k-2}{2}}\right)}{\left(\frac{n+k}{2}r_i^{\frac{n+k-2}{2}} + \nu r_i^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_o^{\frac{n-k-2}{2}} + \nu r_o^{\frac{n-k-2}{2}}\right) - \left(\frac{n+k}{2}r_o^{\frac{n+k-2}{2}} + \nu r_o^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_i^{\frac{n-k-2}{2}} + \nu r_i^{\frac{n-k-2}{2}}\right)} \tag{33}$$

$$C_2 = \frac{\left(-3Kr_o^2 - Lr_o - \nu Kr_o^2 - \nu L + (1+\nu)\alpha T\right)\left(\frac{n+k}{2}r_i^{\frac{n+k-2}{2}} + \nu r_i^{\frac{n+k-2}{2}}\right) - \left(-3Kr_i^2 - Lr_i - \nu Kr_i^2 - \nu L + (1+\nu)\alpha T\right)\left(\frac{n+k}{2}r_o^{\frac{n+k-2}{2}} + \nu r_o^{\frac{n+k-2}{2}}\right)}{\left(\frac{n+k}{2}r_i^{\frac{n+k-2}{2}} + \nu r_i^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_o^{\frac{n-k-2}{2}} + \nu r_o^{\frac{n-k-2}{2}}\right) - \left(\frac{n+k}{2}r_o^{\frac{n+k-2}{2}} + \nu r_o^{\frac{n+k-2}{2}}\right)\left(\frac{n-k}{2}r_i^{\frac{n-k-2}{2}} + \nu r_i^{\frac{n-k-2}{2}}\right)} \tag{34}$$

III. Results and Discussion

In this paper, elastic stress analysis is carried out on annular disc made of bi-material under mechanical and thermo-mechanical loads by using analytic solution. The results are demonstrated under 100°C uniform temperature and inertial force due to rotation having an angular velocity of 75 rad/sn. The inner and outer radii of the discs are r_i=20 mm and r_o=100 mm, respectively.

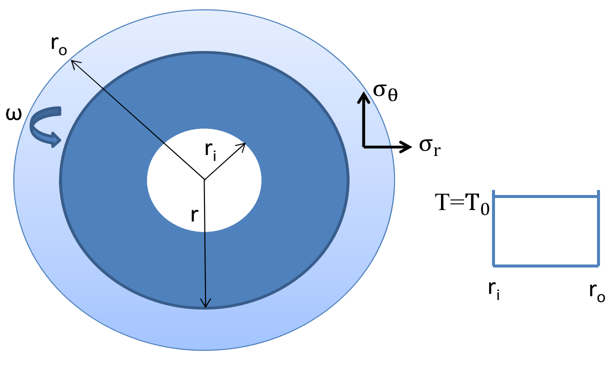

Fig. 1 Bi-material disc under mechanic and thermal load

Mechanical properties of the disc, such as elasticity modulus, thermal expansion coefficient, and Poisson's ratio are given in Table II, respectively.

TABLE II
MATERIAL PROPERTIES OF DISC

Number of Material	E (MPa)	α (1/°C)	ν	ρ (*kg/m³*)
I (20 mm to 80 mm) (Aluminum)	70000	23×10^{-6}	0.3	2700
II (80 mm to 100 mm) (Ceramic)	151000	10×10^{-6}	0.17	5700

A. Results for Mechanical Loading

In Fig. 2-4, the variation of radial stress, circumferential stress and radial displacement of bi-material disc subjected to mechanical loads are presented.

Fig. 2 shows that the highest value of the radial stresses occur between 40mm and 50mm.From those ranges to 80mm, value of the stresses decrease gradually. According to different function of thickness of the disc such as linear, hyperbolic divergent, constant and hyperbolic convergent, the highest radial stress values are 97 MPa, 62 MPa, 40 MPa and 26 MPa, respectively. There is a jump at the start point of the second material (at 80mm).The stresses are tensile stresses.

The radial stresses are zero at point 20mm and 100mm because of the boundary conditions.

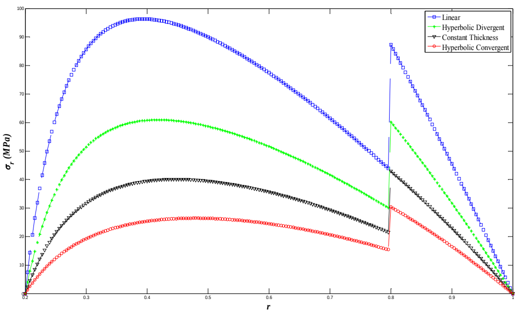

Fig. 2 Variations of the radial stresses along the radial distance of disc subjected to inertial force due to rotating, ɯ=75 rad/sn, for different thickness

The circumferential stress at the beginning of the disc (at 20mm) is at the highest value, as seen in Fig. 3. Subsequently circumferential stresses decrease gradually up to the beginning point (at 80mm) of the second material. There is a sharp increase at start point of the second material. The circumferential stresses are also tensile stresses like radial stresses.

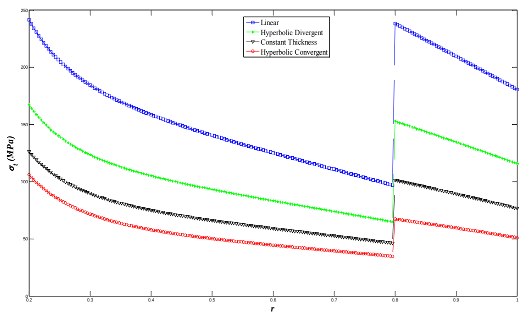

Fig. 3 Variations of the circumferential stresses along the radial distance of disc subjected to inertial force due to rotating, ɯ=75 rad/sn, for different thickness

It is clear from Fig. 4 that, the radial displacement of the disc is the highest value at the end of the disc (at 100mm). Firstly the radial displacement decreases gradually and then increases up to the next material. Except the hyperbolic convergent function, there is a positive jump tendency at the starting point of the next material. When the function is hyperbolic convergent firstly decreases slightly at 80mm and then has stability between 80mm and 100mm.There is a significant change in the displacement at the second material.

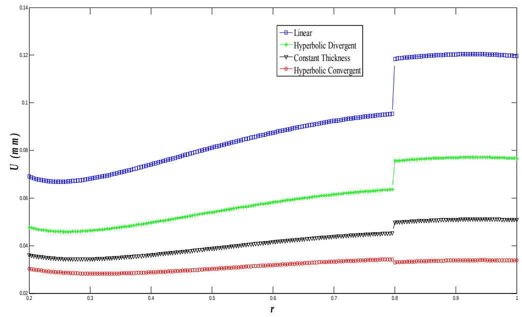

Fig. 4 Variations of the radial displacement along the radial distance of disc subjected to inertial force due to rotating, ɯ=75 rad/sn, for different thickness

B. Results for Thermomechanical Loading

In Figs. 5-7, the variation of radial stress, circumferential stress and radial displacement of bi-material disc subjected to thermo mechanical loads are given. The disc is under 100°Cuniform temperature and angular velocity, ɯ=75 rad/sn.

Fig. 5 shows that the radial stresses between 35mm and 45mmare the highest values. After these ranges, value of the radial stresses decreases up to 80mm. There is a sharp tendency at starting point of the second material (at 80 mm).According to different functions that represent the different thickness values of the disc such as linear, hyperbolic divergent, constant and hyperbolic convergent, the highest radial stress values are 240 MPa, 160 MPa, 75 MPa and 40 MPa, respectively. The stresses are tensile stresses. Because of boundary conditions the radial stresses are zero at points 20mm and 100mm.

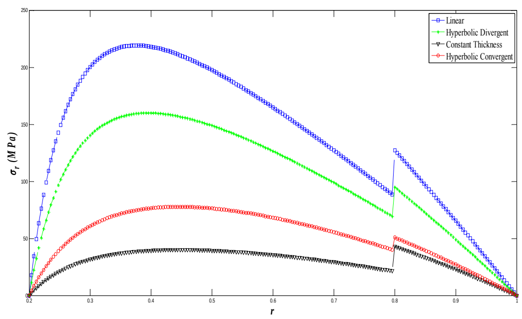

Fig. 5 Variations of the radial stresses along the radial distance of disc subjected to inertial force due to rotating, ɯ=75 rad/sn, and uniformly temperature load, T=100°C, for different thickness

The circumferential stress at the beginning of disc (at 20mm) is the highest value, as seen in Fig. 6. The circumferential stresses decrease gradually from inner surface to the beginning point (at 80mm) of the second material. There is a jump tendency at the starting point of the second material. The stresses are tensile stresses.

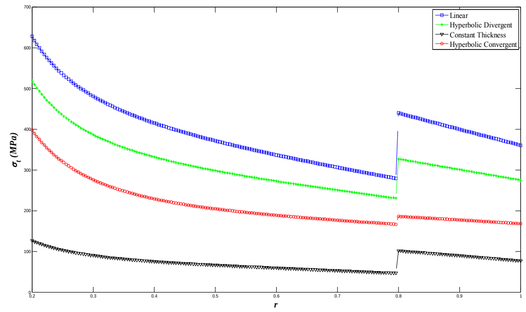

Fig. 6 Variations of the circumferential stresses along the radial distance of disc subjected to inertia force due to rotating, ɯ=75 rad/sn, and uniformly temperature load, T=100◦C, for different thickness

Fig. 7 shows that the radial displacement of the disc at the beginning of the second material (at 80mm) is at the highest value. Firstly, the radial displacement decreases lightly and then increases gradually up to the next material. There is a sharp tendency at the starting point of second material. At the start of the second material, the displacement increases slightly. When the disc has a constant thickness, there is a slightly increase at point 80mm and then has stability between at 80mm to 100mm.

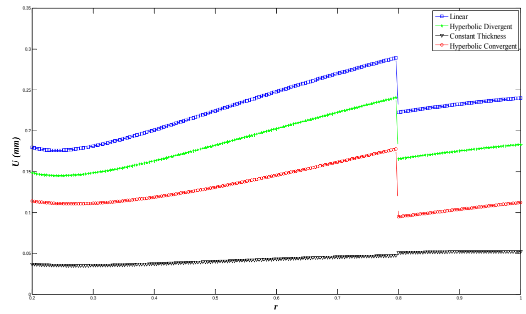

Fig. 7 Variations of the radial displacement along the radial distance of disc subjected to inertial force due to rotating, ɯ=75 rad/sn, and uniformly temperature load, T=100◦C, for different thickness

IV. CONCLUSIONS

In this study, stress analysis of annular bi-material disc subjected to thermal and thermomechanical loads are performed in solutions. The following conclusions can be derived from this study:

- Thickness of the disc and loading type play an important role for the distribution of stresses along the radius of the disc.
- The circumferential stresses take maximum values at the beginning of the inner material and minimum values at the ending of the inner material. Likewise, the circumferential stresses take maximum values at beginning of the second material and minimum values at the ending of the disc.

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